



Semiconductors

## Appendix RF Manual 6<sup>th</sup> edition


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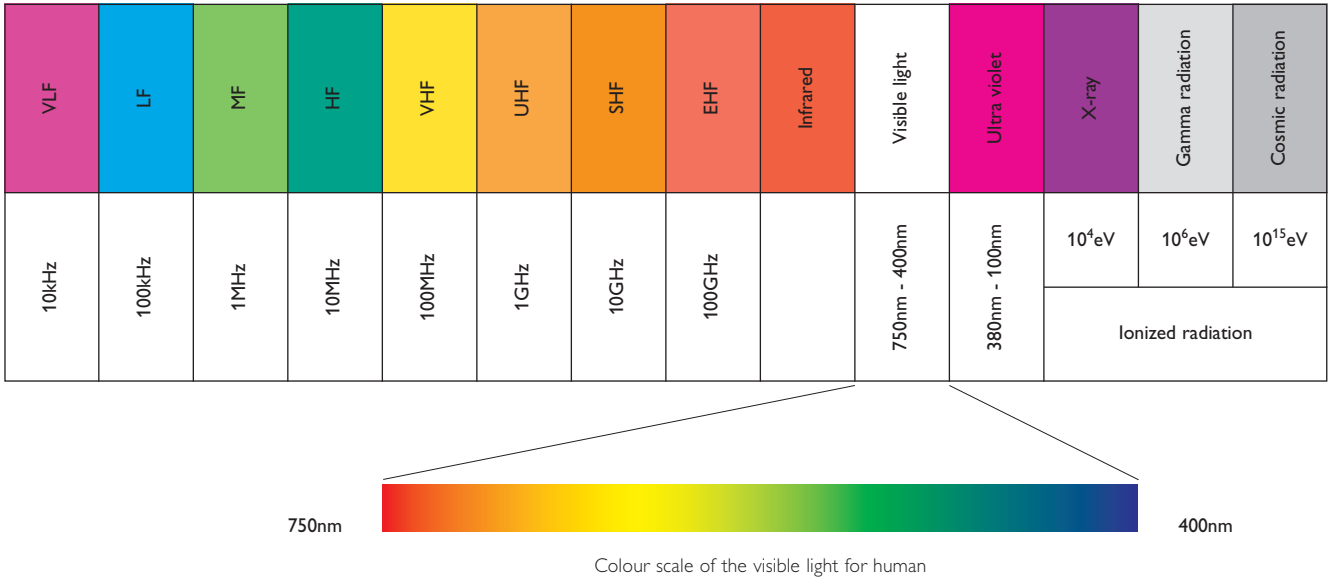
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# 1. RF Application-Basics

## 1.1 Frequency spectrum

### Radio spectrum and wavelengths

Each material's composition creates a unique pattern in the radiation emitted. This can be classified in the "frequency" and "wavelength" of the emitted radiation. As electro-magnetic (EM) signals travel with the speed of light, they do have the character of propagation waves.



A survey of the frequency bands and related wavelengths:

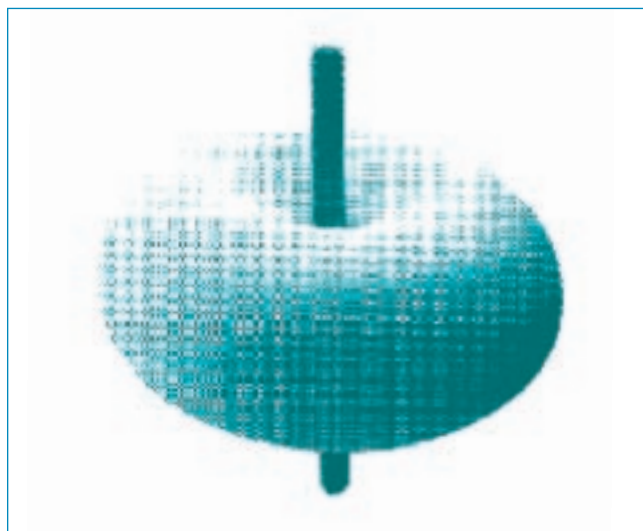
Band	Frequency	Definition	Wavelength - p acc. DIN40015	CCIR Band
VLF	3kHz to 30kHz	Very Low Frequency	100km to 10km	4
LF	30kHz to 300kHz	Low Frequency	10km to 1km	5
MF	300kHz to 1650kHz	Medium Frequency	1km to 100m	6
	1605KHz to 4000KHz	Boundary Wave		
HF	3MHz to 30MHz	High Frequency	100m to 10m	7
VHF	30MHz to 300MHz	Very High Frequency	10m to 1m	8
UHF	300MHz to 3GHz	Ultra High Frequency	1m to 10cm	9
SHF	3GHz to 30GHz	Super High Frequency	10cm to 1cm	10
EHF	30GHz to 300GHz	Extremely High Frequency	1cm to 1mm	11
...	300GHz to 3THz	...	1mm-100µm	12

Literature researches according to the Microwave's sub-bands showed a lot of different definitions with very few or none description of the area of validity. Due to it, the following table will try to give an overview but can't act as a reference.

Source	Nührmann	Nührmann	www.werweiss-was.de	www.atcnea.de	Siemens Online Lexicon	Siemens Online Lexicon	ARRL Book No. 3126	Wikipedia
Validity	IEEE Radar Standard 521	US Military Band	Satellite Uplink	Primary Radar	Frequency bands in the GHz Area	Microwave bands	...	Dividing of Sat and Radar techniques
Band	GHz	GHz	GHz	GHz	GHz	GHz		GHz
A						0,1 - 0,225		
C	4 - 8		3,95 - 5,8	5 - 6	4 - 8	4 - 8	4 - 8	3,95 - 5,8
D		1 - 3						
E		2 - 3					60 - 90	60 - 90
F		2 - 4					90 - 140	
G		4 - 6					140 - 220	
H		6 - 8						
I		8 - 10						
J		10 - 20	5,85 - 8,2					5,85 - 8,2
K	18 - 27	20 - 40	18,0 - 26,5		18 - 26,5	10,9 - 36	18 - 26,5	18 - 26,5
Ka	27 - 40				26,5 - 40	17 - 31	26,5 - 40	26,5 - 40
Ku	12 - 18			≈16	12,6 - 18	15,3 - 17,2	12,4 - 18	12,4 - 18
L	1 - 3	40 - 60	1,0 - 2,6	≈1,3	1 - 2	0,39 - 1,55	1 - 2	1 - 2,6
M		60 - 100						
mm	40 - 100							
P			12,4 - 18,0			0,225 - 0,39	110 - 170	0,22 - 0,3
R			26,5 - 40,0					
Q						36 - 46	33 - 50	33 - 50
S	3 - 4		2,6 - 3,95	≈3	2 - 4	1,55 - 3,9	2 - 4	2,6 - 3,95
U			40,0 - 60,0				40 - 60	40 - 60
V						46 - 56	50 - 75	50 - 75
W							75 - 110	75 - 110
X	8 - 12		8,2 - 12,4	≈10	8 - 12,5	6,2 - 10,9	8 - 12,4	8,2 - 12,4

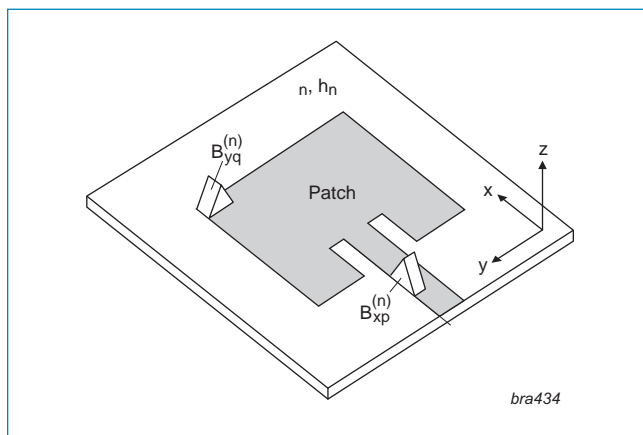
## 1.2 Function of an antenna

In standard application the RF output signal of a transmitter power amplifier is transported via a coaxial cable to a suitable location where the antenna is installed. Typically the coaxial cable has an impedance of  $50\Omega$  ( $75\Omega$  for TV/Radio). The ether, that is the room between the antenna and infinite space, also has an impedance value. This ether is the transport medium for the traveling wireless RF waves from the transmitter antenna to the receiver antenna. For optimum power transfer from the end of the coaxial cable (e.g.  $50\Omega$ ) into the ether (theoretical  $Z=120\pi\Omega=377\Omega$ ), we need a "power matching" unit. This matching unit is the antenna. It does match the cable's impedance to the space's impedance. Depending on the frequency and specific application needs there are a lot of antenna configurations and construction variations available. The simplest one is the isotropic ball radiator, which is a theoretical model used as a mathematical reference.

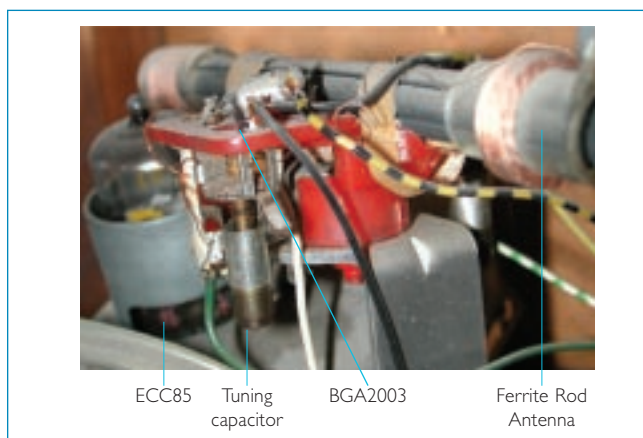


The next simplest configuration and a practical antenna in wide use is the dipole, also called the dipole radiator. It consists of two axial arranged sticks (Radiator). Removal of one Radiator results in to the "vertical monopole" antenna, as illustrated in the adjacent picture. The vertical monopole has a "donut-shaped" field centered on the radiating element.

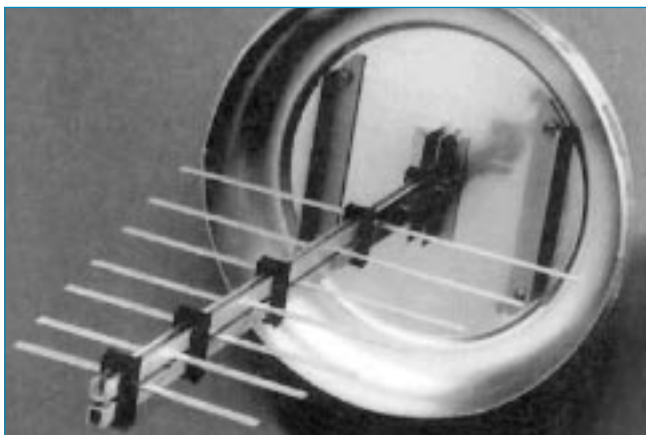
Higher levels of circuitry integration and cost reductions also influence antenna design. Based on the EM field radiation of strip-lines made by printed circuit boards (PCBs), PCB antenna structures were developed called 'patch-antennas' (see diagram). Use of ceramic instead of epoxy dielectric shrinks mechanical dimensions.



In the LF-MF-HF application range, ferrite-rod antennas were commonly used. They compress magnetic fields into a ferrite core, which acts like an amplifier for RF magnetic fields. Coils pick up signals like a transformer. They are a part of the pre-selection LC tank for image rejection and channel selection. The tuner shown is part of a Nordmende Elektra vacuum-tube radio (at least 40 years old and still working). To illustrate its dimensions, a Monolithic Microwave IC is placed in front of a solder point.



BGA2003

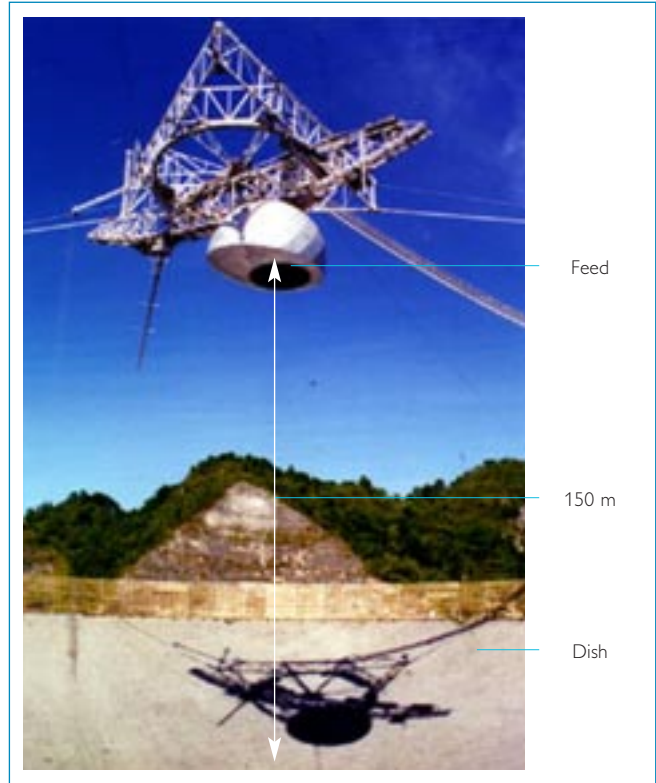


Logarithmic periodic antenna for 406-512 MHz



UHF broadband disccone antenna

The Arecibo observatory, in Puerto Rico, is a radio telescope with a dish antenna 305 m in diameter and 51 m deep. The secondary reflector & receiver are located on a 900-ton platform, suspended 137 m in the air above the dish. This is the feed point of an L-band microwave antenna and 50 MHz - 10 GHz antennas used for the SETI@home project. The receiver is cooled down to 50 K using liquid Helium for low-noise operation, to receive weak, distant signals transmitted (potentially) by extraterrestrial intelligence. The observatory can respond to incoming signals using a transmitter with a balanced klystron amplifier (2.5 kW output peak power; 120 kV / 4.4 A power supply).



The Arecibo observatory, in Puerto Rico

### 1.3 Transistor semiconductor process

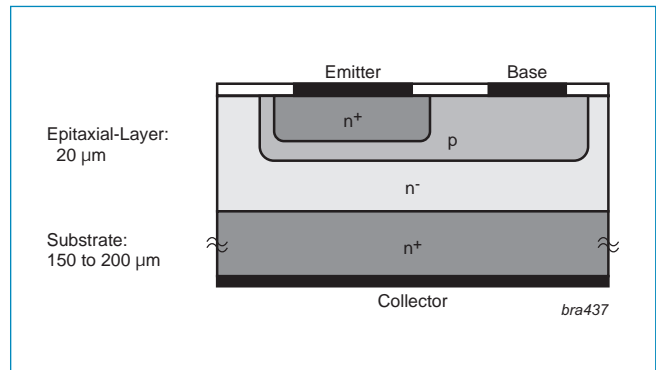
#### 1.3.1 General-purpose small-signal bipolar

The transistor is built up from three different layers:

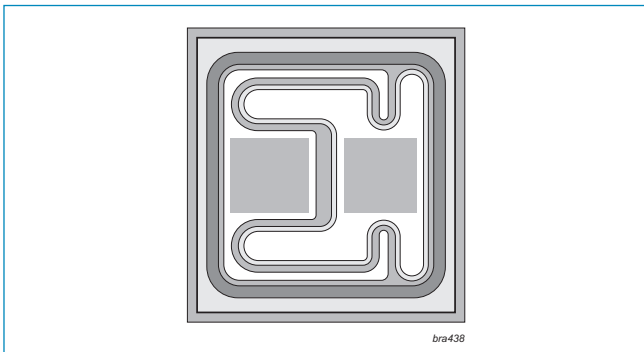
- Highly-doped emitter layer
- Medium-doped base area
- Low-doped collector area.

The highly doped substrate serves as a carrier and conductor only.

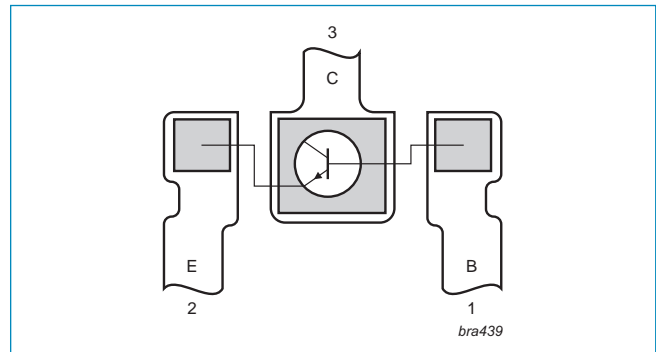
During the assembly process, the transistor die is attached to a lead-frame by gluing or eutectic soldering. The emitter and base contacts are connected to the lead-frame (leads) through bond wires (e.g. gold, aluminum, ...) using, for example, an ultrasonic welding process.



NPN Transistor cross section



Die of BC337, BC817



SOT23 standard lead-frame



### 1.3.2 Double polysilicon

The mobile communications market and the use of ever-higher frequencies mean there is a demand for low-voltage/high-performance RF wideband transistors, amplifier modules and MMICs. To meet that demand, Philips has developed a double-polysilicon process to achieve excellent performance. The 'double-poly' diffusion process uses an advanced transistor technology that is vastly superior to existing bipolar technologies.

#### Advantages of double-poly-Si RF process:

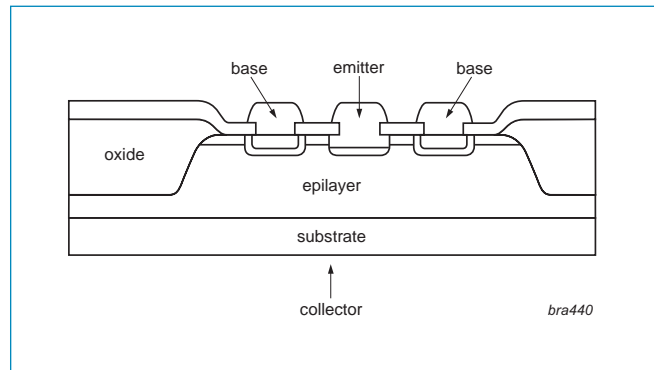
- Higher frequencies (>23GHz)
- Higher power gain  $G_{max}$ , e.g., 22dB/2GHz
- Lower noise operation
- Higher reverse isolation
- Simpler matching
- Lower current consumption
- Optimized for low supply voltages
- High efficiency
- High linearity
- Better heat dissipation
- Higher integration for MMICs (**SSI= Small-Scale-Integration**)

#### Applications

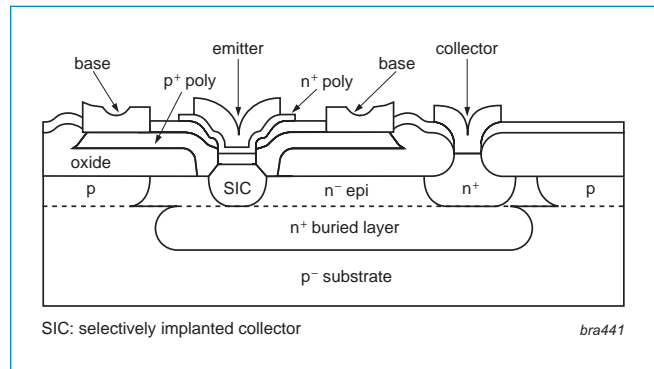
Cellular and cordless markets, low-noise amplifiers, mixers and power amplifier circuits operating at 1.8 GHz and higher), high-performance RF front-ends, pagers and satellite TV tuners.

#### Typical products manufactured in double-poly-Si:

- MMIC Family: BGA20xy, and BGA27xy
- 6<sup>th</sup> generation wideband transistors: BFG403W/410W/425W/480W
- RF power amplifier modules: BGY240S/241/212/280



Existing advanced bipolar transistor



SIC: selectively implanted collector

With double-poly, a polysilicon layer is used to diffuse and connect the emitter while another polysilicon layer is used to contact the base region. Via a buried layer, the collector is brought out on the top of the die. As with standard transistors, the collector is contacted via the back substrate and attached to the lead-frame.

# 2. RF design basics

## 2.1 RF fundamentals

### 2.1.1 RF waves

RF **electromagnetic (EM)** signals travel outward like **waves** in a pond that has had a stone dropped into it. The EM waves are governed by the laws that particularly apply to optical signals. In a homogeneous vacuum, without external influences, EM waves travel at a **speed of  $C_0=299792458$  m/s**. Waves traveling in substrates, wires, or within a non-air dielectric material put into the traveling path slow down and their speed is proportional to the root of the dielectric constant:

$$v = \frac{C_0}{\sqrt{\epsilon_{\text{reff}}}}$$

$\epsilon_{\text{reff}}$  is the **substrate's dielectric constant**.

With 'V' we can calculate the **wavelength**, as:

$$\lambda = \frac{v}{f}$$

Example1: Calculate the speed of an electromagnetic wave in a **Printed Circuit Board (PCB)** manufactured using a FR4 epoxy material and in a metal-dielectric-semiconductor capacitor of an integrated circuit.

Calculation: In a metal-dielectric-semiconductor capacitor, the dielectric material can be Silicon-Dioxide ( $\text{SiO}_2$ ) or Silicon-Nitride ( $\text{Si}_3\text{N}_4$ ).

$$v = \frac{C_0}{\sqrt{\epsilon_{\text{reff}}}} = \frac{299792458 \text{ m/s}}{\sqrt{4.6}} = 139.78 \cdot 10^6 \text{ m/s}$$

FR4	➔	$\epsilon_{\text{reff}} = 4.6$	➔	$v = 139.8 \cdot 10^6 \text{ m/s}$
$\text{SiO}_2$	➔	$\epsilon_{\text{reff}} = 2.7$ to $4.2$	➔	$v = 182.4 \cdot 10^6 \text{ m/s}$ to $139.8 \cdot 10^6 \text{ m/s}$
$\text{Si}_3\text{N}_4$	➔	$\epsilon_{\text{reff}} = 3.5$ to $9$	➔	$v = 160.4 \cdot 10^6 \text{ m/s}$ to $99.9 \cdot 10^6 \text{ m/s}$

Example2: What is the wavelength transmitted from a commercial SW radio broadcasting program (SWR3 in the 49 meter band) at 6030 kHz in air, and within a FR4 PCB?

Calculation: The  $\epsilon_{\text{reff}}$  of air is close to vacuum. ➔  $\epsilon_{\text{reff}} \approx 1$  ➔  $v = c_0$

$$\text{Wavelength in air: } \lambda_{\text{air}} = \frac{C_0}{f} = \frac{299792458 \text{ m/s}}{6030 \text{ KHz}} = 49.72 \text{ m}$$

From Example 1 we take the FR4 dielectric constant to be  $\epsilon_{\text{reff}} = 4.6$ , then  $v = 139.8 \cdot 10^6 \text{ m/s}$  and calculate the wavelength in the PCB as:  $\lambda_{\text{FR4}} = 23.18$  meters

A forward-traveling wave is transmitted (or injected) by the source into the traveling medium (whether it be the ether, a **substrate**, a **dielectric**, wire, **microstrip**, **waveguide** or other medium) and travels to the load at the opposite end of the medium. At junctions between two different dielectric materials, a part of the forward-traveling wave is reflected back towards the source. The remaining part continues traveling towards the load.

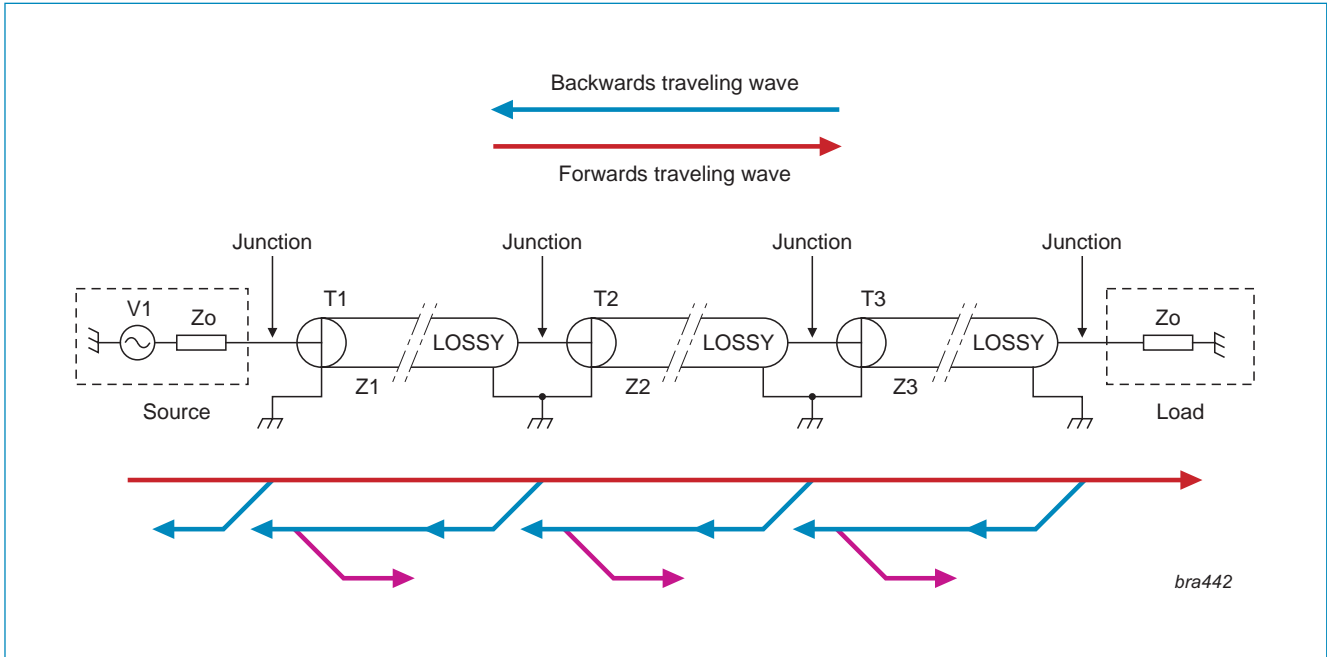
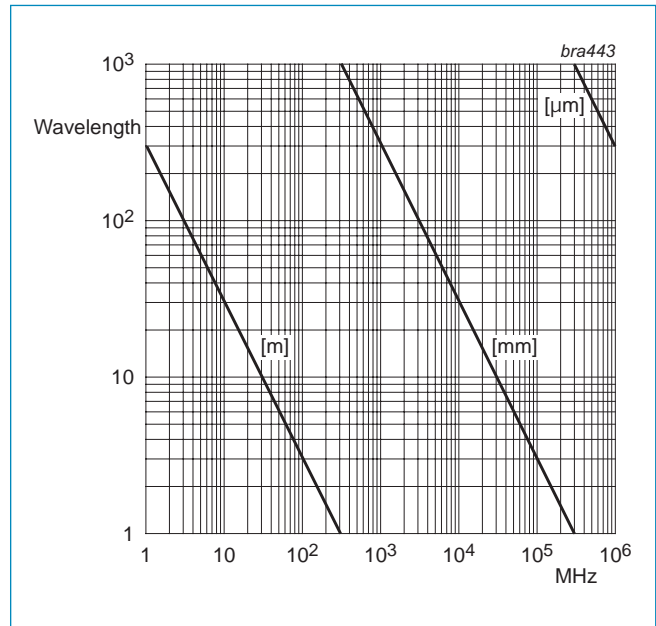


Fig.6: Multiple reflections between lines with different impedances Z1-Z3

In Fig.6, **reflections** of the forward-traveling main wave (red) are caused between materials with different impedance values ( $Z_1, Z_2, Z_3$ ). As shown, a backward-reflected wave (green) can again be reflected into a forward-traveling wave in the direction towards the load (shown as violet in Fig.6). In the case of optimum **matching** between different dielectric mediums, no signal reflection will occur and maximum power is forwarded. The amount of reflection caused by junctions of lines with different impedances, or line **discontinuities**, is determined by the **reflection coefficient**. This is explained in the next chapter.



Example: Select your frequency (ISM433) crossing a trace (blue) you can read the wavelength (70cm)

### 2.1.2 The reflection coefficient

As discussed previously, a forward-traveling wave is partially reflected back at junctions with line impedance discontinuities, or mismatches. Only the portion of the forward traveling wave (arriving at the load) will be absorbed and processed by the load. Because of the frequency-dependent speed of the propagating waves in a dielectric medium, there will be a delay in the arrival of the wave at the load point over what a wave traveling in free space would have (phase shift).

Mathematically this behavior is modeled with a vector in the complex Gaussian space. At each location of the travel medium (or wire), wave-fronts with different amplitude and phase delay are heterodyned. The resulting energy envelope of the waves along the wire appears as a ripple with maximum and minimum values. The phase difference between maximums has the same value as the phase difference between minimums. This distance is termed the **half-wavelength, or  $\lambda/2$  (also termed the normalized phase shift of  $180^\circ$ ).**

**Example:** A line with mismatched ends driven from a source will have standing waves. These will result in minimum and maximum signal amplitudes at defined locations along the line. Determine the approximate distance between worst-case voltage points for a **Bluetooth** signal processed in a printed circuit on a FR4 based substrate.

**Calculation:** Assumed speed in FR4:  $v = 139.8 \cdot 10^6 \text{ m/s}$

$$\text{Wavelength: } \lambda_{air} = \frac{v_{FR4}}{f_{BT}} = \frac{139.78 \cdot 10^6 \text{ m/s}}{2.4 \text{ GHz}} = 58.24 \text{ mm}$$

- At the minimum we have minimum voltage, but maximum current.
- At the maximum we have maximum voltage, but minimum current.
- The distance between a minimum and a maximum voltage (or current) point is equal to  $\lambda/4$ .

The reflection coefficient is defined by the ratio between the backward-traveling voltage wave and the forward-traveling voltage wave:

**Reflection coefficient:**  $r_{(x)} = \frac{U_{b(x)}}{U_{f(x)}}$

**Reflection loss or return loss:**  $r_{dB} = 20 \text{ dB} \cdot \log|r_{(x)}| = 20 \text{ dB} \{ \log|U_{b(x)}| - \log|U_{f(x)}| \}$

The index '(x)' indicates different reflection coefficients along the line. This is caused by the distribution of the standing wave along the line. The return loss (in dB) indicates how much of the wave is reflected, compared to the forward-traveling wave.

Often the input reflection performance of a 50Ω RF device is specified by the **Voltage Standing Wave Ratio (VSWR or just SWR)**.

VSWR:  $s = SWR = VSWR = \frac{U_{\max}}{U_{\min}}$       Matching factor:  $m = \frac{1}{s}$

Some typical values of the VSWR:  
 100% mismatch caused by an open or shorted line:  $r = 1$  and  $VSWR = \infty$   
 Optimum (theoretical) matched line:  $r = 0$  and  $VSWR = 1$   
 In all practical situations 'r' varies between  $0 < r < 1$  and  $1 < VSWR < \infty$

Calculating the reflection factor:  $r = |r_{(x)}| = \frac{SWR - 1}{SWR + 1}$

Using some mathematical manipulation:  $r = \frac{\frac{U_{\max}}{U_{\min}} - 1}{\frac{U_{\max}}{U_{\min}} + 1}$  results in:  $r = \frac{U_{\max} - U_{\min}}{U_{\max} + U_{\min}}$

Reflection coefficients of certain impedances (e.g. a load) leads to:  $r = \frac{Z - Z_0}{Z + Z_0}$

with  $Z_0$  = nominal system impedance (50Ω, 75Ω).

As explained, the standing waves cause different amplitudes of voltage and current along the wire.

The ratio of these two parameters is the impedance  $Z_{(x)} = \frac{V_{(x)}}{I_{(x)}}$  at each location, (x). This means a line with length (L), and a mismatched load  $Z(x = L)$  at the wire-end location (x=L), will show a wire-length dependent impedance at the source location (x=0):

$$Z_{(x=0)} = \frac{V_{(x=0)}}{I_{(x=0)}}$$

Example: There are several special cases (tricks) that can be used in microwave designs.

Mathematically it can be shown that a wire with the length of  $\ell = \frac{\lambda}{4}$  and an impedance  $Z_L$  will be a **quarter wavelength transformer**:

$$\ell = \frac{\lambda}{4} \text{ - impedance transformer: } Z_{(x=\ell)} = \frac{Z_L^2}{Z_{(x=0)}}$$

This can be used in SPDT (single pole, double throw) based PIN diode switches or in DC bias circuits because an RF short (like a large capacitor) is transformed into infinite impedance with low resistive dc path (under ideal conditions).

As indicated in Fig.6, and shown by the RF traveling-wave basic rules, matching, reflection and individual wire performances affect bench measurement results caused by impedance transformation along the wire. Due to this constraint, each measurement set-up must be calibrated by precision references.

Examples of RF calibration references are:

- Open
- Short
- Match
- Through
- Sliding Load

The set-up calibration tools can undo unintended wire transformations, discontinuities from connectors, and similar measurement intrusion issues. This prevents **Device Under Test (DUT)** measurement parameters from being affected by mechanical bench set-up configurations.

Example:

- Determine the input VSWR of **BGA2711 MMIC** wideband amplifier for 2GHz, based on data sheet characteristics.
- What kind of resistive impedance(s) can theoretically cause this VSWR?
- What is the input return loss measured on a 50Ω coaxial cable in a distance of λ/4?

Calculation: **BGA2711** at 2 GHz:  $r_{IN} = 10dB$

$$r = \frac{SWR - 1}{SWR + 1} \Rightarrow r \cdot SWR + r = SWR - 1 \Rightarrow SWR = \frac{1+r}{1-r}$$

$$r = 10^{\frac{-r_{dB}}{20}} = 10^{\frac{-10dB}{20}} = 0.3162 \Rightarrow SWR_{IN} = \frac{1+0.3162}{1-0.3162} = 1.92 \quad r = \frac{Z - Z_0}{Z + Z_0} \Rightarrow$$

$$Z - r \cdot Z = r \cdot Z_0 + Z_0 \Rightarrow Z = Z_0 \frac{1+r}{1-r}$$

Comparison:  $Z = Z_0 \frac{1+r}{1-r}$  &  $SWR = \frac{1+r}{1-r} \Rightarrow Z = Z_0 \cdot SWR$

We know only the magnitude of (r) but not its angle. By definition, the VSWR must be larger than 1. We then get two possible solutions:

$$SWR_1 = \frac{Z_{max}}{Z_0} \quad \text{and} \quad SWR_2 = \frac{Z_0}{Z_{min}} \quad Z_{max} = 1.92 \cdot 50\Omega = 96.25\Omega; \quad Z_{min} = 50\Omega / 1.92 = 25.97\Omega$$

We can then examine r:  $|r| = \left| \frac{96.25 - 50}{96.25 + 50} \right| = \left| \frac{25.96 - 50}{25.96 + 50} \right| = 0.316$

The λ/4 transformer transforms the device impedance to:

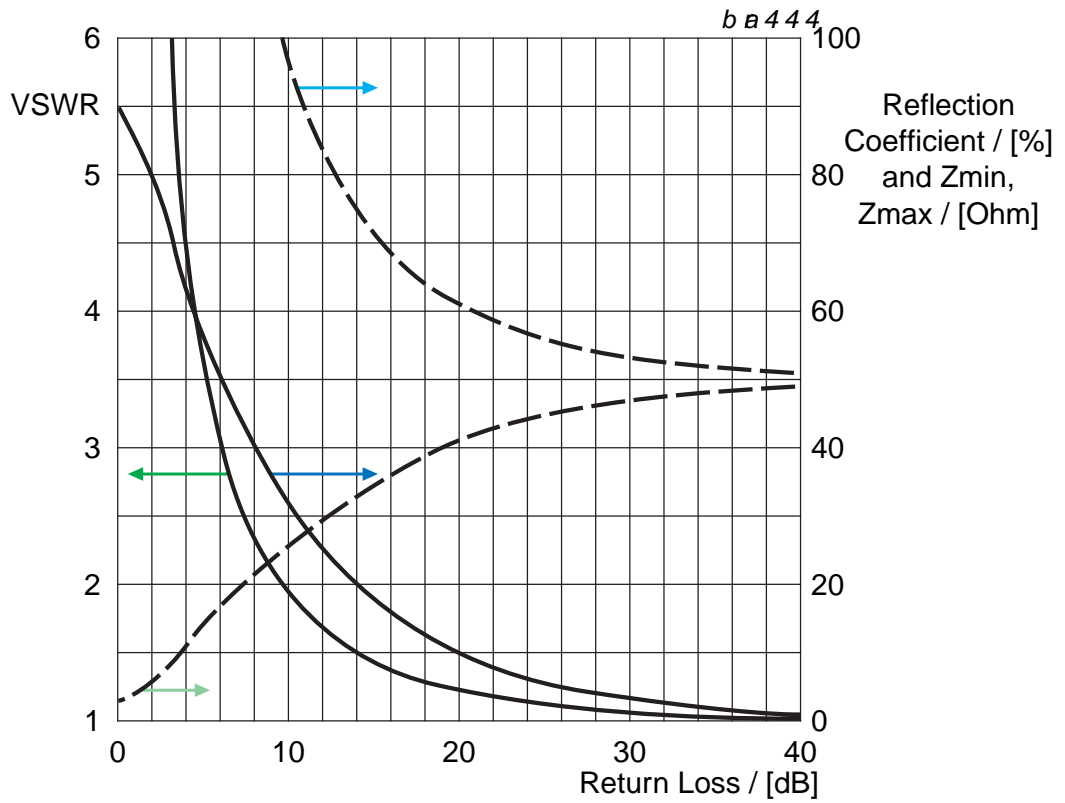
$$Z_{IN1} = 96.25\Omega \Rightarrow Z_{Ende} = \frac{Z_0^2}{Z_{IN}} = \frac{50\Omega^2}{96.25\Omega} = 25.97\Omega \quad \text{and for } Z_{IN2} = 25.97\Omega \Rightarrow 96.25\Omega$$

Results:

At 2GHz, the **BGA2711** offers an input return loss of 10dB or VSWR=1.92. This reflection can be caused by a 96.25Ω or a 25.97Ω impedance. Of course there are infinite results possible if one takes into account all combinations of L and C values.

Measuring this impedance at 2GHz with the use of a non-50Ω cable will cause extremely large errors in λ/4 distance, because the  $Z_{in1} = 96.25\Omega$  appears as 25.97Ω and the second solution  $Z_{in2} = 25.97\Omega$  appears as 96.25Ω!

As illustrated in the above example, the VSWR (or return loss) quickly indicates the quality of a device's input matching without any calculations, but does not tell about its real (vector) performance (missing or phase information). Detailed mathematical network analysis of RF amplifiers depends on the device's input impedance versus output load (S12 issue). The output device impedance is dependent on the impedance of the source driving the amplifier (S21 issue). Due to this interdependence, the use of s-parameters in linear small signal networks offers reliable and accurate results. This s-parameter theory will be presented in the next chapters.



**Example:** Select your interesting return-loss (10dB). Crossing the dark green trace you can find the VSWR ( $\approx 1,9$ ) and crossing the dark blue trace you can find the reflection coefficient ( $r \approx 0,32$ ). There are two (100% resistive) mismatches found either crossing the dashed light green traces ( $Z_{\max} \approx 96\Omega$ ) or crossing the dashed light blue trace ( $Z_{\min} \approx 26\Omega$ ). For further details, please refer to the former algebraic application example.

### 2.1.3 Differences between ideal and practical passive devices

Practical devices have so-called parasitic elements at very high frequencies.

**Resistor** Has an inductive parasitic action and acts like a low-pass filtering function.

**Inductor** Has a capacitive and resistive parasitic, causing it to act like a damped parallel resonant tank circuit with a certain self resonance.

**Capacitor** Has an inductive and resistive parasitic, causing it to act like a damped tank circuit with **Series Resonance Frequency (SRF)**.

The inductor's and the capacitor's parasitic reactance causes self-resonances.

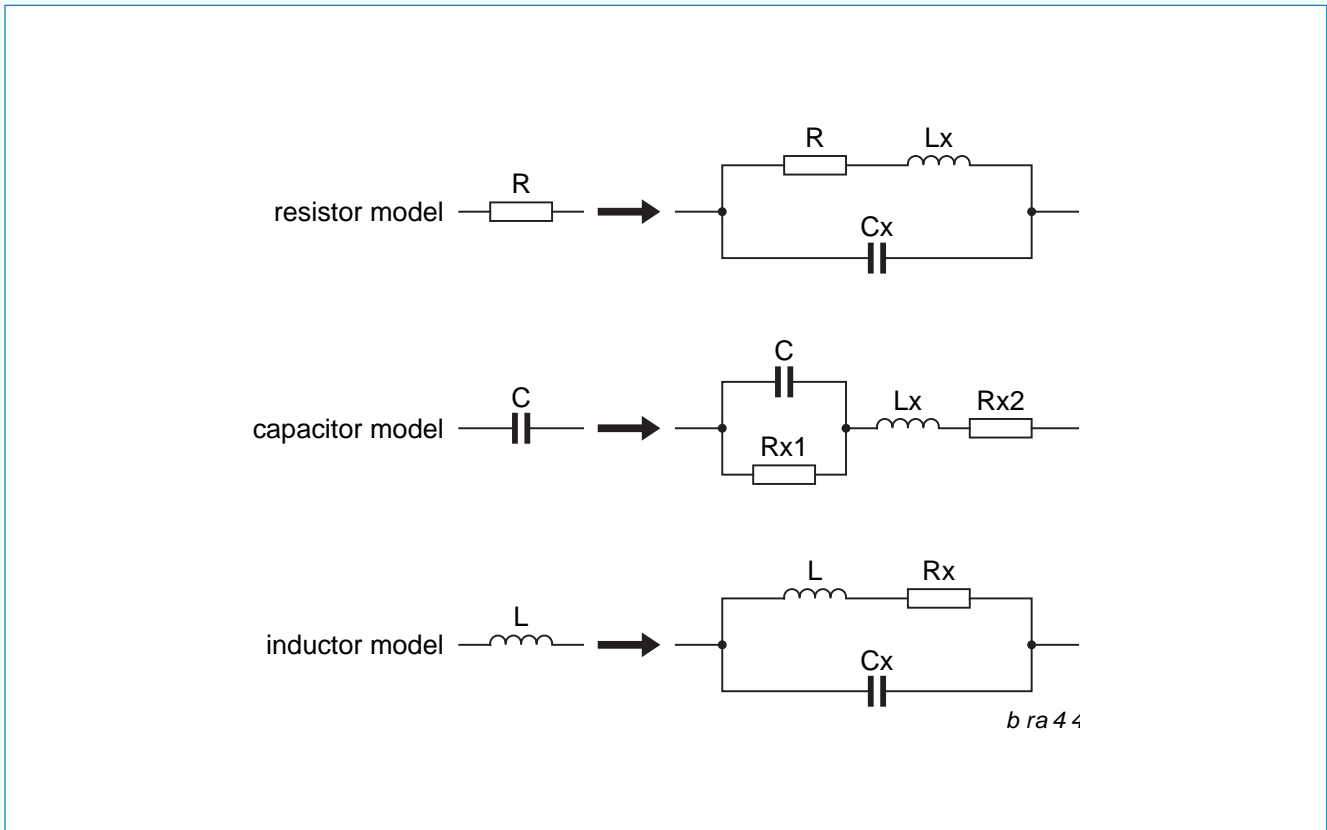


Fig.7 Equivalent models of passive lumped elements

The use of a passive component above its SRF is possible, but must be critically evaluated. A capacitor above its SRF appears as an inductor with DC blocking capabilities.



### 2.1.4 The Smith chart

As indicated in an example in the previous chapter, the impedances of semiconductors are a combination of resistive and reactive parts caused by phase delays and parasitics. RF impedances are best analyzed in the frequency domain under the use of vector algebraic expressions:

Object	➔	into	➔	Frequency domain
Resistor	➔	R	➔	$R =  R  \cdot e^{+j0^\circ}$
Inductor	➔	L	➔	$X_L = +j\omega \cdot L = \omega \cdot L \cdot e^{+j90^\circ}$
Capacitor	➔	C	➔	$X_C = -j \frac{1}{\omega \cdot C} = \frac{1}{\omega \cdot C} \cdot e^{-j90^\circ}$
Frequency	➔	f	➔	$\omega = 2\pi \cdot f$
Complex designator	➔	j	➔	$+j = \sqrt{-1} = \frac{1}{-j} = e^{+j90^\circ}$

Some useful basic vector algebra in RF analysis:

**Complex impedance:**  $Z = \text{Re}\{Z\} + j \text{Im}\{Z\} = |Z| \cdot e^{j\varphi} = |Z| \cdot (\cos\varphi - j \sin\varphi)$

$$\text{Im}\{Z\} = |Z| \sin\varphi ; \text{Re}\{Z\} = |Z| \cos\varphi ;$$

$$\tan = \frac{\sin\varphi}{\cos\varphi} \quad \Rightarrow \quad \tan\varphi = \frac{\text{Im}\{Z\}}{\text{Re}\{Z\}} \quad \text{with; } \varphi = \omega \cdot t$$

Use of angle

➔ **Polar** notation

Use of sum

➔ **Cartesian (Rectangular)** notation

The same rules are used for other issues,

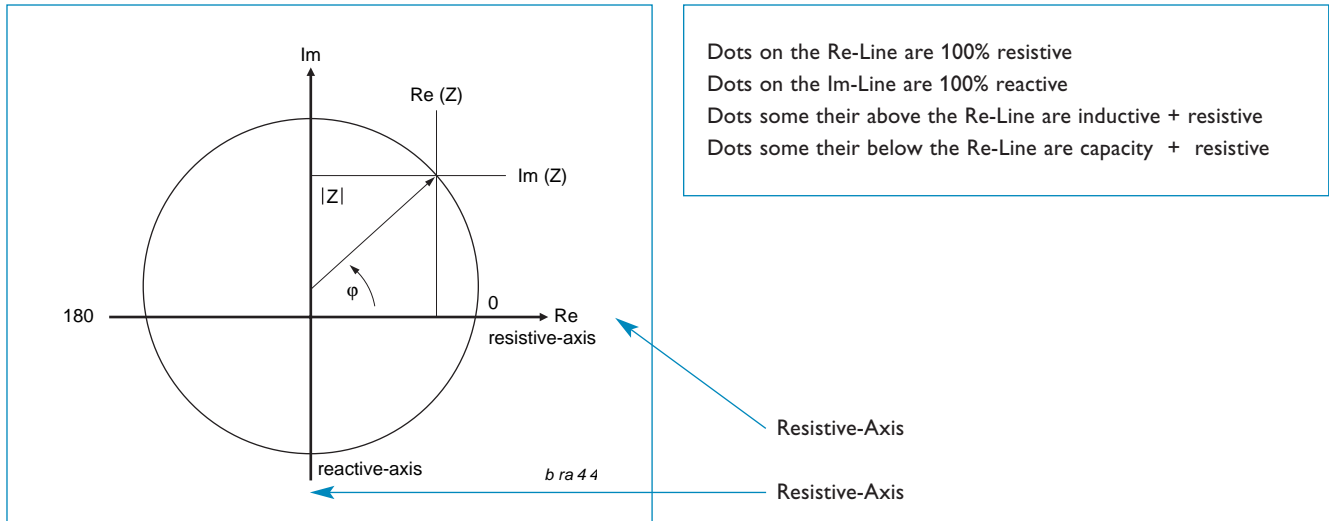
e.g., the **complex reflection coefficient:**

$$r = |r| \cdot e^{j\varphi} = \frac{|U_b| \cdot e^{j\varphi_b}}{|U_f| \cdot e^{j\varphi_f}} = \frac{|U_b|}{|U_f|} \cdot e^{j(\varphi_b - \varphi_f)}$$

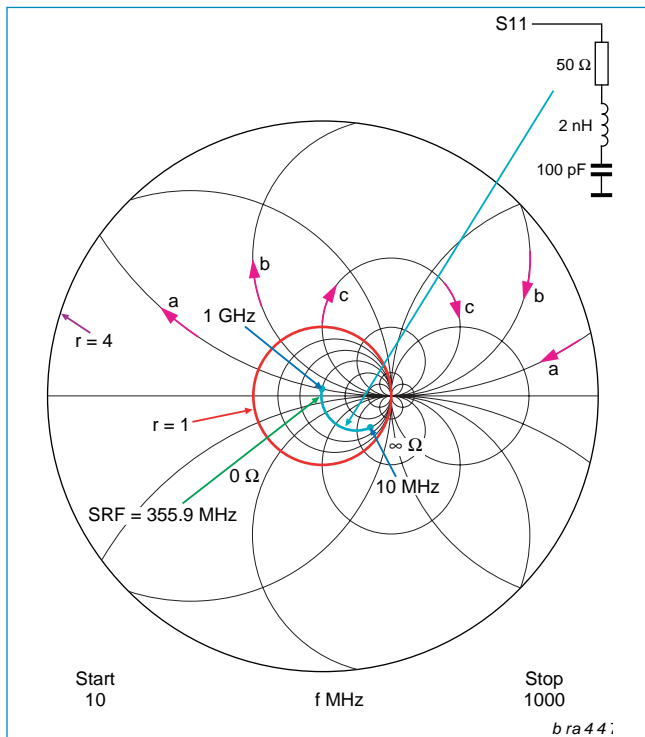
**Special cases:**

- Resistive mismatch:  $\varphi_{(R)} = 0^\circ$  reflection coefficient:  $\varphi_{(r)} = 0^\circ$
- Inductive mismatch:  $\varphi_{(L)} = +90^\circ$  reflection coefficient:  $\varphi_{(r)} = +90^\circ$
- Capacitive mismatch:  $\varphi_{(C)} = -90^\circ$  reflection coefficient:  $\varphi_{(r)} = -90^\circ$

The **Gaussian number area (Polar Diagram)** allows charting rectangular two-dimensional vectors:



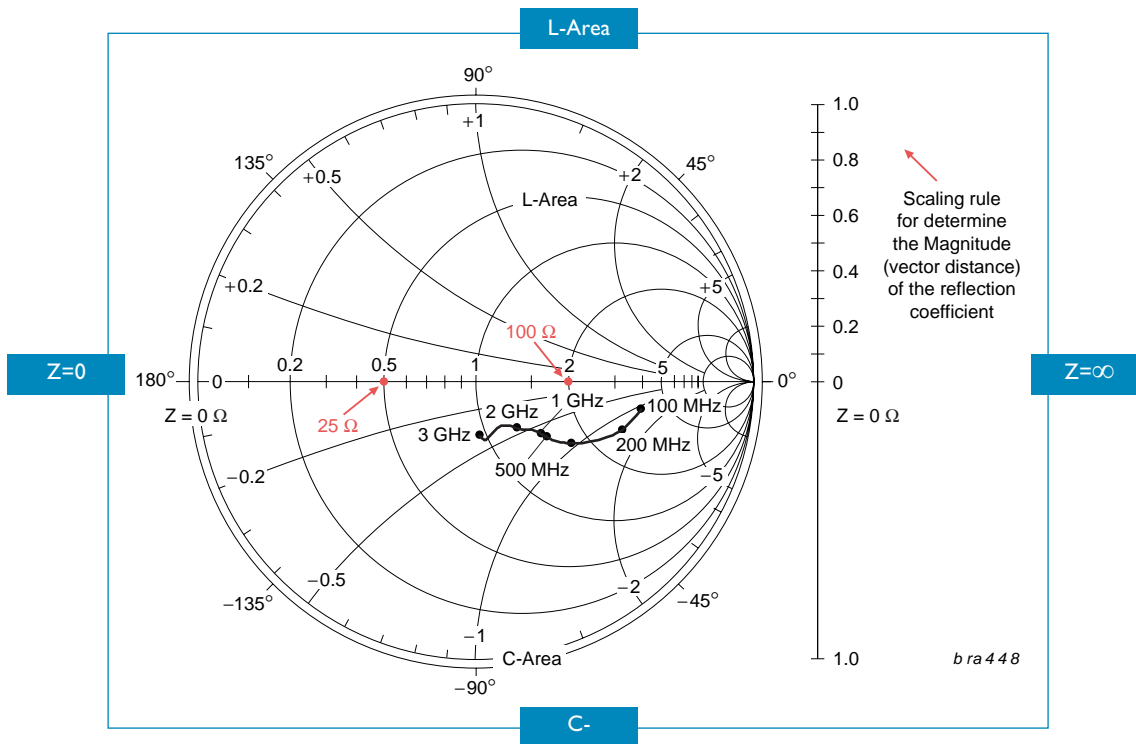
In applications, RF designers try to remain close to a 50 Ω resistive impedance. The polar diagram’s origin is 0 Ω. In RF circuits, relatively large impedances can occur but we try to remain close to 50 Ω by special network design for maximum power transfer. Practically, very low and very high impedances don’t need to be known accurately. The Polar diagram can’t show simultaneous large impedances and the 50 Ω region with acceptable accuracy, because of limited paper size.



Using this fact Mr. Phillip Smith, an engineer at Bell Laboratories, developed the so-called **Smith Chart** in the 1930s. The chart’s origin is at 50Ω. Left and right resistive values along the real axis end in 0Ω and at ∞Ω. The imaginary reactive axis (imaginary axis, or Im-Axis) ends in 100% reactive (L or C). High resolution is provided close to the 50Ω origin. Far away of the chart’s centre the resolution drops. Further from the centre of the chart, the resolution / error increases. The standard Smith Chart only displays **positive resistances** and has a unit radius (r=1). **Negative resistances** generated by **instability** (e.g. **oscillation**) lie outside the unit circle. In this non-linear scaled diagram, the infinite dot of the Re-Axis is ‘theoretically’ bent to the zero point of the Smith Chart. Mathematically it can be shown that this will form the Smith Chart’s unit circle (r=1). All dots lying on this circle represent a reflection coefficient magnitude of 1 (100% mismatch). Any positive L/C combination with a resistor will be mathematically represented by its polar notation reflection coefficient inside the Smith Chart’s unity circle. Because the Smith Chart is a transformed linear-scale polar diagram, we can use 100% of the polar diagram rules. Cartesian-diagram rules are changed due to non-linear scaling.

**Special cases:**

- Dots below the horizontal axis represent impedance with a capacitive part (  $180^\circ < \varphi < 360^\circ$  )
- Dots laying on the horizontal axis (ordinate) are 100% resistive (  $\varphi = 0^\circ$  )
- Dots laying on the vertical axis (abscissa) are 100% reactive (  $\varphi = 90^\circ$  )



**Fig.8: BGA2003 output Smith chart (S<sub>22</sub>)**

The special cases for zero and infinitely large impedance are illustrated (above). The upper half circle is the inductive region. The lower half of the circle is the capacitive region. The origin is the 50Ω system reference (Z<sub>0</sub>). To be more flexible, numbers printed in the chart are normalized to Z<sub>0</sub>.

Normalizing impedance procedure: 
$$Z_{norm} = \frac{Z_x}{Z_o} \quad Z_o = \text{System reference impedance (e.g., 50}\Omega, 75\Omega)$$

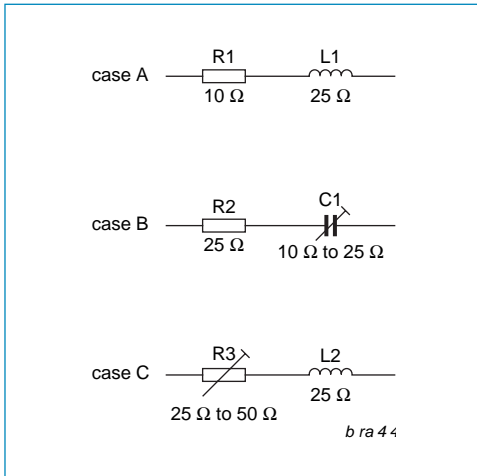
Example: Plot a 100Ω & 50Ω resistor into the upper **BGA2003's** output Smith chart.

Calculation:  $Z_{norm1} = 100\Omega / 50\Omega = 2$ ;  $Z_{norm2} = 25\Omega / 50\Omega = 0.5$

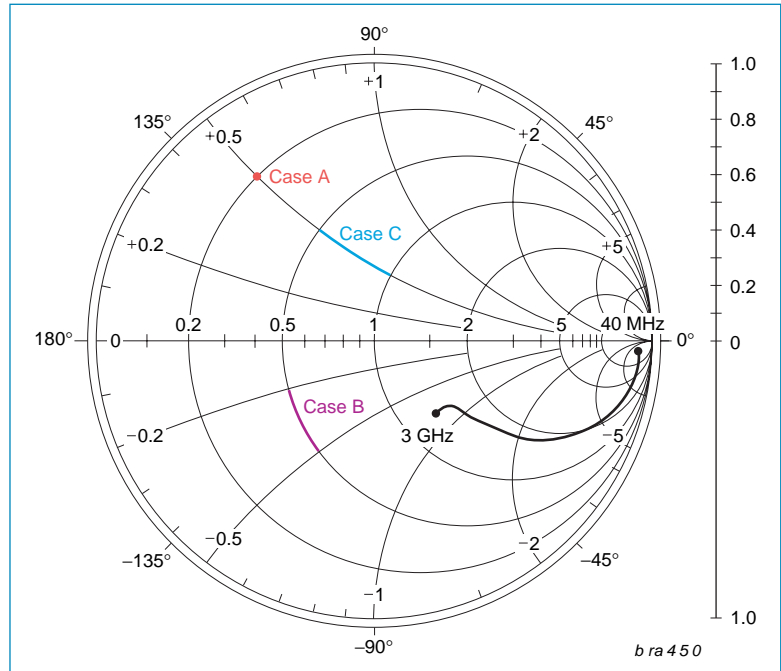
Result: The 100Ω resistor appears as a dot on the horizontal axis at the location 2.  
The 25Ω resistor appears as a dot on the horizontal axis at the location 0.5

Example1: In the following three circuits, capacitors and inductors are specified by the amount of reactance @ their 100MHz design frequency. Determine the value of the parts. Plot their impedance in to the **BFG425W's** output (S<sub>22</sub>) Smith chart.

Circuit:



Result:



Calculation:

**Case A** (constant resistance)

From the circuit ➔  $Z_A = 10\Omega + j25\Omega$  ;

$$L_1 = \frac{25\Omega}{2\pi \cdot 100\text{MHz}} = 39.8\text{nH} \quad Z_{(A)\text{norm}} = Z_A/50\Omega = 0.2 + j0.5 \quad \text{➔ Drawing into Smith chart}$$

Basics:

$$C = \frac{1}{\omega \cdot X_C}$$

$$L = \frac{X_L}{\omega}$$

$$\omega = 2\pi \cdot f$$

**Case B** (constant resistance and variable reactance - variable capacitor)

From the circuit ➔  $Z_B = 10\Omega + j(10 \text{ to } 25)\Omega$

$$C_B = \frac{1}{2\pi \cdot 100\text{MHz} \cdot (10 \text{ to } 25)\Omega} = 63.7\text{pF to } 159.2\text{pF} \quad Z_{(B)\text{norm}} = Z_B/50\Omega = 0.5 - j(0.2 \text{ to } 0.5) \quad \text{➔ Drawing into Smith chart}$$

**Case C** (constant resistance and variable reactance - variable inductor)

From the circuit ➔  $Z_C = (25\Omega \text{ to } 50\Omega) + j25\Omega$

$$L_C = \frac{(25 \text{ to } 50)\Omega}{2\pi \cdot 100\text{MHz}} = 39.8\text{nH to } 79.6\text{nH} \quad Z_{(C)\text{norm}} = Z_C/50\Omega = (0.5 \text{ to } 1) + j0.5 \quad \text{➔ Drawing into Smith chart}$$

Example2:

Determine **BFG425W**'s outputs reflection coefficient (S22) at 3GHz from the data sheet. Determine the output return loss and output impedance. Compensate the reactive part of the impedance.

Calculation: The data in the Smith chart can be read with improved resolution by using the vector reflection coefficient in Polar notation.

Procedure: 1) Mechanically measure the scalar length from the chart origin to the 3GHz (vector distance).  
2) On the chart's right side is printed a ruler with the numbers of 0 to 1. Read from it the equivalent scaled scalar length  $|r| = 0.34$

3) Measure the angle  $\angle(r) = \varphi = -50^\circ$ . Write the reflection coefficient in vector polar notation

$$\underline{r = 0.34e^{-j50^\circ}}$$

$$\text{Normalized impedance: } \frac{Z}{Z_o} = \frac{1+r}{1-r} = 1.513e^{-j30.5^\circ}$$

Because the transistor was characterized in a  $50\Omega$  bench test set-up  $\rightarrow Z_o = 50\Omega$

$$\text{Impedance: } Z_{22} = 75.64\Omega e^{-j30.5^\circ} = (65.2 - j38.4)\Omega$$

$$C = \frac{1}{2\pi \cdot 3\text{GHz} \cdot 38.4\Omega} = 1.38\text{pF}$$

The output of **BFG425W** has an equivalent circuit of  $65.2\Omega$  with  $1.38\text{pF}$  series capacitance.  
Output return loss, not compensated:  $RL_{\text{OUT}} = -20\log(|r|) = 9.36\text{dB}$  resulting in  $VSWR_{\text{OUT}} = 2$

For compensation of the reactive part of the impedance, we take the **conjugate complex** of the reactance:

$$X_{\text{con}} = -\text{Im}\{Z\} = -\{-j38.4\Omega\} = +j38.4\Omega \text{ resulting in}$$

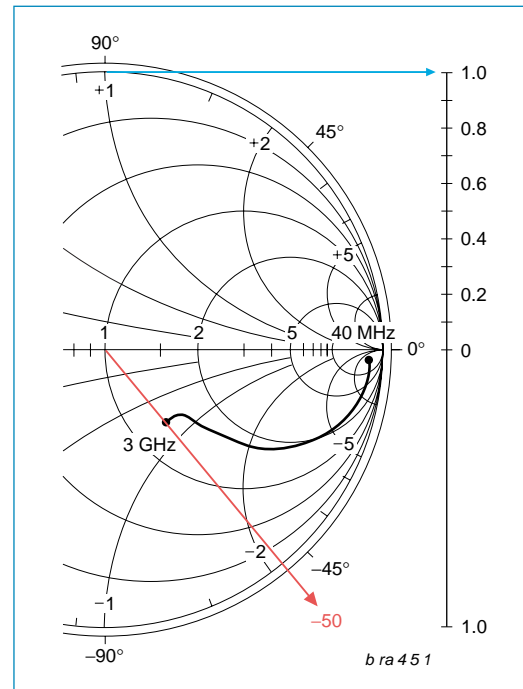
$$L = \frac{38.4\Omega}{2\pi \cdot 3\text{GHz}} = 2\text{nH}$$

A  $2\text{nH}$  series inductor will compensate the capacitive reactance. The new input reflection coefficient is calculated to:

$$r = \frac{65.2\Omega - 50\Omega}{65.2\Omega + 50\Omega} = 0.132$$

Output return loss, compensated:  $RL_{\text{OUT}} = -20\log(0.132) = 17.6\text{dB}$  resulting in  $VSWR_{\text{OUT}} = 1.3$

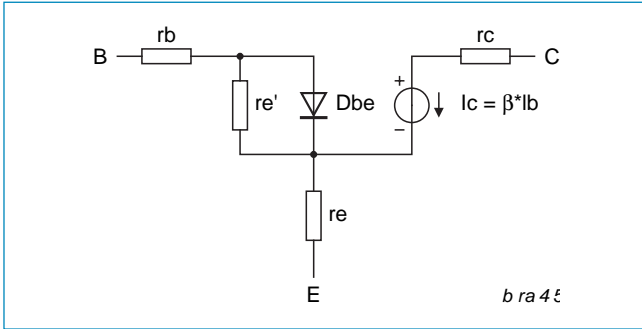
Please note: In practical situations the output impedance is a function of the input circuit. The input and output matching circuits are defined by the **stability** requirements, the need gain and noise-matching. Investigation is done by using network analysis based on **s-parameters**.



## 2.2 Small signal RF amplifier parameters

### 2.2.1 Transistor parameters, DC to microwave

At low DC currents and voltages, one can assume a transistor acts like a voltage-controlled current source with diode clamping action in the base-emitter input circuit. In this model, the transistor is specified by its large-signal DC-parameters, i.e., DC-current gain ( $\beta$ ,  $\beta$ ,  $h_{fe}$ ), maximum power dissipation, breakdown voltages and so forth.



$$I_C = I_{CO} \cdot e^{\frac{U_{BE}}{V_T}} \quad r_e' = \frac{V_T}{I_E}$$

Thermal Voltage:  $V_T = kT/q \approx 26 \text{ mV} @ 25^\circ \text{C}$

$I_{CO}$  = Collector reverse saturation current

$$\text{Low frequency voltage gain} \quad V_u \approx \frac{R_C}{r_e'}$$

$$\text{Current gain} \quad \beta = \frac{I_C}{I_B}$$

Increasing the frequency to the audio frequency range, the transistor's parameters change due to frequency-dependent phase shift and parasitic capacitance effects. For characterization of these effects, small signal **h-parameters** are used. These hybrid parameters are determined by measuring voltage and current at one terminal and using open or short (standards) at the other port.

The **h-parameter** matrix is shown below.

$$\text{h-parameter Matrix:} \quad \begin{pmatrix} u_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} * \begin{pmatrix} i_1 \\ u_2 \end{pmatrix}$$

Increasing the frequency to the HF and VHF ranges, open ports become inaccurate due to electrically stray field radiation. This results in unacceptable errors. Due to this phenomenon, **y-parameters** were developed. They again measure voltage and current, but use only a 'short' standard. This 'short' approach yields more accurate results in this frequency region. The **y-parameter** matrix is shown below.

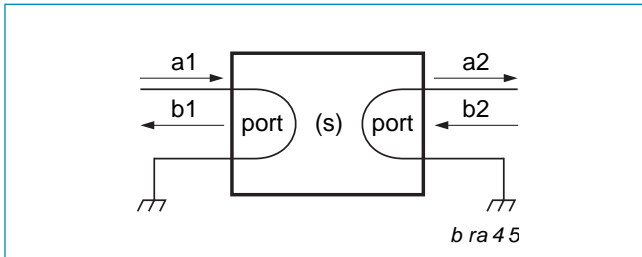
$$\text{y-parameter Matrix:} \quad \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} * \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Further increasing the frequency, the parasitic inductance of a 'short' causes problems due to mechanical-dependent parasitics. Additionally, measuring voltage, current and phase is quite tricky. The scattering parameters, or **s-parameters**, were developed based on the measurement of the forward and backward traveling waves to determine the reflection coefficients on a transistor's terminals (or ports). The **s-parameter** matrix is shown below.

$$\text{s-parameter Matrix:} \quad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} * \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

## 2.2.2 Definition of the s-parameters

Every amplifier has an input port and an output port (a 2-port network). Typically the input port is labeled Port-1 and the output is labeled Port-2.



Matrix: 
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} * \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Equation: 
$$b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2$$
  

$$b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2$$

Fig.10: Two-port network's (a) and (b) waves

The forward-traveling waves (a) are traveling into the DUT's (input or output) ports.

The backward-traveling waves (b) are reflected back from the DUT's ports

The expression 'port Z0 terminate' means the use of a 50Ω-standard.

This is not a conjugate complex power match! In the previous chapter the reflection coefficient was defined as:

**Reflection coefficient:** 
$$r = \frac{\text{back running wave}}{\text{forward running wave}}$$

Calculating the **input reflection factor** on port 1: 
$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$
 with the output terminated in Z<sub>0</sub>.

That means the source injects a forward-traveling wave (a1) into Port-1. No forward-traveling power (a2) injected into Port-2. The same procedure can be done at Port-2 with the

**Output reflection factor:** 
$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$
 with the input terminated in Z<sub>0</sub>.

**Gain** is defined by: 
$$\text{gain} = \frac{\text{output wave}}{\text{input wave}}$$

The **forward-traveling wave** gain is calculated by the wave (b2) traveling out of Port-2 divided by the wave (a1) injected into Port-1.

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

The **backward traveling wave** gain is calculated by the wave (b1) traveling out of Port-1 divided by the wave (a2) injected into Port-2.

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

The normalized waves (a) and (b) are defined as:

$$a_1 = \frac{1}{2\sqrt{Z_0}} (V_1 + Z_0 \cdot i_1) = \text{signal into Port-1}$$

$$a_2 = \frac{1}{2\sqrt{Z_0}} (V_2 + Z_0 \cdot i_2) = \text{signal into Port-2}$$

$$b_1 = \frac{1}{2\sqrt{Z_0}} (V_1 - Z_0 \cdot i_1) = \text{signal out of Port-1}$$

$$b_2 = \frac{1}{2\sqrt{Z_0}} (V_2 - Z_0 \cdot i_2) = \text{signal out of Port-2}$$

**Forward transmission:**  
 FT = 20log(S<sub>21</sub>)dB

**Isolation:**  
 S12(dB) = -20log(S<sub>12</sub>)dB

**Input return loss:**  
 RL<sub>in</sub> = -20log(S<sub>11</sub>)dB

**Output return loss:**  
 RL<sub>out</sub> = -20log(S<sub>22</sub>)dB

**Insertion loss:**  
 IL = -20log(S<sub>21</sub>)dB

The normalized waves have units of  $\sqrt{Watt}$  and are referenced to the system impedance  $Z_o$ , shown by the following mathematical analyses:  
The relationship between U, P and  $Z_o$  can be written as:

$$\frac{u}{\sqrt{Z_o}} = \sqrt{P} = i \cdot \sqrt{Z_o}$$

Substituting:  $\frac{Z_o}{\sqrt{Z_o}} = \sqrt{Z_o}$

$$a_1 = \frac{V_1}{2\sqrt{Z_o}} + \frac{Z_o \cdot i_1}{2\sqrt{Z_o}} = \frac{\sqrt{P_1}}{2} + \frac{Z_o \cdot i_1}{2\sqrt{Z_o}}$$

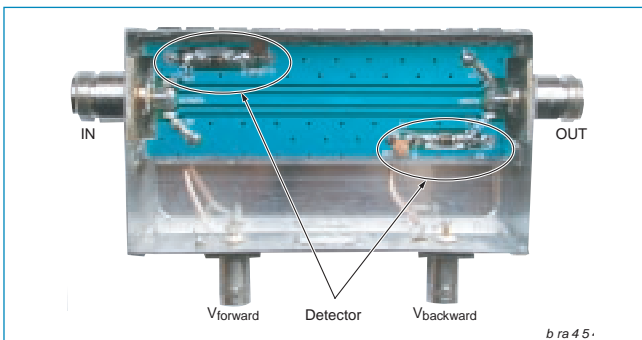
$$a_1 = \frac{\sqrt{P_1}}{2} + \frac{\sqrt{Z_o} \cdot i_1}{2} = \frac{\sqrt{P_1}}{2} + \frac{\sqrt{P_1}}{2} \rightarrow a_1 = \sqrt{P_1} \quad (\rightarrow \text{Unit} = \sqrt{Watt} = \frac{Volt}{\sqrt{Ohm}})$$

Rem:

$$\frac{Z_o}{\sqrt{Z_o}} = \frac{Z_o \cdot \sqrt{Z_o}}{\sqrt{Z_o} \cdot \sqrt{Z_o}} = \frac{Z_o \cdot \sqrt{Z_o}}{Z_o} = \sqrt{Z_o}$$

$$P = U \cdot I = \frac{U^2}{R} \rightarrow \sqrt{P} = \frac{U}{\sqrt{R}} = I \cdot \sqrt{R}$$

Because  $a_1 = \frac{V_{forward}}{\sqrt{Z_o}}$ , the normalized waves can be determined by the measuring the voltage of a forward-traveling wave referenced to the system impedance constant  $\sqrt{Z_o}$ . Directional couplers or VSWR bridges can divide the standing waves into the forward- and backward-traveling voltage wave. (Diode) Detectors convert these waves to the  $V_{forward}$  and  $V_{backward}$  DC voltage. After easy processing of both DC voltages, the VSWR can be read.



A 50Ω VHF-SWR-meter built from a kit (Nuova Elettronica). It consists of three strip-lines. The middle line passes the main signal from the input to the output. The upper and lower strip-lines select a part of the forward and backward traveling waves by special electrical and magnetic cross-coupling. Diode detectors at each coupled strip-line end rectify the power to a DC voltage, which is passed to an external analog circuit for processing and monitoring of the VSWR. Applications include: power antenna match control, PA output power detector, vector voltmeter, vector network analysis, AGC, etc. These kinds of circuit kits are discussed in amateur radio literature and in several RF magazines.

### 2.2.2.1 2-Port Network definition

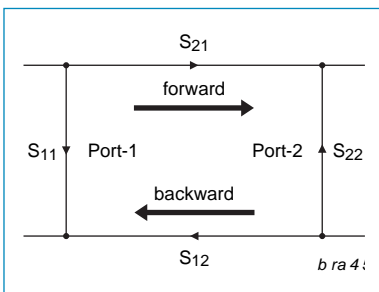


Fig.11: S-parameters in the two-port network

Philips' data sheet parameter **Insertion power gain**

$$|S_{21}|^2: 10dB \cdot \log|S_{21}|^2 = 20dB \cdot \log|S_{21}|$$

#### Input return loss

$$S_{11} = \sqrt{\frac{\text{Power reflected from input port}}{\text{Power available from generator at input port}}}$$

#### Output return loss

$$S_{22} = \sqrt{\frac{\text{Power reflected from output port}}{\text{Power available from generator at output port}}}$$

#### Forward transmission loss (insertion loss)

$$S_{21} = \sqrt{\text{Transducer power gain}}$$

#### Reverse transmission loss (isolation)

$$S_{12} = \sqrt{\text{Reverse transducer power gain}}$$



Example: Calculate the insertion power gain for the **BGA2003** at 100MHz, 450MHz, 1800MHz, and 2400MHz for the bias set-up  $V_{VS-OUT}=2.5V$ ,  $I_{VS-OUT}=10mA$ .

Calculation: Download the s-parameter data file [2\_510A3.S2P] from the Philips website page for the Silicon MMIC amplifier **BGA2003**.

This is a section of the file:

# MHz S MA R 50

! Freq	S11		S21		S12		S22 :	
100	0.58765	-9.43	21.85015	163.96	0.00555	83.961	0.9525	-7.204
400	0.43912	-28.73	16.09626	130.48	0.019843	79.704	0.80026	-22.43
500	0.39966	-32.38	14.27094	123.44	0.023928	79.598	0.75616	-25.24
1800	0.21647	-47.97	4.96451	85.877	0.07832	82.488	0.52249	-46.31
2400	0.18255	-69.08	3.89514	76.801	0.11188	80.224	0.48091	-64

Results:

100MHz ➔  $20\log(21.85015) = 26.8 \text{ dB}$

450MHz ➔  $20dB \log \left| \frac{16.09626e^{130.48^\circ} + 14.27094e^{123.44^\circ}}{2} \right| = 23.6dB$

1800MHz ➔  $20\log(4.96451) = 13.9 \text{ dB}$

2400MHz ➔  $20\log(3.89514) = 11.8 \text{ dB}$

### 2.2.2.2 3-Port Network definition

Typical products for 3-port s-parameters are: directional couplers, power splitters, combiners, and phase splitters.

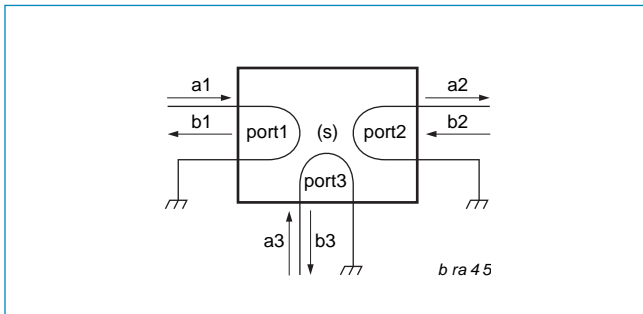


Fig.12: Three-port networks (a) and (b) waves

#### 3-Port s-parameter definition:

##### • Port reflection coefficient / return loss:

Port 1 ➔  $S_{11} = \frac{b_1}{a_1} |_{(a_2=0; a_3=0)}$

Port 2 ➔  $S_{22} = \frac{b_2}{a_2} |_{(a_1=0; a_3=0)}$

Port 3 ➔  $S_{33} = \frac{b_3}{a_3} |_{(a_1=0; a_2=0)}$

##### • Transmission gain:

Port 1=>2 ➔  $S_{21} = \frac{b_2}{a_1} |_{(a_3=0)}$

Port 1=>3 ➔  $S_{31} = \frac{b_3}{a_1} |_{(a_2=0)}$

Port 2=>3 ➔  $S_{32} = \frac{b_3}{a_2} |_{(a_1=0)}$

Port 2=>1 ➔  $S_{12} = \frac{b_1}{a_2} |_{(a_3=0)}$

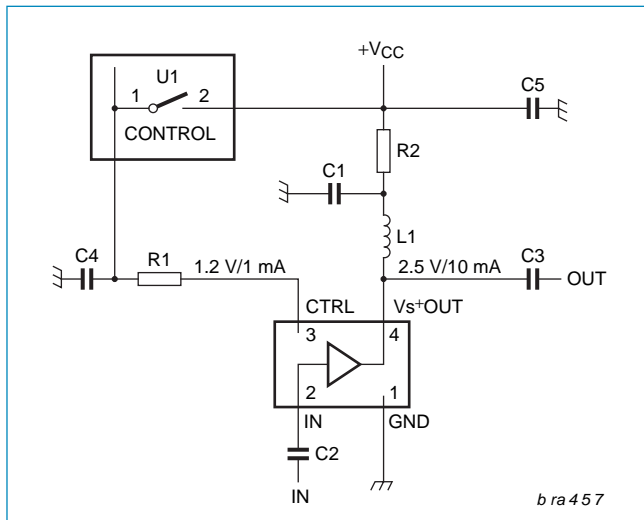
Port 3=>1 ➔  $S_{31} = \frac{b_1}{a_3} |_{(a_2=0)}$

Port 3=>2 ➔  $S_{23} = \frac{b_2}{a_3} |_{(a_1=0)}$

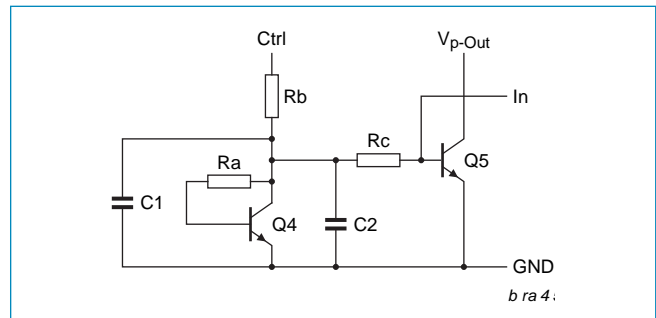
## 2.3 RF amplifier design fundamentals

### 2.3.1 DC bias point adjustment for MMICs

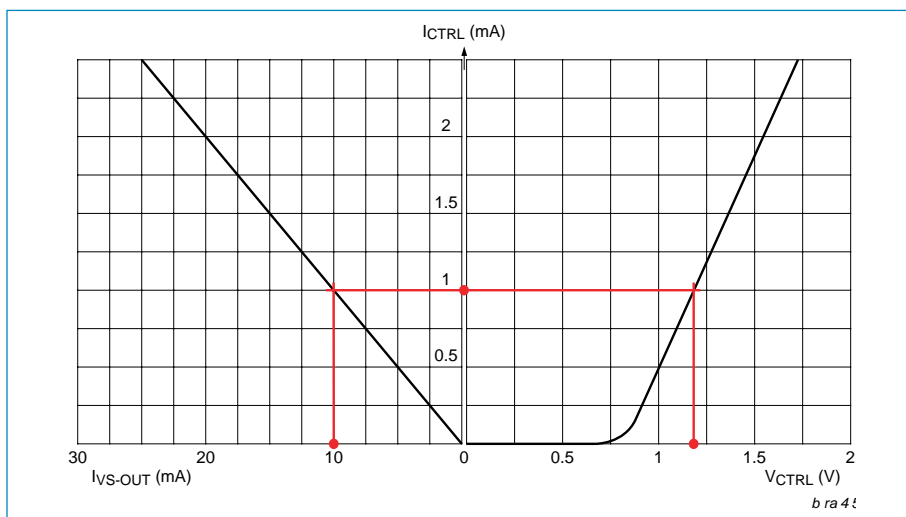
S-parameters are dependent on the bias point and the frequency, as shown in the previous chapter. Consequently, s-parameter files do include the DC bias-setting data. It's recommended to use this setup because the s-parameter will not be valid for a different bias point. An example of DC bias-circuit design is illustrated with the **BGU2003** for  $V_S=2.5V$ ;  $I_S=10mA$ . The supply voltage is chosen to be  $V_{CC}=3V$ .



LNA DC bias setup



**BGA2003** equivalent circuit: Q5 is the main RF transistor. Q4 forms a current mirror with Q5. The input current of this current mirror is determined by the current into Ctrl.pin. Rb limits the current when a control voltage is applied directly to the Ctrl input. Rc, C1, and C2 decouple the bias circuit from the RF input signal. Because Q4 and Q5 are located on the same die, Q5's bias point is very temperature-stable.



DC bias point adjustment for transistors

From the **BGA2003** datasheet, Figs 4 and 5 were combined (see adjacent graph) to better illustrate the MMIC's I/O DC relationship.

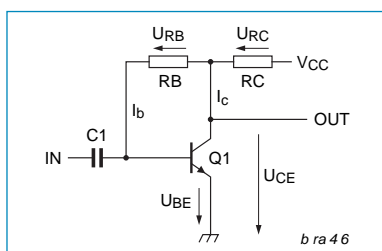
The red line shows the graphical construction starting with the requirement of  $I_{VS-OUT}=10mA$ , automatically crossing the ordinate  $I_{CTRL}=1mA$ , and finishing into the abscissa at  $V_{CTRL}=1.2V$

$$R_2 = \frac{V_{cc} - V_S}{I_{VS-OUT}} = \frac{3V - 2.5V}{10mA} = 50\Omega$$

$$R_1 = \frac{V_{cc} - V_{CTRL}}{I_{VS-OUT}} = \frac{3V - 1.2V}{1mA} = 1.8k\Omega$$

### 2.3.2 DC bias point adjustment for transistors

In contrast to the easy bias setup for MMICs, here is the design of a setup used, for example, in audio or IF amplifiers.



$$h_{FE} = \beta = B = \frac{I_C}{I_b}$$

$$R_C = \frac{V_{CC} - U_{CE}}{I_b + I_C} = \frac{V_{CC} - U_{CE}}{I_C \left( \frac{h_{FE} + 1}{h_{FE}} \right)}$$

$$R_C = \frac{V_{CC} - U_{CE}}{I_C (h_{FE} + 1)} \cdot h_{FE}$$

$$; V_{CC} - I_C \cdot R_C = V_{CE} = I_b \cdot R_B + U_{BE}$$

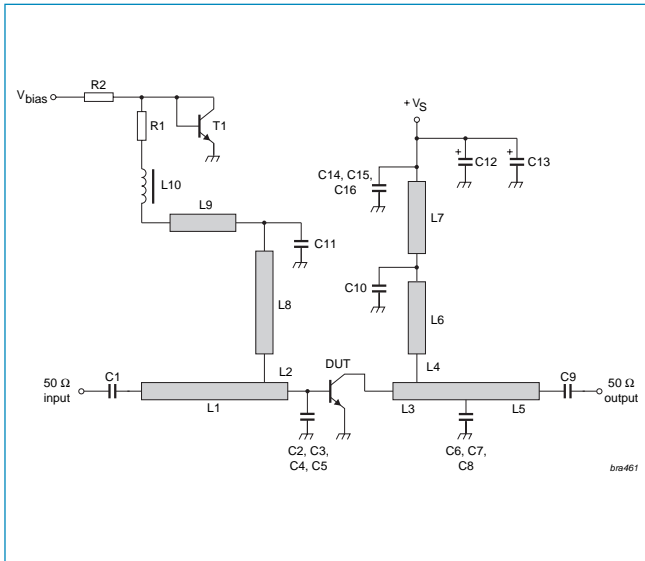
$$R_B = h_{FE} \cdot \left( \frac{V_{CC} - U_{BE}}{I_C} - R_C \right) = h_{FE} \cdot \frac{V_{CE} - V_{BE}}{I_C}$$

**DC bias setup with stabilization via voltage feedback**

The advantage of this setup is a very highly resistive, resistor RB. Its lowering of the input impedance at terminal [IN] can be negated, and the IF-band filter is less loaded. Because there is no emitter feedback resistor, high gain is achieved from Q1. This is needed for narrow bandwidth, high-gain IF amplifiers. The disadvantage is a very low stability of the operating point caused by the Si BE-diodes' relatively linear negative

temperature coefficient of ca.  $V_{BE} \approx -2.5mV/K$  into amplified 
$$I_C = \frac{V_C - V_{BE}}{R_B} \cdot h_{FE}$$

This can be lowered by adding an extra resistor between ground and the emitter.



An emitter resistor has the disadvantage of gain loss or the need for a bypassing capacitor. Additionally, the transistor will loose quality in its gnd performance (instability) and will have an emitter heat sinking into the gnd plane. At medium output power, the bias setup must be stabilized due to the increased junction temperature causing DC drifting. Without stabilization the transistor will burn out or distortion can rise. A possible solution is illustrated in the adjacent picture (BFG10). Comparable to the BGA2003, a current mirror is designed together with the DC transistor T1. T1 works like a diode with a  $V_{CE}$  ( $V_{BE}$ ) drift close to the RF transistor (DUT) in the case of close thermal coupling. With  $\beta_1 = \beta_{DUT}$  and  $V_{BE1} = V_{BE-DUT}$  we can do a very simplified algebraic analysis:

$$V_T \cdot \ln\left(\frac{I_{C-1}}{I_{CO}}\right) \approx V_T \cdot \ln\left(\frac{I_{C-DUT}}{I_{CO}}\right)$$

finalizing into a very temperature-independent relation ship of  $I_{C-DUT} \approx I_{C1} \approx (V_{bias} - V_{BE})/R_2$  For best current imaging, the BE die structure areas should have similar dimensions.

**2.3.3 Gain definitions**

The gain of an amplifier is specified in several ways depending on how the (theoretical) measurement is implemented, on stability conditions, and on way of matching (e.g. best power processing, max. gain, lowest noise figure or a certain stability performance). Often certain power gains are calculated for the upper and lower possible parameter extremes. Additionally calculating circles in the smith chart (power gain circles, stability circles) can be used to select a useful working range in the input or output. The algebraic expressions used can vary from one literature source to the other. In reality S12 cannot be neglected, causing the output being a function of the required source and the input being a function of the required load. This makes matching complicated and is a part of the GA and GP design procedure.

**Transducer power gain:** 
$$G_T = \frac{P_L}{P_{AVS}} = \frac{\text{power delivered to the load}}{\text{power available from the source}}$$

This includes the effect of I/O matching and device gain but doesn't take into account the losses in components.

**Power gain or operating power gain:** 
$$G_P = \frac{P_L}{P_{IN}} = \frac{\text{power delivered to the load}}{\text{power input to the network}}$$

Used in the case of non-negligible S12, GP is independent of the source impedance.

**Available power gain:** 
$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{\text{power available from the network}}{\text{power available from the source}}$$

GA is independent of the load impedance.

**Maximum available gain (MAG):** 
$$MAG = G_{T,max} = 10 \log \left( \frac{|S_{21}|}{|S_{12}|} \cdot \left| K \pm \sqrt{K^2 - 1} \right| \right)$$

The MAG you could ever hope to get from a transistor is under simultaneous conjugated I/O match with a **Rollett stability** factor of  $K > 1$ .  $K$  is calculated from the s-parameters in several sub steps. At a frequency of unconditional stability,  $MAG (G_{T,max} = GP_{,max} = GA_{,max})$  is plotted in transistor data sheets.

**Maximum stable gain:** 
$$MSG = \frac{|S_{21}|}{|S_{12}|}$$

MSG is a figure of merit for a potentially unstable transistor and valid for  $K=1$  (subset of MAG). At a frequency of potential instability, MSG is plotted in transistor data sheets.

**Further examples of used definitions in the design of amplifiers:**

- $G_{T,max}$  = Maximum transducer power gain under simultaneous conjugated match conditions
- $G_{T,min}$  = Minimum transducer power gain under simultaneous conjugated match conditions
- $G_{TU}$  = Unilateral transducer power gain
- $GP_{,min}$  = Minimum operating power gain for potential unstable devices
- Unilateral figure of merit  $\frac{G_T}{G_{TU}}$  determine the error caused by assuming  $S_{12}=0$ .

The adjacent example shows the **BGU2003's** gain as a function of frequency

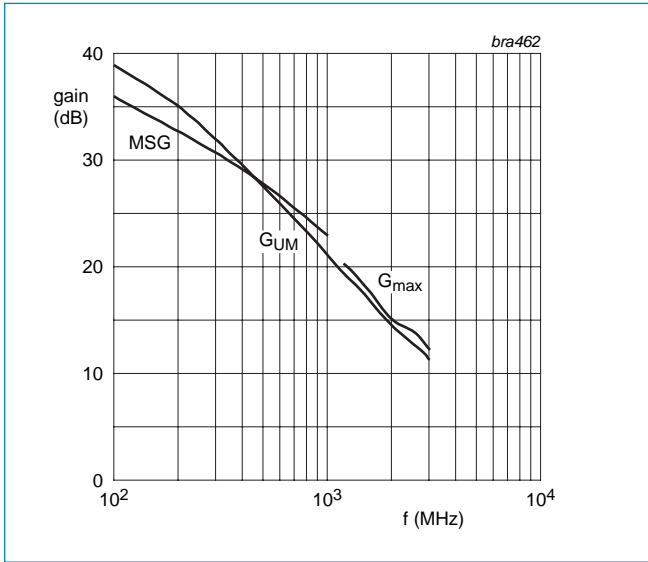
In the frequency range of 100 MHz to 1 GHz the MMIC is potentially unstable. Above 1.2 GHz the MMIC is unconditionally stable (within the 3 GHz range of measurement)

$G_{UM}$  is the maximum unilateral transducer power gain assuming  $S_{12}=0$  and a conjugated I/O match: A  $S_{12}=0$  (=unilateral figure of merit) specify an unilateral 2-port network resulting in  $K=\infty$  and  $D_s=S_{11} \cdot S_{22}$

$$G_{TU,max} = 10 \log \left( \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)} \right) dB$$

$$G_{UM} = 10 \log(G_{TU,max})$$

For further details please refer to books e.g. Pozar, Gonzalez, Bowick, etc.



**2.3.4 Amplifier stability**

All variables must be processed with complex data. The evaluated  $K$ -factor is only valid for the frequency and bias setup for the selected s-parameter quartet  $[S_{11}, S_{12}, S_{21}, S_{22}]$

Determinant: 
$$D_s = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

Rollett stability factor: 
$$K = \frac{1 + |D_s|^2 - |S_{11}|^2 - |S_{22}|^2}{2 \cdot |S_{21} \cdot S_{12}|}$$

In some literature sources, the size of  $D_s$  isn't take into account for dividing into the following stability qualities.

### **$K > 1$ & $|D_s| < 1$**

Unconditionally stable for any combination of source and load impedance

### **$K < 1$**

Potentially unstable and will most likely oscillate with certain combinations of source and load impedance. It does not mean that the transistor will not be useable for the application. It means the transistor is more tricky to use. A simultaneous conjugated match for the I/O isn't possible.

### **$-1 < K < 0$**

Used in oscillator designs

### **$K > 1$ & $|D_s| > 1$**

This potentially unstable transistors with the need  $SWR_{(IN)} = SWR_{(OUT)} = 1$  are not manufactured and do have a gain of  $G_{T,min}$ .

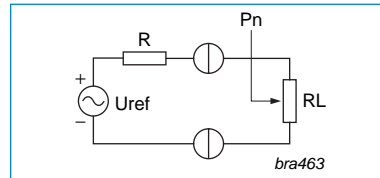
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# 3. Introduction to noise

## 3.1 Definition of the equivalent noise source and noise temperature

In a resistor, a broadband white (Nyquist) noise voltage is caused by an environmental temperature of  $T_0 > 0$  Kelvin. The noise voltage source,  $U_{ref}$  in the circuit diagram, causes the same noise voltage as a heated resistor. Resistor R has the same resistance as the heated one, but is assumed to be noise less.



• The power of this noise voltage source delivered into the load is:

$$P_N = K \cdot T \cdot B \quad 0 \text{ Kelvin} = -273^\circ\text{C}$$

T = Temperature in Kelvin

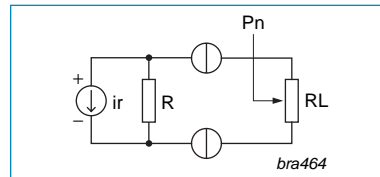
B = The bandwidth

PN = Noise power in the bandwidth B injected into load RL

K =  $1.3806226 \cdot 10^{-23} \text{Ws/K}$  the Boltzmann constant

R = Noise resistance

RL = Load resistor



In the same way, we can define the noise current source:

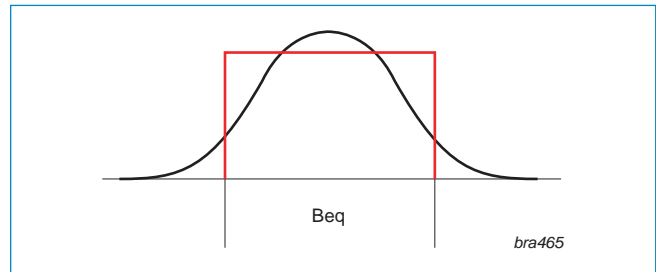
$i_r$  = Noise current source, causing a noise voltage across the parallel noiseless resistor R.

• The statistical power distribution over frequency of thermal noise is constant and called 'white-noise' (Nyquist-noise).

• The noise power is referenced to 1 Hz noise-bandwidth.

The measured system bandwidth must be converted into the rectangular Gauß-Filter [9, p622]:  $B_{noise} \approx 1.2 \cdot B_{(-3dB)}$

Knowing the -6dB bandwidth of a system gives a rough  $B_{noise}$ .



**Equivalent noise bandwidth:**  $B_{noise} \approx B_{(-6dB)}$

• We define the accessory noise factor  $F_z$ :

$$\text{Two-port accessory noise factor: } F_z = F - 1$$

(In some literature, this is used as  $T_z$  in equations involving  $F_z$ )

• In the introduction, we transformed a noisy heated resistor into equivalent noise sources. The noisy resistor has the so-called noise temperature T.

**Noise temperature:**  $T = F \cdot T_0$

$T_0$  = 290 K the absolute reference temperature

([2] SPICE default is  $300.15\text{K} \approx 27^\circ\text{C}$ )

T = The noise temperature of the noisy resistor.

The noise temperature causes a fictive resistor generating a thermal noise power density equal to the former noise source.

**Antenna noise temperature:**

e.g. used in scanning radar antennas 
$$T_{ANT} = \frac{P_{N-ANT}}{K \cdot B}$$

### 3.2 Determine the equivalent noise sources

Normally, power matches are used in RF designs:

$$(1) P = U_{eff} \cdot I_{eff} \quad \text{and} \quad (2) P = U_{eff} \cdot I_{eff}$$

At power match, the voltage delivered to the load is the half of the generator quantity and the delivered current is the half of the shunted source, resulting in the maximum power delivered into the load.

Maximum current  $I_{L(max)}$  (current match) is found at  $R_L=0$  generating

$$P_0 = \frac{U_0^2}{R_g}$$

This  $P_0$  100% grinded in  $R_g$ . Maximum available power from the source

is found at  $R_L=R_g$  with  $P_L = \frac{P_0}{2}$ .  $P_{R_g}=P_0-P_L$  is burned by  $R_g$ .

$$(3) P_L = \frac{P_0}{2} = \frac{U_0}{2} \cdot \frac{I_0}{2} = \frac{1}{4} \cdot \frac{U_0^2}{R} \quad \text{with } U_0 \text{ and } I_0 \text{ in RMS quantity}$$

$$(4) U_0 = \sqrt{4 \cdot P_L \cdot R_L}$$

$$(5) U_L = \frac{U_0}{2} = \frac{\sqrt{4 \cdot P_L \cdot R_L}}{2} = \sqrt{P_L \cdot R_L} \quad \text{RMS load noise voltage; } U_L=0.5 \cdot U_0 \text{ valid for power match}$$

$$(6) \hat{U}_L = \sqrt{2} \cdot U_L$$

$$(7) \hat{U}_L = \sqrt{2 \cdot K \cdot T_N \cdot B \cdot R_L} \quad \text{At power match: } R_N=R_L \text{ consequently } T_N=T_L$$

$$(8) \hat{U}_L = \sqrt{2 \cdot K \cdot T_N \cdot B \cdot R_L} \quad \text{Noise peak-voltage across the load}$$

$$(9) U_0 = \sqrt{4 \cdot K \cdot T_N \cdot B \cdot R_N} \quad \text{RMS voltage of the equivalent noise source's generator}$$

In the introduction, we mentioned  $U_L$  dependence on the shunt, open and matched source. This indicates that the noise available into a two-port is a function of the return loss (load and source impedance relationship). In LNAs, the input impedance must be matched to the equivalent noise-source impedance specified in the datasheets by the characteristic noise-parameters. For cascaded amplifiers, typically the rating of the noise-figure or noise temperature for the ideal (noise matched) condition is given. A mismatch is a noise-source too. This mismatch can be seen as a loss of delivered power into a two-port. Furthermore, loss of power can be caused in e.g. a resistive power attenuator. At such attenuator building blocks, the noise-figure is identical to the attenuation (explained in a later chapter).

The resulting equation (12) is confirmed by [10, p161] but without explanations and algebraic rooting.

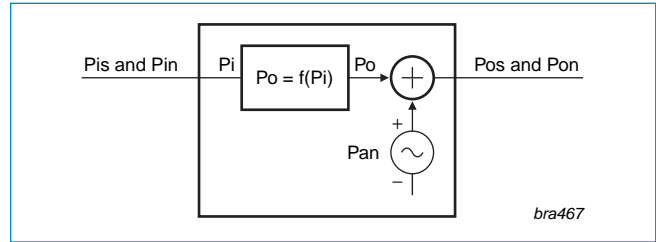
Some literature e.g. for operational amplifiers, uses the unit expressions  $nV/\sqrt{Hz}$  and  $nA/\sqrt{Hz}$

$$\frac{U_0}{\sqrt{B}} = \frac{\sqrt{4 \cdot K \cdot T_N \cdot B \cdot R_N}}{\sqrt{B}} \quad \text{Normalized to a 1 Hz bandwidth gives the bandwidth independent normalized noise voltage quantity:}$$

$$U_0/\sqrt{Hz} = \sqrt{4 \cdot K \cdot T_N \cdot R_N} \quad \text{for simplification the comparison of noise performances measured under different conditions and applications.}$$

### 3.3 Noisy two-port device: the noise figure and SNR

A two-port device (amplifier, attenuator, detector, filter), just loaded with the characteristic impedances at the input and output, generates an outgoing noise power towards the load  $R_L$  without any two-port input signal. This noise power is found at a temperature  $T_0 > 0K$ . Replacing the input-matching resistor by a source shows that this noise power adds to the device's output signal as shown in the diagram. The noise power  $P_{an}$  is caused by the amplifier itself (e.g. semiconductor noise)



There is  $P_{on} = P_{in} \cdot f(PI) + P_{an}$

- $P_{on}$  = Sum of all noise power out of the two-port
- $P_{in}$  = Noise power caused by the input source
- $P_{an}$  = Additional noise power caused by the two-port itself
- $f(PI)$  = Two-port transfer-function (=frequency-dependent gain)

**Noise-Factor:**  $F = \frac{SNR_{(IN)}}{SNR_{(OUT)}} = \frac{\text{Input Signal to Noise Ratio}}{\text{Output Signal to Noise Ratio}}$

**Signal to Noise Ratio:**  $SNR = \frac{P_s}{P_N} = \frac{\text{Signal power on the port}}{\text{Noise power on the port}}$

$$SNR_{(IN)} = \frac{P_{is}}{P_{in}} \qquad SNR_{(OUT)} = \frac{P_{os}}{P_{on}}$$

$$F = \frac{SNR_{(IN)}}{SNR_{(OUT)}} = \frac{\frac{P_{is}}{P_{in}}}{\frac{P_{os}}{P_{on}}} = \frac{P_{is}}{P_{os}} \cdot \frac{P_{on}}{P_{in}} = \frac{P_{on}}{P_{in}} \cdot \frac{G_n}{G_s} \qquad \Rightarrow \qquad G_n = F \cdot G_s$$

- $G_n$  = Noise gain
- $G_s$  = Signal gain

**Noise Figure:**  $NF = 10 \cdot \log(F)$

**Two-port's equivalent noise temperature**

can be found from the noise factor by:  $F = 1 + \frac{T}{T_0}$

vice versus  $T = (F - 1) \cdot T_0 = F_z \cdot T_0$

acc. [23]

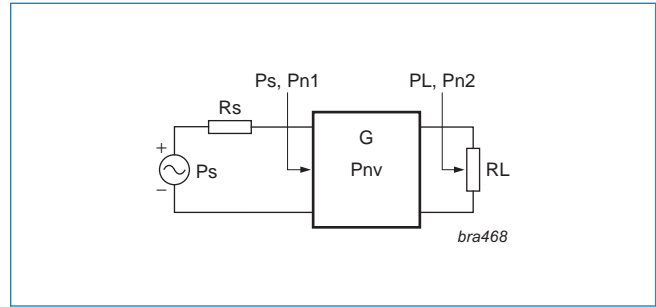
SNR / [dB]	Quality
0	MDS = minimum detectable signal
10	Minimal quality for understanding of voice
20	Good quality of understanding the voice
30	Minimum quality need for music

The expression noise temperature is used in e.g. extremely low noise amplifiers like Radar applications (amplifier; antenna), in cooled CCD-image cameras, in infrared-emission-microscopes (used in failure analysis labs for investigating semiconductors), in infrared cameras, etc. The cooling is made by Peltier elements down to about  $-50^\circ C$  or by liquid nitrogen down to about  $-196^\circ C$ .



### 3.4 Noise Figure terminated by the amplifiers own semiconductor noise

- $P_S$  = Power of a signal generator
- $R_S$  = The generator's source resistance
- $P_{N1}$  = Noise power injected into the amplifier
- $R_L$  = Amplifier's output load
- $P_L$  = The power delivered into the load =  $P_{OUT}$
- $P_{N2}$  = The noise power available from the amplifier output delivered into the load  $R_L$
- $SNR_{(IN)}$  = SNR at the amplifier input
- $SNR_{(OUT)}$  = SNR at the amplifier output
- $B$  = Bandwidth of the amplifier



$$(1) SNR = \frac{S}{N} = \frac{P_{signal}}{P_{noise}}$$

$$(2) F = \frac{SNR_{(IN)}}{SNR_{(OUT)}} = \frac{P_S}{P_{N1}} \cdot \frac{P_{N2}}{P_L} = \frac{1}{G} \cdot \frac{P_{N2}}{P_{N1}} = \text{Input related Noise-Factor according to Friis equation}$$

The amplifier's self-generated output noise power is:  $P_{NV(OUT)} = G \cdot P_{NV}$

with  $P_{NV}$  as the equivalent amplifier self generated input noise power. That means the noisy amplifier with its output noise  $P_{NV(OUT)}$  is replaced by a noiseless two-port (black-box) and a heated resistor  $R_N$  connected to the noiseless box input. This noise-resistor  $R_N$  has the noise temperature  $T_{NV}$  causing an equivalent input noise quantity  $P_{NV}$ . To determine the two-port Noise-Factor  $F$ , the source generator is seen as generating the signal  $P_S$  and a reference-noise power of:

$$(3) P_{NS} = K \cdot T_0 \cdot B \quad P_{NS} \text{ is injected into the two-port input. } P_{N1} = P_{NS} \text{ and (3) into (2) gives:}$$

$$(4) F = \frac{1}{G} \cdot \frac{P_{N2}}{K \cdot T_0 \cdot B}$$

The noise power delivered to the load is the sum of the gained input noise  $P_{N1}$  plus the amplifiers self-generated noise output power quantity  $P_{NV(OUT)}$ .

$$(5) P_{N2} = P_{NV(OUT)} + G \cdot P_{N1} \quad (5) \text{ into (4) gives:}$$

$$(6) G \cdot F = \frac{P_{NV(OUT)} + G \cdot P_{N1}}{K \cdot T_0 \cdot B} \quad (3) \text{ shoot into (6) gives:}$$

$$(7) G \cdot F = \frac{P_{NV(OUT)} + G \cdot K \cdot T_0 \cdot B}{K \cdot T_0 \cdot B} = \frac{P_{NV(OUT)}}{K \cdot T_0 \cdot B} + G$$

$$(8) F = 1 + \frac{P_{NV(OUT)}}{K \cdot T_0 \cdot B} \cdot \frac{1}{G} \quad \text{with } P_{NV(OUT)} = G \cdot P_{NV} \quad F = 1 + \frac{G \cdot K \cdot T_{NV} \cdot B}{K \cdot T_0 \cdot B} \cdot \frac{1}{G}$$

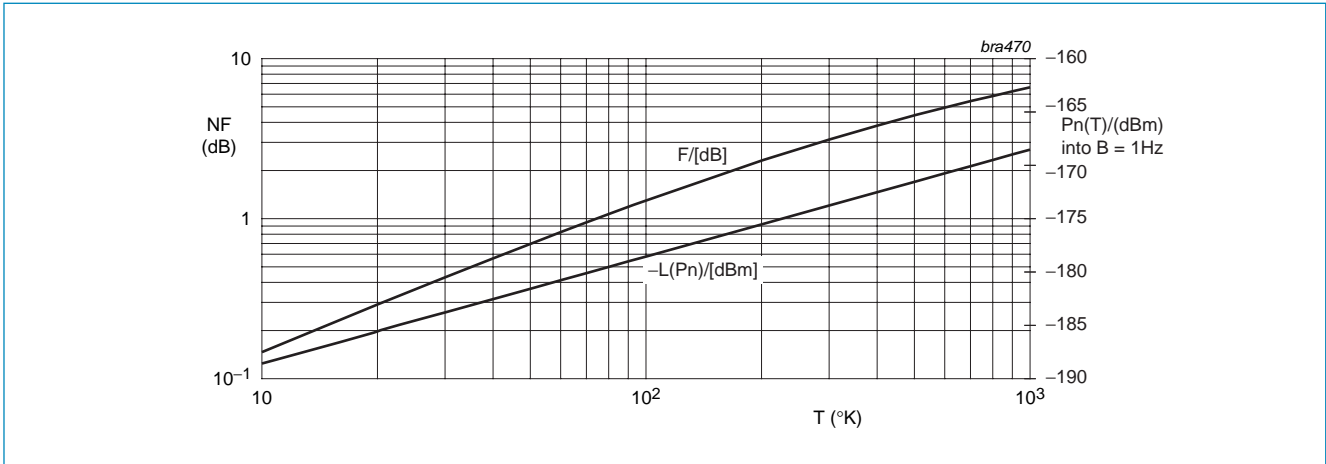
$$F = 1 + \frac{T_{NV}}{T_0} \quad \frac{T_{NV}}{T_0} = F - 1 = F_Z$$

- $T_{NV}$  = Amplifier input related noise temperature
- $T_0$  = 290K = the normal- or reference-temperature

The noise temperature  $T_{NV}$  of a two-port given by a data sheet, is input related.

The noise factor  $F$  (= noise figure  $NF$  expressed in dB) of a two-port, is referenced to the normal temperature  $T_0$  (290 Kelvin).

### 3.5 Noise Figure versus noise temperature

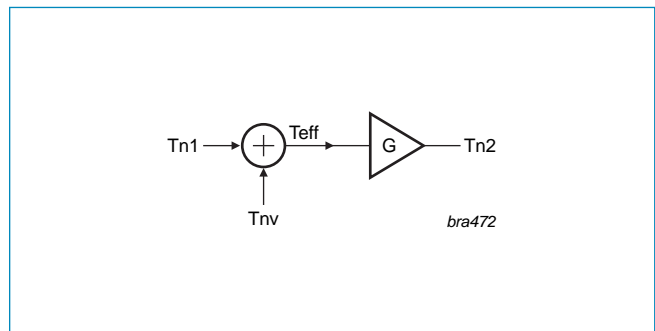
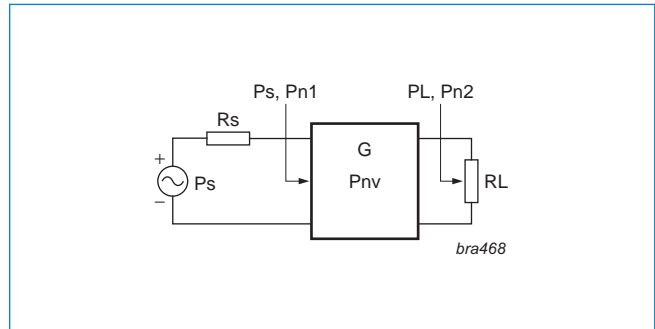


### 3.6 Noise Figure versus noise temperature

In the previous section the output noise power on a noisy two port was evaluated:

- (1)  $P_{N2} = P_{NV(OUT)} + G \cdot P_{N1}$
- (2)  $P_{N2} = P_{NV(OUT)} + G \cdot P_{N1}$       (2) in (1) gives:
- (3)  $P_{N2} = G \cdot P_{NV} + G \cdot P_{N1}$
- (4)  $P_{N2} = G \cdot (P_{NV} + P_{N1}) = G \cdot K \cdot B \cdot (T_{NV} + T_{N1})$
- (5)  $P_{N2} = K \cdot B \cdot T_{N2}$       (5) in (4) gives
- (6)  $K \cdot B \cdot T_{N2} = G \cdot K \cdot B \cdot (T_{NV} + T_{N1})$
- (7)  $T_{N2} = G \cdot (T_{NV} + T_{N1}) = G \cdot T_{eff}$  and  $T_{IN(eff)} = T_{NV} + T_{N1}$

From equation (7) we can see, that the output noise temperature  $T_{N2}$  is the gained effective input noise temperature  $T_{eff}$  (7) and shows that noise temperatures of different sources on the input port can be added. The amplifier noise temperature  $T_{NV}$  is converted into the noise factor by referencing to the normal temperature  $T_{N1} = T_{N0}$ . Please note adding is only allowed with linear quantities. Do not add dBs!



### 3.7 Noise temperature of a lossy device (attenuator, cable etc.)

The attenuator is a two-port with the gain: (1)  $G = \frac{1}{D}$      D= Attenuation factor

Its noise factor is: (2)  $F = \frac{SNR_{(IN)}}{SNR_{(OUT)}} = \frac{1}{G} \cdot \frac{P_{N2}}{P_{N1}}$  (for details refer to previous chapters)

Because the Friis Noise-Factor is referenced to  $T_0$ : (3)  $P_{N1} = K \cdot T_0 \cdot B$

The attenuator is a passive two-port. It does not generate additional pink-noise into the pass-band bandwidth B, as happens in a semiconductor device. Due to its resistive behavior working on the system impedance  $Z_0$ , the output Nyquist noise power is: (4)  $P_{N2} = K \cdot T_{ATN} \cdot B$

(3) and (4) into (2) gives: (5)  $F = \frac{1}{G} \cdot \frac{K \cdot T_{ATN} \cdot B}{K \cdot T_0 \cdot B} = \frac{1}{G} \cdot \frac{T_{ATN}}{T_0}$

**For an attenuator with a temperature  $T_{ATN}=T_0$  follows the noise factor:**

From the definition  $\frac{T_{NV}}{T_0} = F - 1 = F_Z$  is found  $T_{Att} = (D-1) \cdot T_0$

That means e.g. a cable in a system adds white noise, modeled by a noise factor equal to the damping (attenuation) factor D. For example, a filter with 3 dB insertion-loss has a noise figure of NF = 3 dB.

**This behavior can be explained in the following practical way too:**

An ideal signal generator injects a clean signal into the attenuator. This signal generator has the impedance  $Z_0$  heated with the temperature  $T_0$  causing a certain  $SNR_{(IN)}$  referenced to the system  $Z_0$  noise power  $N_0$ . The attenuator drops down the signal power by its attenuation. The attenuator does not create self-noise power but its output is again referenced to  $Z_0$  causing the equal reference noise power  $N_0$ , because only the signal power is changed by the attenuation factor by the same,  $N_0$  at the input and output ( $S_{(in)}=D \cdot S_{(out)}$ ;  $N_{(in)}=N_{(out)}$ )

$$SNR_{(OUT)} = \frac{SNR_{(IN)}}{D} \quad \Rightarrow \quad F = D = \frac{SNR_{(IN)}}{SNR_{(OUT)}} \quad \text{for linear quantity [u]}$$

**At a resistively lossy two-port:**     (6)  $SNR_{(OUT)} = SNR_{(IN)} - \text{Losses}$      for quantity in [dB]

**At a two-port is defined:**     (7)  $SNR_{(OUT)} = SNR_{(IN)} + NF$      for quantity in [dB]

Subtracting (6) - (7) gives again:     **NF=Losses in [dB]**

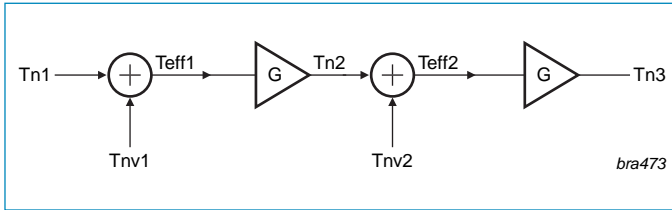
**Cables and attenuators are sources of white noise!**

The problem of noise caused by resistive loss is valid for a lot of circuits, like passive filters, resonators used in oscillators, strip-lines, and so on. In strip-lines, there are frequency-dependent conductive losses and dielectric losses. In some CAD programs, these can be separately defined.

### 3.8 Noise temperature of a resistor

The noise temperature of a resistor is equal to its body temperature. Its noise factor is  $F = \frac{T_{(body)}}{T_0}$

### 3.9 Cascading noisy blocks



$$(1) \quad T_{N2} = G_1 \cdot T_{eff1} = G_1 \cdot (T_{N1} + T_{NV1}) \quad \rightarrow \quad (2) \quad T_{N3} = G_2 \cdot T_{eff2} = G_2 \cdot (T_{N2} + T_{NV2})$$

(1) in (2) gives

$$(3) \quad T_{N3} = G_2 \cdot (G_1 \cdot (T_{N1} + T_{NV1}) + T_{NV2}) \quad \rightarrow \quad (4) \quad T_{N3} = G_2 \cdot G_1 \cdot (T_{N1} + T_{NV1}) + T_{NV2} \cdot G_2$$

$$(5) \quad T_{N3} = G_2 \cdot G_1 \cdot T_{N1} + G_2 \cdot G_1 \cdot T_{NV1} + T_{NV2} \cdot G_2$$

$$(6) \quad T_{N3} = G_2 \cdot G_1 \cdot \left( T_{N1} + T_{NV1} + \frac{T_{NV2}}{G_1} \right)$$

This is equal to an amplifier (eff. gain =  $G_1 \cdot G_2$ ) with its own equal input noise temperature  $T_{NV1} + T_{NV2}/G_1$ .  
 $T_{N1}$  is the reference temperature injected into the cascaded amplifier to determine the Friis Noise factor.

The resulting amplifier noise temperature of a cascaded amplifier results in:

$$(7) \quad T_{NVeff} = T_{NV1} + \frac{T_{NV2}}{G_1} + \frac{T_{NV3}}{G_1 \cdot G_2} + \dots$$

$$(8) \quad T_N = F_Z \cdot T_0 \quad (8) \text{ into } (7) \text{ results in the effective system access noise factor. } T_0 \text{ can be canceled out.}$$

$$(9) \quad F_{Zveff} = F_{ZV1} + \frac{F_{ZNV2}}{G_1} + \frac{F_{ZV3}}{G_1 \cdot G_2} + \dots$$

$$(10) \quad F_Z = F_N - 1 \quad \rightarrow \quad (11) \quad F_{NVeff} - 1 = F_{NV1} - 1 + \frac{F_{NNV2} - 1}{G_1} + \frac{F_{NV3} - 1}{G_1 \cdot G_2} + \dots$$

The resulting effective system noise factor becomes:

$$(12) \quad F_{NVeff} = F_{NV1} + \frac{F_{NNV2} - 1}{G_1} + \frac{F_{NV3} - 1}{G_1 \cdot G_2} + \dots \quad \rightarrow \quad \text{Noise Figure: } (13) \quad NF_{eff} = 10dB \cdot \log(F_{NVeff})$$

### 3.10 Example: a main satellite receiver system design

A receiver system ( $T_{SYS(eff)}$ ) is build by a dish ( $T_{ANT}$ ) with built-in LNA ( $G_1; T_{LNA}$ ), followed by a lossy cable (damping factor  $D_{cabel}=1/G_2$ ; temperature  $T_{cabel}$ ) ending in the SAT receiver ( $G_3; T_{SATR}$ ):

Scheme of cascading noise temperatures: (0)  $T_{NVeff} = T_{NV1} + \frac{T_{NV2}}{G_1} + \frac{T_{NV3}}{G_1 \cdot G_2} + \dots$   
 Applied to the present case:

$$(1) \quad T_{SYS(eff)} = T_{ANT} + T_{LNA} + \frac{T_{cabel}}{G_1} + \frac{T_{SATR}}{G_1 \cdot \frac{1}{D_{cabel}}} \quad (T_{ANT}=T_{N1} \text{ from the previous section})$$

$$(2) \quad T_{SYS(eff)} = T_{ANT} + T_{LNA} + \frac{T_{cabel}}{G_1} + \frac{T_{SATR}}{G_1 \cdot \frac{1}{D_{cabel}}}$$

Noise factor of the cable is (3)  $T_{cabel} = T_0 \cdot (D_{cabel} - 1)$

$$(4) \quad D_{cabel} = 10^{\left(\frac{\text{Loss in dB}}{10dB}\right)}$$

The resulting input-related noise temperature of the satellite system is:

$$(5) \quad T_{SYS(eff)} = T_{ANT} + T_{LNA} + \frac{T_0 \cdot (D_{cabel} - 1)}{G_1} + \frac{D_{cabel} \cdot T_{SATR}}{G_1} \quad (\text{in linear quantity [u]})$$

The effective system noise figure is: (6)  $F_{SYS(eff)} = 1 + \frac{T_{SYS(eff)}}{T_0}$  (antenna dish included)

For a certain allowed maximum bit error rate BER (demodulated signal) at the baseband processor output, the equivalent min baseband  $SNR_{(SATRBB)}$  can be determined. The relationship for BER versus SNR depends on the modulation used. The  $SNR_{(ANT)}$  at the dish input must be better by at least the factor  $F_{SYS(eff)}$ .

$$(7) \quad SNR_{(ANT)} = F_{SYS(eff)} \cdot SNR_{(SATBB)} \quad (\text{in linear quantity [u]})$$

The effective noise power at the SAT-dish can be determined by:

$$(8) \quad P_{N(ANT)} = K \cdot B \cdot T_{SYS(eff)}$$

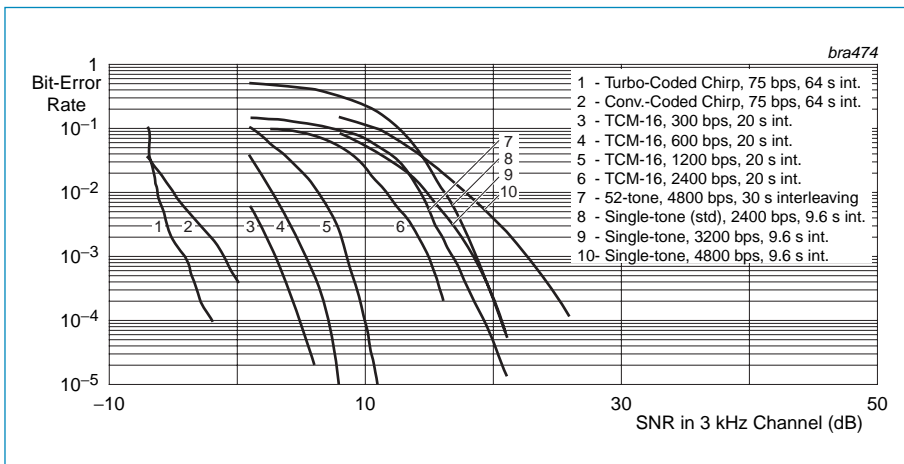
The min. signal for operation is then easily found by:

$$(9) \quad P_{Sant(min)} = SNR_{(ANT)} \cdot P_{N(ANT)}$$

An antenna signal power of  $>P_{Sant(min)}$  ensures the min. BER in the SAT-receiver's baseband processor, and this quantity appears to be the SAT-system sensitivity for the requested BER.

The level of the noise floor at the baseband processor output is given by: (10)  $P_{Nflor(BB)} = \frac{P_{Sant(min)}}{SNR_{(SATBB)}} \cdot \frac{G_1 \cdot G_3}{D}$

In [25, p8] the BER is given as a function of SNR by ... 'the Defense Science and Technology Organization (DSTO) to support the Modernized High Frequency Communications System (MHFCS) [1], (also referred to by its project nomenclature JP2043), to be built to serve the Australian Defense Force.'... 'This work covers a wide range of topics including characterization of expected HF noise and channel distortion, waveform design, protocol design, radio access scheme design, provision of HF Internet services, overall system design, and modeling and simulation of end-user service performance. ...'



### 3.11 Antenna noise

Antenna noise is sometimes called sky noise. The antenna receives noise from several sources [20, p5]:

- Terrestrial noise (man-made noise) sources
- Solar noise sources
- Galactic sources
- Noise caused by the antenna radiation food impedance

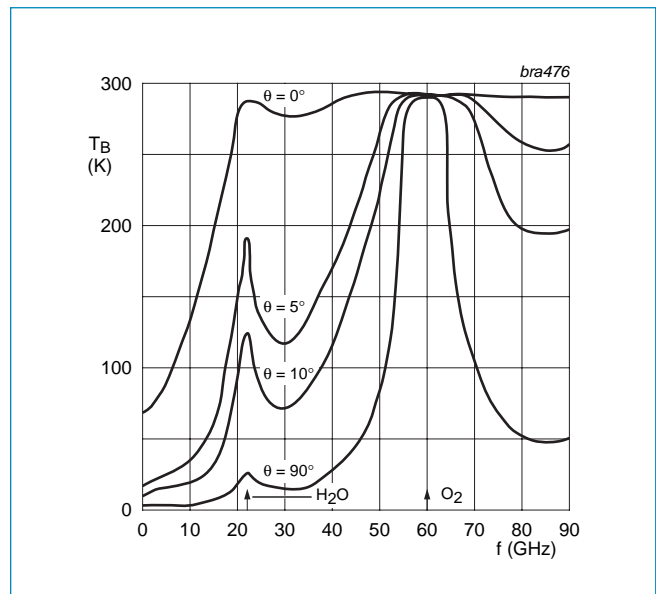
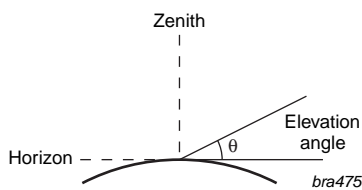
The size of the noise source depends on the antenna elevation angle, time of day, sun activity, and the frequency. These noises are modeled as an increased thermal-noise temperature of the antenna.

Frequency Range	Sky temperature	Root cause
30KHz - 300KHz	>108K	Very high atmospheric noise
300KHz - 3MHz	>108K	High atmospheric noise
3MHz - 30MHz	108 - 105K	Atmospheric noise
30MHz - 300MHz	105 - 103K	High Galactic noise
300MHz - 3GHz	103 - 10K	Galactic noise and cosmic background noise
3GHz - 30GHz	10 - 100K	Atmospheric thermal noise, O <sub>2</sub> , H <sub>2</sub> O resonance
<30MHz		Noise due to lightning discharges or 'atmospherics'
30MHz to 1GHz		Galactic or cosmic noise
1GHz to 10GHz		Noise is generated in the atmosphere. A vertical antenna will receive less noise than a horizontal antenna. The sky noise temperature can approach the minimum of 3K set by cosmic background radiation (relic of the 'Big Bang')
2GHz to 8GHz		The low-noise window used in radio-telescopes and space telemetry
>10GHz		The noise temperature rises in peaks due to resonance effects in water vapor and oxygen molecules (O <sub>2</sub> H <sub>2</sub> O resonance), finally reaching a steady value of around 290Kelvin.

[21, p2] says 'antennas radiate broadband 'blackbody' noise corresponding to their surface temperature. If the beam of an antenna is narrower than the noise', it 'sees' the background with noise temperature  $T_b=290k$ . A satellite dish aimed at the earth surface only receiving the earth's surface black body noise will have an antenna temperature of  $T_{ANT}=290K$ . If the antenna's beam loop sees only the earth's noise, the effective antenna temperature is the rated share sum of all responsible temperature noise sources in the main loop:

$$T_{ANT(eff)} = k_1 \cdot T_b + k_2 \cdot T_{SKY} + \dots$$

Example for TSKY is given in the upper table.



The International Telecommunication Union (ITU) has defined (in the CCIR report 322) the frequency dependent atmospheric interferences and (in CCIR report 258-4) man-made noise.

For further details see at e.g.:

<http://www.veron.nl/tech/noise/noiserefs.htm>

<http://www.spawar.navy.mil/sti/publications/pubs/td/2813/nradtd2813homepg.html>

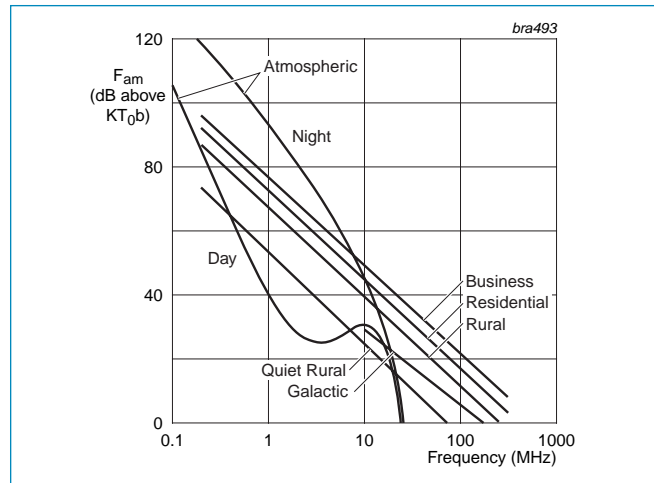
According to [24, p5-6]: The statistical behavior of the antenna noise factor,  $F_a$ , can be shown by plotting the distribution on a normal probability graph where random variables that are Gaussian distributed form a straight line with a slope equal to its standard deviation and a median equal to its mean. The graph is used to determine the median antenna noise figure  $F_{am}$  of a rural, residential, or business environment. Further analysis of  $F_a$  includes determining within-the-hour-, hour-to-hour-, and location-to-location-variability.'

$$F_a = F - F_r - 1$$

$F$  is the measured noise factor

$F_r$  is the receiver noise factor

$F_a$  is the antenna noise factor

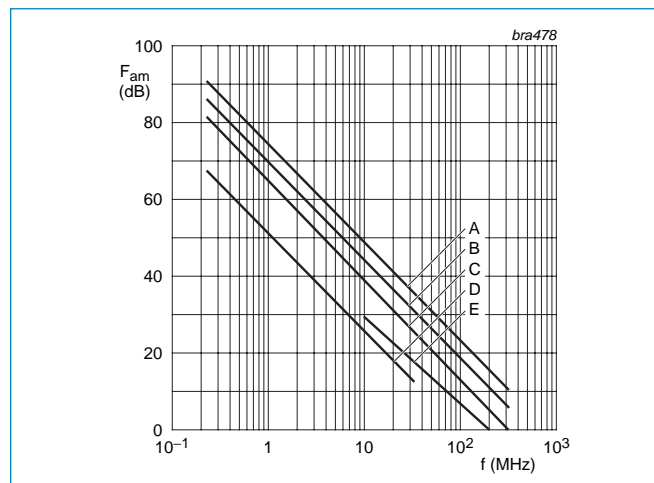


Adjacent is the average interference power produced by man-made radiation, received by a short vertical antenna with ideal GND earth [26, p5] (acc. ITU-R P.372-7 'Radio noise', Figure 10)

$$F_{am} = c - d \cdot \log(f)$$

$F_{am}$  in dB

$f$  in MHz



Environment	c	d
Business area (trace A)	76.8	27.7
Residual area (trace B)	72.5	27.7
Countrified area (trace C)	67.2	27.7
Calm countrified area (trace D)	53.6	28.6
Galactic noise (trace E)	52	23

### 3.12 Example: A radar system

Antenna:  $T_Q=350k$   
 Receiver:  $T_V=380k$   
 Bandwidth:  $BW=1MHz$   
 Gain:  $G=100dB$   
 Baseband:  $SNR_{(OUT)}=10dB$

Effective system input related temperature:  $T_{SYS} = T_Q + T_V = 350K + 380K = 730K$

The noise temperatures can be un-weighted added, because  $T_q$  is a source.

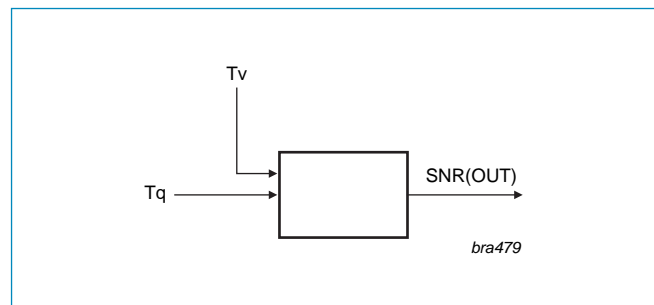
$$F = \frac{1}{G} \cdot \frac{P_{N2}}{P_{M1}}$$

$$G \cdot F = \frac{G \cdot K \cdot B \cdot T_q + G \cdot K \cdot B \cdot T_v}{K \cdot B \cdot T_q}$$

$$F = 1 + \frac{T_v}{T_q} = \frac{T_{SYS}}{T_q}$$

$T_v$ =Amplifier

$T_q$ =Antenna



$$F = \frac{730K}{350K} = 2.09$$

$$SNR_{(IN)} = F \cdot SNR_{(OUT)} = 2.09 \cdot 10 = 20.9 \quad \text{■ } 13.2\text{dB}$$

$$F_{IN} = \frac{P_{sig}}{P_n} \quad P_n = K \cdot T_{sys} \cdot BW = 1.381 \cdot 10^{-23\text{Ws/K}} \cdot 730K \cdot 1\text{MHz} = 10.08 \cdot 10^{-15}\text{W}$$

$$P_{sig} = P_n \cdot F_{IN} = 10.08 \cdot 10^{-15}\text{W} \cdot 20.9 = 210 \cdot 10^{-15}\text{W} \quad \text{■ } -96.8\text{dBm}$$

$$P_{sig(OUT)} = P_{sig(IN)} \cdot G = -96.8\text{dBm} + 100\text{dB} = -3.2\text{dBm}$$

### 3.13 Input and output related noise temperature

Available is the effective input noise temperature  $T_{eff}$  of an amplifier. Its  $T_{eff}$  is the sum of the source noise temperature  $T_{source}$  and the amplifier itself generate equivalent input noise temperature  $T_v$ .

$$F = \frac{T_{eff(IN)}}{T_{source}} = \frac{T_{source} + T_v}{T_{source}} = 1 + \frac{T_v}{T_{source}}$$

$T_{eff}$  is the sum of all system related noise sources referenced to the noise free gain block.

This gain block was made noise free by using the equivalent input related noise temperature  $T_v$ .

The output related effective noise temperature is:

$$T_{eff(OUT)} = G \cdot T_{eff(IN)}$$

for the equivalent gain block with  $NF=0\text{dB}$ . The effective output noise power is:

$$P_{Neff(OUT)} = K \cdot T_{eff(IN)} \cdot B \cdot G$$

with  $G$  as the gain of the amplifier.

### 3.14 Amplifier sourced by a noisy generator

$$T_{eff} = T_g + T_v$$

$$F_{sys} = \frac{T_{eff}}{T_g} = 1 + \frac{T_v}{T_g}$$

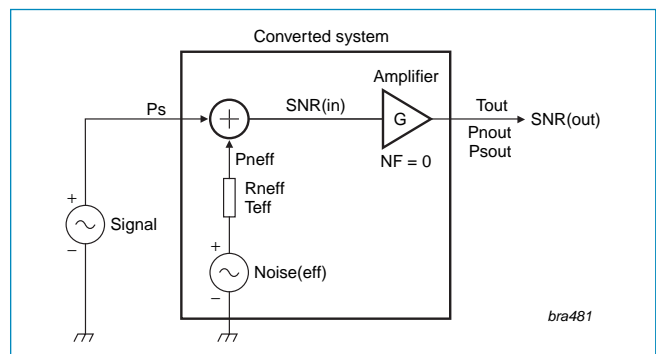
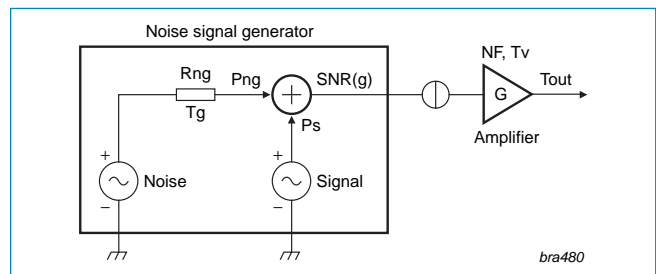
$$T_{OUT} = G \cdot T_{eff}$$

$$SNR_{(IN)} = \frac{P_{S(IN)}}{P_{Neff(IN)}}$$

$$P_{Nout} = G \cdot (K \cdot B \cdot T_{eff})$$

$$SNR_{(OUT)} = \frac{G \cdot P_{S(IN)}}{P_{Nout}} = \frac{G \cdot P_{S(IN)}}{G \cdot (K \cdot B \cdot T_{eff})}$$

$$SNR_{(OUT)} = \frac{P_{S(IN)}}{K \cdot B \cdot T_{eff}}$$





### 3.15 Noise Figure, noise temperature & sensitivity of a receiver

The equivalent noise temperature of a two-port (e.g. receiver) is  $T_e$ . The Noise factor  $F$  of the two-port (four pole device) is determined by referring to the reference temperature (room temperature) of typically 290K (sometimes 300k).

$$(1) \quad F = 1 + \frac{T_e}{T_0} \qquad (2) \quad NF = 10dB \cdot \log(F)$$

$T_e$  is the temperature of the theoretically resistor causing the same two-port output noise power than the noisy two-port itself. The noise power into 1 Hz bandwidth is:

$$P_{N(1Hz)} = k \cdot T = 1.3806226 \cdot 10^{-23} Ws/K \cdot 290K = 4 \cdot 10^{-18} mW \qquad \approx \quad -174dBm$$

A receiver with a noise figure  $NF$  (in dB) does have an equivalent input related noise power of:

$$P_{NiRec} / dBm = -174dBm + 10dB \cdot \log(B_{noise} / 1Hz) + NF / dB$$

This is called the receivers noise floor. The term ' + NF ' in the equation comes from the SNR's definition:

$$SNR_{(IN)} = SNR_{(OUT)} + NF$$

A minimum detectable signal MDS must at least break through the receivers noise floor:  $MDS = P_{NiRec}$

$$MDS / dBm = -174dBm + 10dB \cdot \log(B_{noise} / 1Hz) + NF / dB$$

Third order intermodulation-free dynamic range, IMD3, is the range where a signal can be processed and detected without distortion:

$$SFDR = IMD3 = IP3 - MDS \qquad \text{in dBm}$$

In analog to digital converters (ADCs) the term spurious free dynamic range (SFDR) is used too. The noise floor in ADCs is evaluated from the binary resolution terminated quasi-random quantization noise. That means an ADC does have a noise figure too. This  $NF$  causes an ADC output signal to noise relation  $SNR_{(OUT)} = SNR_{(IN)} - NF$  in dB. This is used for receiver system analysis with a digital baseband processing DSP. The ADC can either sit behind the demodulator or (at modern designs) in the end of the IF rail replacing the analog demodulator.

In some literature, the term noise floor power density (NFPD) is used. That's the equivalent noise temperature terminated receivers noise floor in a theoretically 1 Hz wide Gauß-filter.

$$NFPD = -174dBm + NF \qquad \text{in dBm}$$

$$NFPD = MDS - 10 \log(B_{noise}) \qquad \text{in dBm}$$

### 3.16 Noise sources in semiconductor devices

The noise source can be defined by the noise figure:

$$\text{Noise voltage source: } F = \frac{\overline{U_r^2}}{4 \cdot k \cdot T \cdot B \cdot R} \qquad \text{Noise current source: } F = \frac{\overline{I_r^2} \cdot R}{4 \cdot k \cdot T \cdot B}$$

$U_r^2$  and  $I_r^2$  are the square quantity of the equivalent noise sources [2].

$R$  is the noise source's generator resistance.

$u$  and  $i$  come out of the relation:  $P_N = \bar{i} \cdot \bar{u} = k \cdot T \cdot B$  with  $\bar{u} = \bar{i} \cdot R$  we do get

$$P_N = \bar{i} \cdot \bar{u} = \bar{i}^2 \cdot R = k \cdot T \cdot B$$

The amount of noise power delivered into the load is different, because of power matching the current and the voltage; each one is the half quantity:

$$P_{an} = \frac{i_n}{2} \cdot \frac{u_n}{2} = \frac{1}{4} \cdot i_n \cdot u_n$$

**Noise source current:**  $\bar{i} = \sqrt{\frac{4 \cdot k \cdot T}{R}} \cdot \sqrt{B}$

**Noise source voltage:**  $\bar{u} = \sqrt{4 \cdot k \cdot T \cdot R} \cdot \sqrt{B}$

**Effective noise voltage:**  $\bar{v}_n^2$  unit [V<sup>2</sup>]

**Power noise spectral density:**  $\frac{P_{an}}{B}$

**Voltage noise spectral density:**  $S_{nv} = \frac{\bar{v}_n^2}{B} = 4 \cdot K \cdot T \cdot R$  unit [V<sup>2</sup>/Hz] [5]

**Equivalent noise voltage:**  $E_{nv} = \sqrt{S_{nv}} = \sqrt{\frac{\bar{v}_n^2}{B}}$  unit [V/<sup>√</sup>Hz] [5]

**Current noise spectral density:**  $S_{mi} = \frac{\bar{i}_n^2}{B} = \frac{\left(\bar{i} = \sqrt{\frac{4 \cdot K \cdot T}{R}} \cdot \sqrt{B}\right)^2}{B} = \frac{4 \cdot K \cdot T \cdot B}{B} = 4 \cdot K \cdot T \cdot \frac{1}{R}$

**Noise spectra:**  $C_{i(f)}$

**Shot noise:**  $C_{i(f)}^{NF} \approx 2 \cdot e \cdot I$  NF = model parameter

**Shot noise current:**  $I_N = \sqrt{2 \cdot q \cdot I_D \cdot B}$  caused in a PN junction [33, p42]

**Short noise voltage:**  $V_N = I_N \cdot r_{PN} = I_N \cdot \frac{K \cdot T}{q \cdot I_E} \approx \frac{I_N}{g_m}$  for  $r_{PN}$  ref. to [34, p77]  
 $r_{PN}$ =dynamic junction impedance (base emitter diode)

**Electron charge:**  $e=q=1.6 \cdot 10^{-19}$ Coulombs;  $1C=1J/Kelvin$

**Transistor forward transconductance:**  $g_m = \frac{q \cdot I_C}{K \cdot T} = \frac{I_C}{I_E \cdot r_{PN}} = \frac{h_{fe}}{r_{PN} \cdot (1 + h_{fe})} \approx \frac{1}{r_{PN}}$  (small signal):

**Generation-recombination noise:**  $C_{(f)} = KB \cdot \frac{I^{AB}}{1 + \left(\frac{f}{FB}\right)^2}$  KB, AB, FB = model parameters

The generation-recombination noise is alternatively called combination-, recombination-, burst- or popcorn-noise.

**Flicker noise:**  $C_{\left(\frac{1}{f}\right)} = KF \cdot \frac{I^{AF}}{f^b}$  AF, KF, b=model parameter 'B'  
 Normally B=1

Sometimes the Shot noise and the Flicker noise are called excess noise.

**Flicker noise current:**  $I_N = \sqrt{m \cdot \frac{I^{AF}}{f^b} \cdot B}$  [34, p44-45]

Flicker Noise = 1/f noise

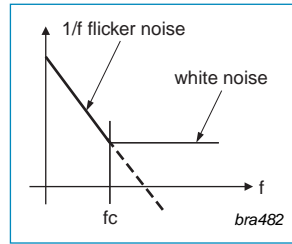
A coarse model of the heterodyned 1<sup>st</sup> order flick noise (1/f-noise) and broadband white noise versus frequency is given by [10, p149]:

$$P_{\Sigma} = P_{N(white)} + P_{N(flicker)} \approx K \cdot T \cdot B \cdot \left( 1 + \frac{f_c}{f} \right)$$

$f_c$  = Flicker noise corner frequency

**White noise is:**  $P_{N(white)} = K \cdot T \cdot B$

The diagram shows the noise power versus frequency for e.g. a transistor. In oscillators this noise distribution envelopes the carrier.



More accurate modeling is used by a higher order polynomial power density equation [10, p149]:

$$F_{SC} = F_{SemicNoise} \approx \frac{1}{\delta f^0} + \frac{1}{\delta f^1} + \frac{1}{\delta f^2} + \frac{1}{\delta f^3} + \frac{1}{\delta f^4}$$

$$F_{SC} \approx 1 + \frac{1}{\delta f^1} + \frac{1}{\delta f^2} + \frac{1}{\delta f^3} + \frac{1}{\delta f^4}$$

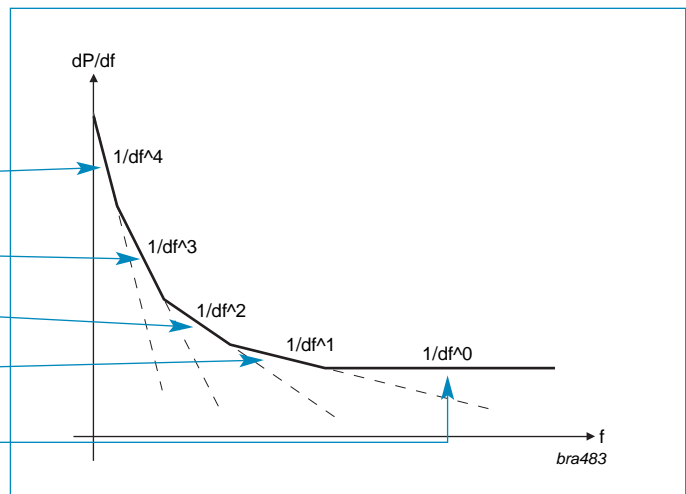
**FM random walk noise:** (1/δf<sup>4</sup>)

**FM flicker noise:** (1/δf<sup>3</sup>)

**Random walk noise & white FM noise:** (1/δf<sup>2</sup>)

**PM Flicker noise:** (1/δf<sup>1</sup>)

**White Nyquist noise floor:** (1/δf<sup>0</sup>)



**Root cause of the different noise envelope selections:**

PM noise in a resonator causes an FM-modulation with a 1/f<sup>2</sup> frequency response. This 1/f<sup>2</sup> FM modulation can be converted back into a PM modulation with 1/f<sup>3</sup> trace. Temperature instability and microphonic (mechanical) noise cause the so-called random walk noise with 1/f<sup>2</sup> noise sidebands. This can be converted into 1/f<sup>3</sup> FM noise.

According to [19, p250-252]

Spectrum envelope tangent	Name	Root cause
1/f <sup>4</sup>	Random walk FM	External physical environment influencing the oscillator: mechanical shock, vibration, temperature, ...
1/f <sup>3</sup>	Flicker FM	Physical resonance mechanism of the active oscillator, the design or choice of parts used for electronic, power supply or environmental properties. In high quality oscillators marked by 1/f <sup>2</sup> or 1/f noise.
1/f <sup>2</sup>	White FM	White frequency, random walk of frequency. Noise in passive-resonator frequency standards like cesium and rubidium frequency standards
1/f <sup>1</sup>	Flicker φM	Flicker modulation of phase. Common in high-quality oscillators. Introduced by next-stage noisy electronics. Amplifiers for gain the oscillator carrier or multiplier.
1/f <sup>0</sup>	White φM	Produced in the same way like Flicker φM. The late amplifier stage is usually responsible. Narrow-band filter at the output can help.

### 3.17 Frequency range of the noise contributions

Noise type	Root cause of the noise phenomena	Frequency range	Envelope of the noise spectra
White- (Thermal-) (Nyquist-)	Random movement of thermal charged carriers in resistive elements		constant
Pink-	Frequency dependent $P_N$		$F(f)$
Shot- (Schottky-)	DC current causes random space charge areas with individual current impulses, due to random transport of charge carriers. From [33, p42] caused by the quantum nature of electron flow through a potential barrier (PN junction).	$10^4\text{Hz}$ to $10^9\text{Hz}$	$F \sim I_C$ $\approx \text{white}$
Popcorn- (Burst-)	Random generation / regeneration of carriers and fluctuation between different semiconductor potentials	below 100Hz	$F=1/f^2$
Flicker- 1/f- (Contact-) (Johnson-)	Random recombination effects at defects in the semiconductor crystal barred; the borders of diffusion areas or material surface	100Hz to 1KHz most interesting up to $\approx 10\text{Hz}$ to 100Hz	$F=1/f$
Barkhausen-	Noise in DC biased magnetic substances [11, p61]		
Avalanche- [18]	Created by a PN junction operating in the reverse breakdown mode. Electrons with a very high kinetic energy collide with crystal atoms to produce electron holes.		like shot noise but higher intensity

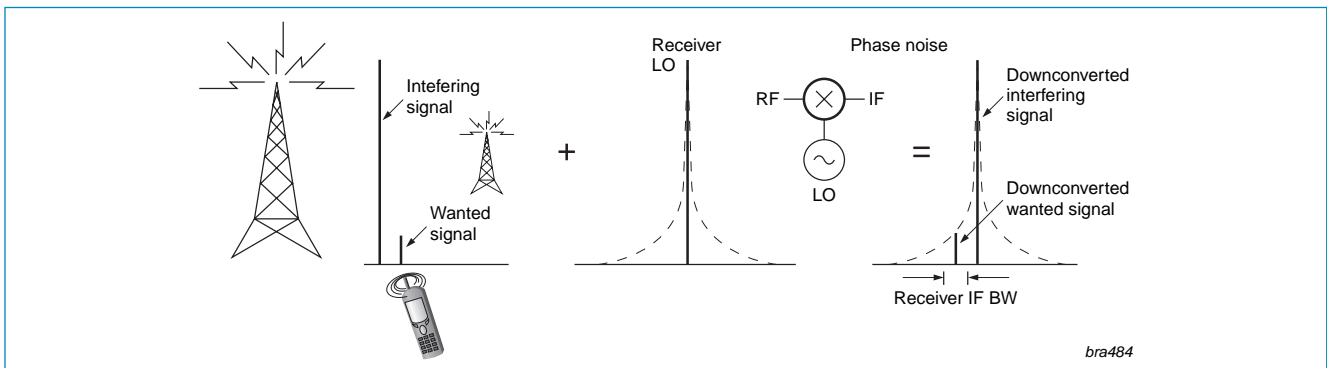
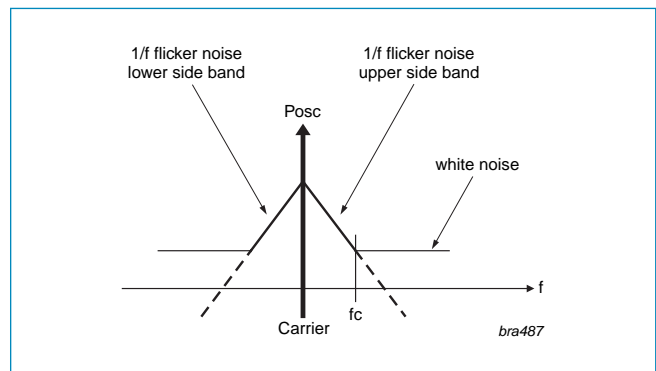
The accessory noise factor is used to model a theoretically noiseless two-port model. This model has an input signal from the source and an input noise source causing the same output noise power as the former noise two-port.

### 3.18 Sideband noise in oscillators and mixers

In oscillators and mixers, the single sideband SSB noise and the double sideband DSB noise can be measured. In reality, both sidebands don't need to be equivalent.

#### 1/f (Flicker-Noise) Corner Frequency $f_c$ :

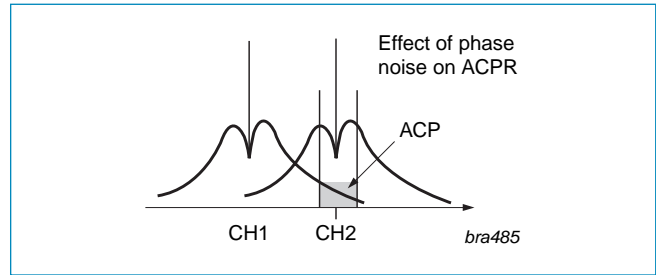
Found in op amps (Si transistors), about 10Hz to 100Hz. Crossing the  $f_c$  border will show that the 1/f Noise ends in the white Nyquist noise floor. The corner frequency is dependent on the Fab-process, application conditions, temperature and DC bias [34, p46]. At MOS  $f_c \sim \frac{1}{\text{Channel length}}$ , III-V group devices like GaAs and IGaP have a much larger  $f_c$  than Si devices. Consequently, Philips 5<sup>th</sup> (e.g. **BFG425W**) and 4.5<sup>th</sup> (e.g. **BFG325W**) family Si-WBT are preferred for use in microwave oscillators. That's because the close-to-the-carrier sideband-dominant flicker noise is much better in double-poly Si transistors than in III-V semiconductor products.  $f_{c(Si)} \ll f_{c(III-V)}$ . A good synthesizer can particularly improve the sideband noise enveloping the oscillator carrier. This partially depends on the PLL filter bandwidth.



The influence of the oscillator's sideband noise on the receiver selectivity is shown in the figure above [35, p2].

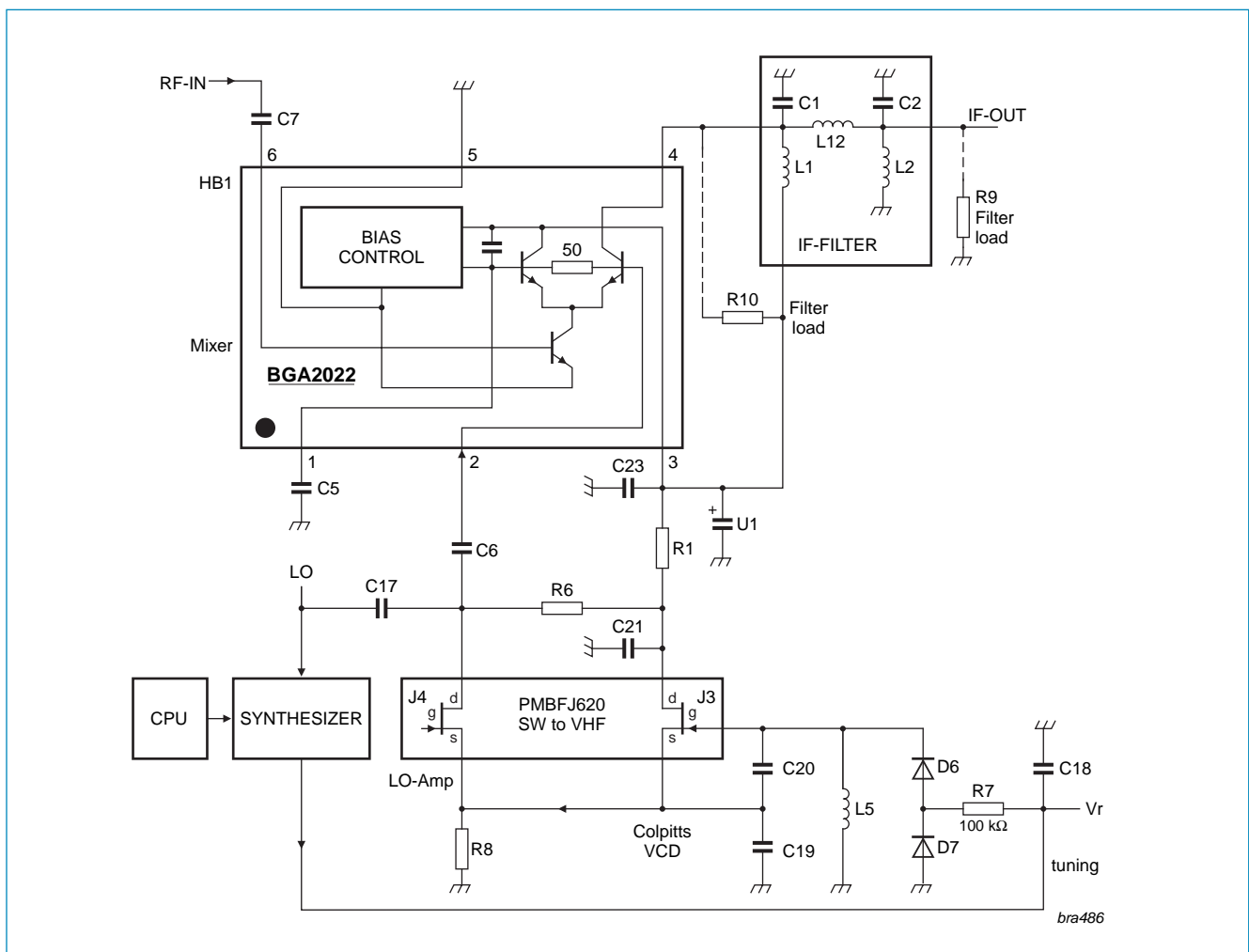
A clean oscillator carrier will only cause the converted wanted signals to pass the IF filter. Due to sideband noise, the unwanted signal is converted into the IF band too. The sideband noise causes jitter with the consequence of increased BER.

In transmitter oscillators, the carrier noise causes channel power injection into the next closed channel, as illustrated [35, p2].



Mixers and oscillators can be built with our dual gate MOSFETs, JFETs., MMICs and RF transistors. As an example, Philips Semiconductors offers the MMIC Mixer **BGA2022** (half balanced structure).

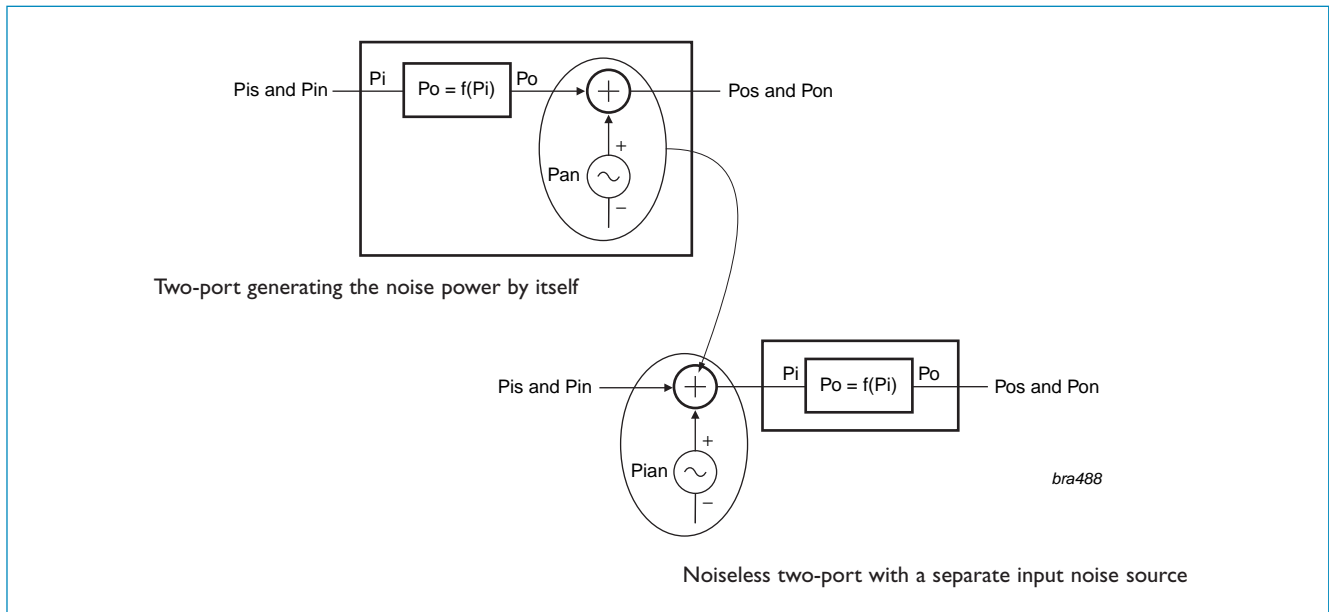
Adjacent is an idea for a mixer with a varicap-tuned JFET VCO. This example uses the **PMBFJ620**, which can be used in various applications from DC through audio into the VHF range.



The oscillator J3 works in the classical common Darin-Colpitts circuit, with low output impedance. The LO buffer and amplifier J4 work in a common gate circuit like a differential amplifier circuit. Common source would be possible too. Both FETs are self-biasing with  $V_{GS} < 0V$ . The symmetrically D6, D7 Varactor tuning circuit reduces harmonics, drift by DC rectifying, temperature effects and offers a tuning voltage range down to  $V_r = 0V$ . Eventually a diode will be needed from Gate-J3 to GND. The tapped C19 - C20 - L5 transformer resonator sets the positive feedback. The C19, C20 are parallel to D6, D7 and shrink the frequency variation range. To reduce phase noise, the Varactor voltage tuning range should be maximum. A parallel capacitor reduces the required frequency variation range for L5, and C19, C20, CGS and CGD the parallel quantity for J3. At very high frequencies, a Hartley configuration (taped L) is interesting because it allows the removal of R8 — a potential source for Nyquist noise.

A passive highly-linear double-balanced mixer (DBM) based on two **PMBF1620** JFETs is shown. Because  $V_{DS}=0$ , the FETs work in the linear (resistive) mode. The LO amplitude must drive the FETs clear into the switch mode. Applications include up- and down- conversion mixers, phase comparators, and frequency doublers. The Baluns Tx1 to Tx3 are used to convert the none-balanced signal into an balanced one and for impedance transformation. If a balanced signal (without DC) is available, it can be directly connected to the FET-quartet without the affected Balun. For further improved IP3, several DBMs can be combined. Because this is a passive mixer, it has insertion loss IL. The Noise Figure approximately equals the IL and because there is no DC supply current, the amount of semiconductor Shot-Noise is reduced.

### 3.19 Equivalent input related noise source



The equivalent noise power of the two-port is:  $P_N = K \cdot T \cdot B$

$$P_N = K \cdot T \cdot B \quad \text{with } K = 1.3806226 \cdot 10^{-23} \text{Ws/k}$$

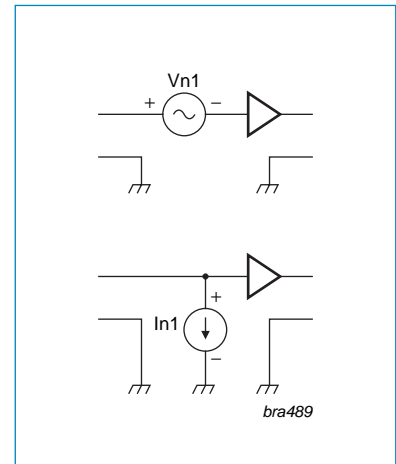
$$\frac{P_N}{1mW} = 1.3806226 \cdot 10^{-23} \text{Ws/K} \cdot 300 \text{Kelvin} \cdot B \cdot \frac{1}{1mW} \quad |10 \log()$$

$$\frac{P_N}{dBm} = -174 \text{ dBm/Hz} + 10 \log(B_{noise} / \text{Hz})$$

Input related equivalent noise floor of a two-port:

$$\frac{P_{Ni}}{dBm} = -174 \text{ dBm/Hz} + 10 \log(B_{noise} / \text{Hz}) + NF / dB$$

This relationship is often used in receivers and spectrum analyzers for specifying the sensitivity of these systems.



The minimum power of a signal for demodulation must at least break through the noise floor. This quantity is the so-called Minimum

$$\text{Detectable Signal MDS [10, p118]. } MDS = -174 \text{ dBm/Hz} + 10 \log(B_{noise} / \text{Hz}) + NF / dB$$

$$\text{The two-tone dynamic range of a receiver is [12, p113]: } DR / dB = \frac{2}{3} (IP3 / dBm - MDS / dBm)$$

This DR equation shows that the dynamic range lower limit is determined by the receiver sensitivity and the upper limit is caused by distortions.

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# 4. Performance of cascaded RF blocks

## 4.1 Receiver dynamic range

The minimum power of a signal needed for demodulation must break through the noise floor [10, p118]. This input signal quantity is the so called

**Minimum Detectable Signal:** 
$$MDS/dBm = -174 \text{ dBm/Hz} + 10 \log(B_{noise}/\text{Hz}) + NF/dB$$

**Two tone dynamic range of a receiver:** 
$$DR/dB = \frac{2}{3} (IP3/dBm - MDS/dBm) \quad [12, p113]$$

## 4.2 Cascaded gain

n = running index of the involved gain affecting stage

**System gain:** 
$$G_{system} = \sum_1^{x=n} G_X \quad (\text{Quantities in dB})$$

## 4.3 Cascaded noise

All factors in linear quantity [u].

Example Amplifier-1:

Gain: G1 in [u]

Gain: L1 in [dB]

Noise Temperature: T<sub>N1</sub> in [Kelvin]

Noise Figure: NF<sub>1</sub> in [dB]

Noise Factor: F<sub>N1</sub> in [u]

**System noise temperature:** 
$$T_{Neff} = T_{N1} + \frac{T_{N2}}{G_1} + \frac{T_{N3}}{G_1 \cdot G_2} + \dots$$
  
with  $T_N = F_Z \cdot T_0$

**System access noise factor:** 
$$F_{Zeff} = F_{Z1} + \frac{F_{Z2}}{G_1} + \frac{F_{Z3}}{G_1 \cdot G_2} + \dots$$
  
with  $F_Z = F_N - 1$

**System noise factor:** 
$$F_{Neff} = F_{N1} + \frac{F_{N2} - 1}{G_1} + \frac{F_{N3} - 1}{G_1 \cdot G_2} + \dots$$

**System noise figure:** 
$$NF_{eff} = 10dB \cdot \log(F_{Neff})$$

**System gain:** 
$$G_{eff} = G_1 \cdot G_2 \cdot G_3 \cdot \dots \quad \text{in [u]}$$

$$L_{eff} = L_1 + L_2 + L_3 + \dots \quad \text{in [dB]}$$

The algebraic root cause is shown in the chapter 'Cascading of noise specified devices'



## 4.4 Cascaded intermodulation

References: [1], [2, p120], [3, p2], [4, p1], [5, p1, p24-25], [9], [11, p6]

Third order intermodulation products are caused by two signal sources,  $f_1$  and  $f_2$ , interfering on a nonlinear transfer function and a device's nonlinear inputs. This transfer function can be successively approximated by an  $n^{\text{th}}$  order Taylor series polynomial:

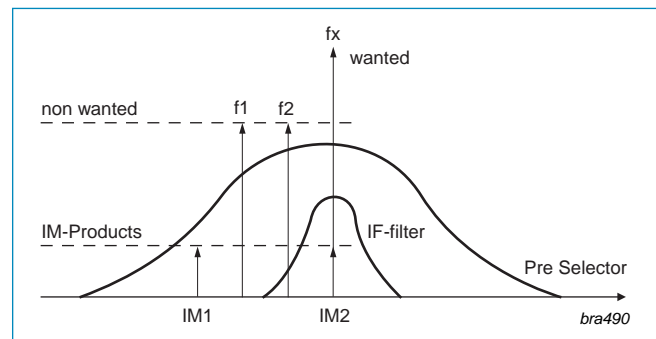
$V_O = k_1V_i + k_2V_i^2 + k_3V_i^3 + k_4V_i^4 + \dots$  An applied single tone will be converted into a simple be filtered out mesh of harmonics. Two applied signals start interfering (mixing). On the nonlinear transmission function's cubic term  $k_3$ , new 3<sup>rd</sup> order harmonic signals (intermodulation products) are generated with the frequencies:  $2 \cdot f_1 + f_2$ ;  $2 \cdot f_1 - f_2$ ;  $2 \cdot f_2 + f_1$ ;  $2 \cdot f_2 - f_1$ ;  $3f_1$ ;  $3f_2$  e.g.:  $3 \cdot f_2$  is the third harmonic of  $f_2$ . The two-tone intermodulation product's frequencies relation is given by:  $M \cdot f_1 \pm N \cdot f_2$

with  $M, N = 0, 1, 2, 3, \dots$  The final order of the distortion product is: e.g. at  $2 \cdot f_1 + f_2$  is  $M=2$  and  $N=1 \Rightarrow M+N=3 \equiv \text{THD} = 3^{\text{rd}}$  order harmonic distortions. The difference signals  $2 \cdot f_1 - f_2$  and  $2 \cdot f_2 - f_1$  ( $M+N=3$ ) are most dangerous, because the effective mixed-out difference frequency signals are very close to the original tones. Due to this, the THD passes the receiver's filters carrying the information of both original signals. The principle function of a mixer is based on the interfering of two signals (LO and RF) on the second order quadratic term  $k_2$  [15, p235-236]. That means a mixer should have an infinitely long quadratic transfer function causing the intermodulation products:  $f_1 \pm f_2$ ;  $2 \cdot f_2$  and  $2 \cdot f_1$ . So, a mixer can be used as a frequency doubler by sourcing LO and RF input ports with the same signal. A phase-shift between both will cause an output DC offset used in e.g. PLL phase detectors.

Example:

Input signals	$f_2 - f_1$	$\text{IM}_1 = 2 \cdot f_1 - f_2$	$\text{IM}_2 = 2 \cdot f_2 - f_1$	$f_1 - \text{IM}_1$	$\text{IM}_2 - f_2$
$f_1 = 99.95\text{MHz}$	100KHz	99.85MHz	100.15MHz	100KHz	100KHz
$f_2 = 100.05\text{MHz}$					
$f_x = 100.15\text{MHz}$			➔ !		

Two tones  $f_1, f_2$  with a difference of  $\Delta f$  causes 3<sup>rd</sup> order IM signals in a distance of  $\Delta f$  to each tone  $f_1, f_2$ . In the example, the wanted signal  $f_x$  and the non-wanted signals  $f_1, f_2$  passes the pre-selection filter.  $f_1$  and  $f_2$  caused 3rd order IM products  $\text{IM}_1$  and  $\text{IM}_2$  in the front-end amplifier. The problem:  $\text{IM}_2$  is heterodyning the wanted signal  $f_x$  with the information of  $f_1$  and  $f_2$  carriers. A THD distortion product like  $\text{IM}_2$  falling into the IF pass-band can no longer be filtered out. This signal must be rejected through demodulation processing gain, limiter suppression (FM-systems) or digital error correction algorithms.



[15] Increasing the input tones by 1dB will cause the 2<sup>nd</sup> order IMDs rising by 2dB and the 3<sup>rd</sup> order IMDs by 3dB. In general [2, p121]: with  $n = \text{order of the } \Delta \text{IM}$ ,  $\Delta P_{IN}$  the change of fundamental tone in dB and  $\Delta \text{IM}_n$  change of the  $n^{\text{th}}$  order IM in dB. In high-linearity devices, like Philips' **BAP70** family PIN-diodes, there is a need to take care and test equipment self-generated harmonics and distortions. In a cascade of amplifiers of e.g. three amplifiers, the last (3<sup>rd</sup>) amplifier will start clipping first. By further increasing the system input power, the 2<sup>nd</sup> amplifier starts clipping and by total input overload, the 1<sup>st</sup> amplifier clips. Because the last amplifier of the rail is the first overdrive one, its  $\text{IP}_3$  quality has the primary degree of responsibility on the effective system linearity.

The 3<sup>rd</sup> order intercept point can be input related ( $i\text{IP}_3$ ) and output related ( $o\text{IP}_3$ ):  $o\text{IP}_3 = i\text{IP}_3 + \text{Gain}$

$n^{\text{th}}$  order input intercept point:  $\text{NOI} = i\text{IP}_n = P_{IN} + \frac{\Delta \text{IM}_n}{n-1}$  all quantities in dBm and dB.

Example 3<sup>rd</sup> order:  $\text{IP}_3 = 10\text{dBm} \cdot \left( \frac{\text{IP}_3/[W]}{1\text{mW}} \right)$

**Cascaded input related IP3:** 
$$\frac{1}{iIP3_{eff}} = \frac{1}{iIP3_{(1)}} + \frac{G_1}{iIP3_{(2)}} + \frac{G_1 \cdot G_2}{iIP3_{(3)}} + \dots$$

The first amplifier's iIP3 is unit weighted. Each amplifiers iIP3 coming more close to the end is higher weighted by multiplying with the gain factor of the former blocks.

**Cascaded output related IP3:** 
$$\frac{1}{oIP3_{eff}} = \frac{1}{oIP3_{(1)} \cdot G_{(2)} \cdot \dots \cdot G_{(n)}} + \frac{1}{oIP3_{(2)} \cdot G_{(3)} \cdot \dots \cdot G_{(n)}} + \dots + \frac{1}{oIP3_{(n-1)} \cdot G_{(n)}} + \frac{1}{oIP3_{(n)}}$$

The equation shows the output IMD of the first amplifier is amplified by all following amplifiers. This happens at each following amplifier. At the final cascaded output, all IMD signals are heterodyning.

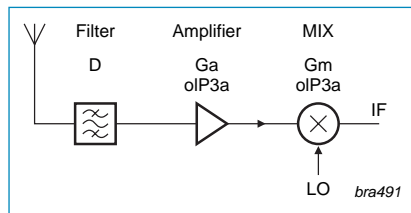
Example: 
$$\frac{1}{oIP3_{eff}} = \frac{1}{oIP3_{(1)} \cdot G_{(2)} \cdot G_{(3)}} + \frac{1}{oIP3_{(2)} \cdot G_{(3)}} + \frac{1}{oIP3_{(3)}}$$

Note: The shown equations are only valid for in-phase (coherent) heterodyning intermodulation products.

**Conclusion:**

- The gain and noise figure of the first amplifier determines the system's noise performance. The higher the gain of the first one, and lower its noise, the better the overall noise system performance.
- The last amplifiers 3<sup>rd</sup> order intermodulation determines the system's IP3. The lower the IP3 and lower gain of all amplifiers, the better the overall linearity system performance for a certain input power level.
- Both conclusions can be applied to cascaded systems from e.g. audio up to microwave applications.
- Mixers are multiplying devices with a quadratic transfer function cause 2<sup>nd</sup> order intermodulation.
- Two input tones applied on nonlinear device causes problematic 3<sup>rd</sup> order difference intermodulation products very close to the originator tones.

**Example: a front-end**



D is the preselector (image rejection filter) pass-band insertion loss.

$$\frac{1}{oIP3_{eff}} = \frac{1}{oIP3_{(F)} \cdot G_{(a)} \cdot G_{(m)}} + \frac{1}{oIP3_{(a)} \cdot G_{(m)}} + \frac{1}{oIP3_{(m)}}$$

Preselector output 3<sup>rd</sup> order intercept point: oIP3(f)=∞

In CAD simulation something like oIP3=40 or 50dBm is used.

$$oIP3_{eff} = \frac{1}{\frac{1}{oIP3_{(a)} \cdot G_{(m)}} + \frac{1}{oIP3_{(m)}}$$

$$\frac{1}{iIP3_{eff}} = \frac{1}{iIP3_{(F)}} + \frac{G_{(D)}}{iIP3_{(a)}} + \frac{G_{(D)} \cdot G_{(a)}}{iIP3_{(m)}} \approx G_{(D)} \cdot \left( \frac{1}{iIP3_{(a)}} + \frac{G_{(a)}}{iIP3_{(m)}} \right)$$

$$iIP3_{eff} = D \cdot \left( \frac{1}{iIP3_{(a)}} + \frac{G_{(a)}}{iIP3_{(m)}} \right)^{-1}$$

**Example:** An input attenuator in front of the front-end does increase the input related IP3 by its attenuation factor. This relationship can be used by measurement with e.g. spectrum analyzers. It is used in shortwave receivers e.g. the actual received man-made noise causes intermodulation problems. The disadvantage is an increase of the noise floor proportional to the attenuation factor.

Out-of-phase (incoherent) heterodyning intermodulation products are added by the discrete quadratic quantities below the overall summing square root [5, p1, p24-25].

$$iIP3_{eff} = D \cdot \left( \left[ \frac{1}{iIP3_{(a)}} \right]^2 + \left[ \frac{G_{(a)}}{iIP3_{(m)}} \right]^2 \right)^{-0.5}$$

Working with voltage quantities instead of measured power will change the equations' form [6, p21]:

$$\text{Cascaded noise factor: } F_{Neff} = F_{N1} + \frac{F_{N2} - 1}{G_1^2} + \frac{F_{N3} - 1}{G_1^2 \cdot G_2^2} + \dots$$

$$\text{Cascaded input IP3: } \frac{1}{iIP3_{eff}} = \frac{1}{iIP3_{(1)}} + \frac{G_1^2}{iIP3_{(2)}} + \frac{G_1^2 \cdot G_2^2}{iIP3_{(3)}} + \dots$$

## 4.5 Cascaded compression

A signal reaching the 1dB input compression point  $P_{L1}$  will compress the receiver front-end and lower the gain for the wanted signal. This desensitizing of the receiver causes a loss in sensitivity and in limiter margin at FM and PM based demodulators and distortions in AM demodulators. As shown before, the gain has a positive effect on reducing the system's noise figure and sensitivity (MDS). On the other side, a larger gain causes the RF blocks located at end of the rail to clip much earlier. A trade-off between noise figure and input intercept point must be made in receiver. As an example, low noise amplifiers have relatively low supply current. Increasing the transistor collector current improves its linearity but raises the proportion of Shot noise. The front-end linearity must be designed to handle input signals as large as necessary and as low-noise as necessary for the worse case in the application. The range between these specification limit borders is the so-called intermodulation free dynamic range DR. In analog to digital converters, as used for digitizing the IF for processing in the following DSP, there is spoken from the spurious free dynamic range SFDR. Offense this noise and linearity spec limits will cause problems. For example, the design may have great sensitivity and transmission distance for small signals, but when the transmitter and receiver get close, they will not work because of saturation. Alternatively, the front-end can fight every income signal but work only close to the transmitter. A solution is the use of high-linearity devices in the front-end rail, like the **BGA6589**, and use of a voltage-controlled variable gain amplifier **BGA2031/1**. This VGA can be a block within the automatic gain loop (AGC) to prevent saturation at high antenna field-strength signals.

Because there is a linear approximation relationship between the 1dB compression point and the IP3 of  $oIP3 = oPL1dB + 10.63dB$ , the form of the IP3 can be converted into the cascaded PL1 [11, p6].

$$\text{Linear to log conversion: } P_{L1} = 10dBm \cdot \log\left(\frac{P_{L1}/W}{1mW}\right)$$

$$\text{Cascaded input related } iP_{L1}: \frac{1}{iP_{L1_{eff}}} = \frac{1}{iP_{L1(1)}} + \frac{G_1}{iP_{L1(2)}} + \frac{G_1 \cdot G_2}{iP_{L1(3)}} + \dots \quad \text{linear qty}$$

Using the next by shown I/O compression relation can determine the  $oP_{L1}$  from the cascaded IP3.

$$\text{Output 1dB compression point: } oP_{L1} = iP_{L1} + Gain \quad \text{logarithmic quantity [dB] and [dBm]}$$

$$oP_{L1} = iP_{L1} \cdot Gain \quad \text{linear quantity [W] and [u]}$$

$$\text{Cascaded output related } oP_{L1}: \frac{1}{oP_{L1_{eff}}} = \frac{G_1}{oP_{L1(1)}} + \frac{G_1 \cdot G_2}{oP_{L1(2)}} + \frac{G_1 \cdot G_2 \cdot G_3}{oP_{L1(3)}} + \dots \quad \text{linear qty}$$

## 4.6 Transmission distance

There are several ways of increasing the transmission distance in a wireless system:

- Better antenna (gain, beam, etc.)
- Higher sensitivity of the receiver (MDS, noise floor, used modulation, demodulator efficiency)
- Higher output power at the transmitter
- Other operation frequency
- Improved front-end selectivity (filter)
- Improved front-end linearity (PL1, IP3)
- Improved noise-figure (LNA gain and NF)

This chapter discusses increasing the transmission distance by using an additional gain block [14, p5] based on the theory of an isotropic antenna (3D homogenous round around field radiation by an ideal spherical dot). The following law describes the theoretical power-density of damped traveling waves, radiated to the reference-isotropic antenna at a certain distance:

$$(1) P_{E(r)} = P_S \cdot A_E \cdot \frac{1}{4\pi \cdot r^2} \cdot e^{-\chi \cdot r}$$

$P_{E(r)}$	= Receiver power after the distance 'r' to the transmitter's isotropic antenna
r	= Distance receiver-transmitter
$P_S$	= Transmitter power
c	= Atmospheric attenuation exponent
$A_E$	= Receiver antenna surface

The law is used in all kinds of spherical wave and energy radiation topics like in optics, acoustics, thermal, electromagnetic and so on. The antenna is power matching the cable impedance (50Ω, 75Ω, ...) to the space's impedance with the (ideal) electromagnetic far-field impedance of  $120 \cdot \pi \cdot \Omega$ . The received normalized power/unit area  $P_r$  at the receiver, transmitted with the power  $P_t$ , from distance d, and neglecting atmospheric attenuation ( $\chi=0$ ) is:

$$(2) P_{RX} = \frac{P_{TX}}{4\pi \cdot d^2}$$

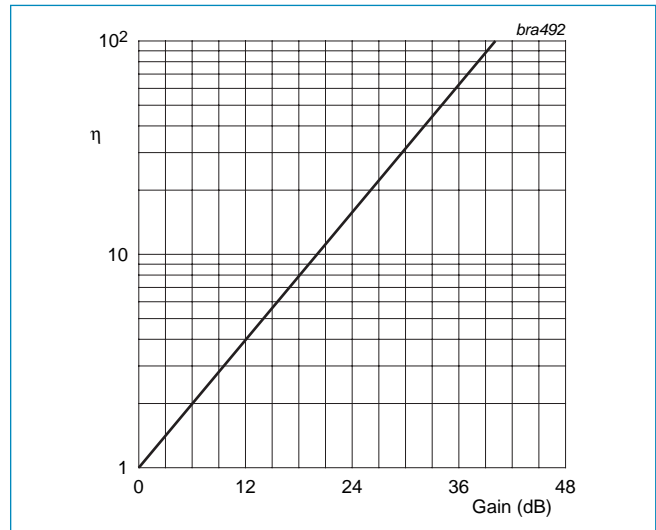
$$\text{TX-RX-distance relation: (3) } d = \sqrt{\frac{P_t}{4\pi \cdot P_r}} \quad (\chi=0)$$

$$\text{Without extra PA relation: (4) } d_1 = \sqrt{\frac{P_{t1}}{4\pi \cdot P_r}}$$

Expanded distance by the extra PA with the same received RX power:

$$(5) d_2 = \sqrt{\frac{P_{t2}}{4\pi \cdot P_r}}$$

$$(6) \eta = \frac{d_2}{d_1} = \frac{\sqrt{\frac{P_{t2}}{4\pi \cdot P_r}}}{\sqrt{\frac{P_{t1}}{4\pi \cdot P_r}}} = \sqrt{\frac{P_{t2}}{P_{t1}}} \quad \eta = \sqrt{|Gain|^2}$$



Using an extra gain block, like our medium power MMICs **BGA6289**, **BGA6489** or **BGA6589**, increases the actual transmission distance by the factor of h, assuming no compression of the amplifiers and an isotropic antenna radiator. In reality we have to take into account the amplifier input/output matching circuits.

## 4.7 Example: transmission distance limited by frequency and receiver quality

A receiver with a sensitivity of  $0.1\mu\text{V}$  for 20dB SNR ( $=S/N$ ) uses an antenna with effective surface of  $3\text{ cm}^2$ , 10 MHz. Determine the necessary transmitter power for 1000 km distance neglecting the effects of atmospheric ionization anomalies, atmospheric attenuation and free space propagation.

$$P_{IN} = \frac{V_{IN}^2}{Z_0} = \frac{(0.1\mu\text{V})^2}{50\Omega} = 200 \cdot 10^{-18} \text{ W} \cong -127\text{dBm}$$

$$P_{E(r)} = P_S \cdot A_E \cdot \frac{1}{4\pi \cdot r^2} \cdot e^{-\chi \cdot r}$$

$$P_{TX} = \frac{P_{RX}}{A_E} \cdot 4\pi \cdot r^2 = \frac{200 \cdot 10^{-18} \text{ W} \cdot 4\pi \cdot (1000000\text{m})^2}{0.03\text{m}^2}$$

$$P_{TX} = 84\text{mW} \cong +19.2\text{dBm} \quad (\chi=0)$$

The propagation loss caused by isotropic radiation is  $A_{RX} = P_{TX} - P_{IN} = [19.2\text{dBm} - (-127\text{dBm})] = 146.2\text{dB}$

The median noise figure is  $F_{am} \approx 50\text{dB}$  at 10MHz. The receiver has an IF bandwidth of 10KHz. This gives a bandwidth terminated receiver Nyquist noise floor of:

$$P_{NRfloor} = K \cdot T \cdot B = 1.38 \cdot 10^{-23} \text{Ws/K} \cdot 290\text{K} \cdot 10\text{KHz}$$

$$P_{NRfloor} = -134\text{dBm} + 39.8 \approx -94.2\text{dBm}$$

$$P_{med} = P_{NRfloor} + F_{am} = -94.2\text{dBm} + 50\text{dB} = -44.2\text{dBm}$$

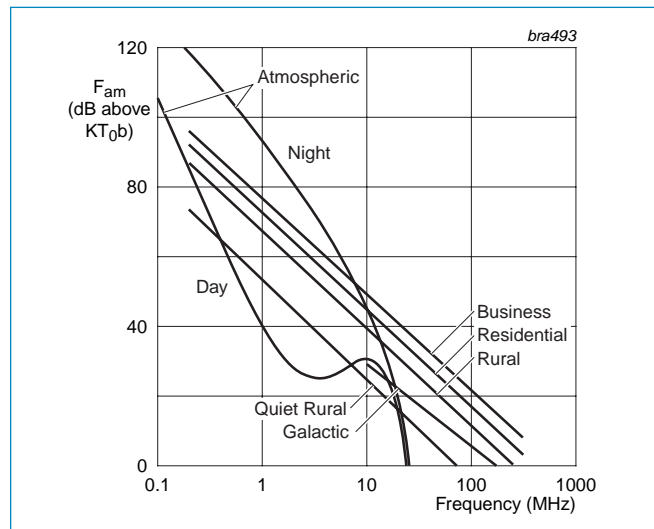
At the present receiver, the effective equivalent front-end input noise floor is specified to be 20dB below  $0.1\mu\text{V}$ :

$$P_{NRE} = -127\text{dBm} - 20\text{dB} = -147\text{dBm} + 2 \approx -145\text{dBm}$$

The effective resulting receiver input noise power is:

$$P_{FR} = P_{NRE} + P_{med} \approx P_{med} \approx -44\text{dBm}$$

Resulting  $\text{SNR}_{(med)} = P_{IN} - P_{FR} = -127\text{dBm} - (-44\text{dBm}) = -83\text{dB}$  and without man-made median noise +7dB



Two ideas for SNR improvement: Decrease of the BW, increase of the transmit power. Selected case: Increase of the transmitter power in order of maintenance a SNR=20dB above the resulting median noise floor at the receiver's location:

$$P_{INc} = P_{FR} + \text{SNR} = -84\text{dBm} + 20\text{dB} = -64\text{dBm}$$

$$P_{TXc} = P_{INc} + A_{RX} = -64\text{dBm} + 146.2\text{dBm} = +82.2\text{dBm}$$

$$P_{TXc} = 166\text{KW} \text{ for a } 10\text{KHz} \text{ bandwidths and } \text{SNR}=20\text{dB} \text{ (Music)}$$

$$P_{TXc} = 166\text{W} \text{ for } 100\text{Hz} \text{ bandwidth, } \text{SNR}=10\text{dB} \text{ (e.g. Morse Code)}$$

### Conclusion:

In shortwave (SW) bands, the necessary transmitter power is determined by the high amount of man-made noise. Due to that, the receiver's noise figure in SW is not important. More important is the receiver's input linearity (IP3) to handle the high-power man-made noise and intermodulation signals. Very important is the IF bandwidth and used modulation (min. bandwidth). Moving into the microwave range will dramatically reduce the man-made noise but the amount of galactic noise becomes the bottom-limiting factor. At such high frequencies, atmospheric absorption caused by water, oxygen and other gas molecules causes excessive rising of propagation losses. So a receiver's noise figure, gain and narrow bandwidth become increasingly important with rising frequency.

## 4.8 Filters in the receiver rail

The filters used and their primary responsibilities are:

- A band-pass filter in front of the LNA, for image frequency band rejection
- A band-pass filter after the LNA, for image noise suppression
- A filter in the IF circuit, for selecting the RF transmission channel
- A filter in the baseband, for selecting the baseband relevant frequency spectrum.

In spectrum analyzers and high-quality broadband short wave receivers, the first IF (Yig-Filters at SPA) is far above the received RF spectrum. Due to it, there is used a low pass filter preventing of front-end tuned tracking filters for image rejection purpose. Practically independent of the front-end pre-selection filter, the equivalent noise bandwidth is determined by the IF filter bandwidth.

## 4.9 Relationships and conversion of distortion parameters

$oIP3 \approx oPL_{1dB} + 10.63dB$	in dBm
$iIP3 \approx oIP3 - Gain$	in dBm
$iPL_{1dB} \approx oPL_{1dB} - Gain$	in dBm

The offset factor of 10.63dB [1] can slightly vary. In other literature, 9.6dB is quoted. Ref. [1] is preferred, because of arithmetical detail explaining the reason behind this value.

$oIP3$	= Output third order intercept point in dBm
$iIP3$	= Input related third order intercept point in dBm
$oPL1$	= Output 1dB gain compression point in dBm
$iPL1$	= Input related 1dB gain compression point in dBm
Gain	= Gain in dB

$oIP3$  typically used in transmitter systems

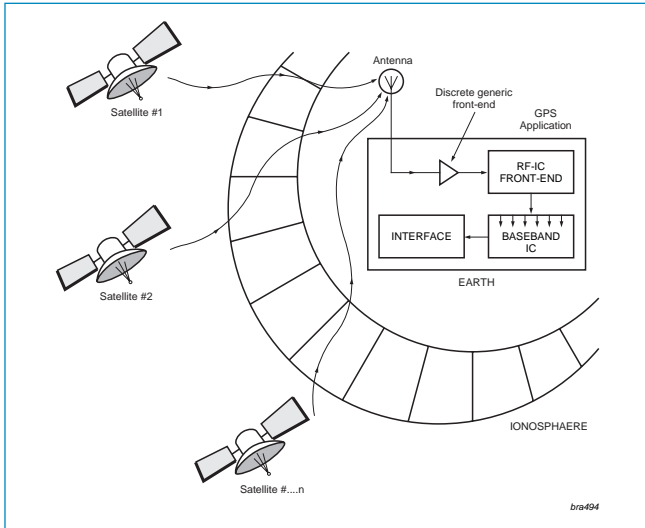
$iIP3$  typically used in receiver systems

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# 5. Introduction to GPS front-ends

Due to the continuous size reductions and attractive pricing of semiconductor devices, GPS applications have become very popular in the past few years. A GPS navigation system is based on measuring and evaluating RF signals transmitted by GPS satellites. At least 24 active satellites are necessary, at a distance of 20200 km above the Earth's surface. All satellites transmit their civilian-use L1 signal simultaneously, down to users at 1575.42 MHz in the so-called microwave L-band. Each satellite has its own **C/A (Coarse Acquisition) code**.



This satellite identifier C/A code is Pseudo Random and appears like Noise in the frequency spectrum (=PRN C/A code). The L1 carrier is **BPSK** (Binary Phase Shift Keying) modulated by the C/A data code, by the navigation data message and the encrypted P(Y)-code. Due to C/A's PRN modulation, the carrier is **DSSS** modulated (Direct Sequence Spread Spectrum modulation). This DSSS spreads the former bandwidth signal to a satellite internal limited width of 30 MHz. A GPS receiver must know the C/A code of each satellite for selecting it out of the antenna's RF spectrum. Because a satellite is selected using an identification code, GPS is a **CDMA** system (Code Division Multiple Access). This RF signal is transmitted with enough power to ensure a minimum signal power level of -160 dBW on the Earth's surface. The absolute minimum receiver bandwidth is  $> 2$  MHz.

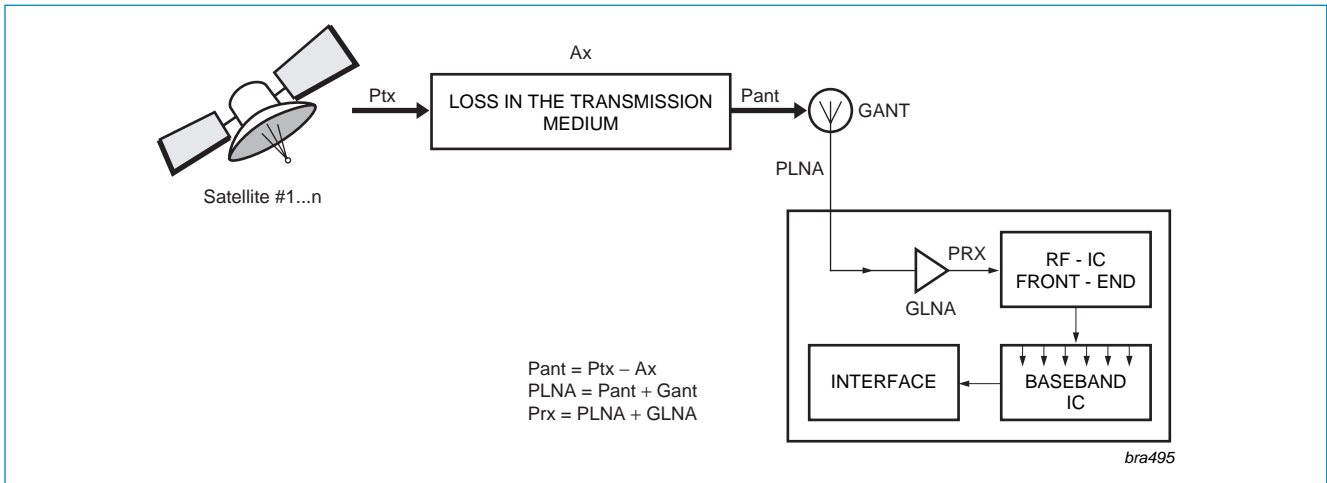
The L1 carrier based GPS system uses:  
**CDMA - DSSS - BPSK modulation**

Available GPS carrier frequencies		
L1	Link 1 carrier frequency	1575.42 MHz
L2	Link 2 carrier frequency	1227.6 MHz
L3	Link 3 carrier frequency	1381.05 MHz
L4	Link 4 carrier frequency	1379.913 MHz
L5	Link 5 carrier frequency	1176.45 MHz

The U.S. navigation system, GPS, was originally started by the U.S. military in 1979. It will be updated to supply the L2 & L5 carriers for increased performance in civil applications, while still providing the standard L1 RF carrier. GPS uses BPSK modulation on the L1 carrier and, beginning with launch of the modernized Block IIR, also on the L2 carrier. The L5 signal that will appear with the Block IIF satellites in 2006, will use **QPSK** modulation (Quadrature Phase Shift Keying).

Performance overview of current and up-coming GPS systems:

Topic	Used Codes	Need of a second reference base station	Resolution	Comments
Today: basic positioning	C/A Code on L1	No	Before May 2000: 25-100m Today 6-10m (resolution controlled by US)	---
Tomorrow: basic positioning	C/A Code on L1 L2C Code on L2 New Code on L5	No	1-5m	Eliminates need for costly DGPS in many non-safety applications.
Today: advance positioning	L1 Code and Carrier L2 Carrier Data Link	Yes	2cm	max. distance to reference 10km
Tomorrow: advanced positioning	L1 Code and Carrier L2 Code and Carrier L5 Code and Carrier Data Link	Yes	2cm	max. distance to reference 100km; faster recovery following signal interruption



The spread-spectrum modulated signal's field strength is very weak and causes a negative SNR in the receiver input circuit — caused by Nyquist noise determined by the analog front-end IF bandwidth:

Satellite Generation	Channel	C/A Loop peak
II/IIA/IIR	L1	-158.5dBW
	L2	-164.5dBW
IIR-M/IIF	L1	-158.5dBW
	L2	-160.0dBW

$$dBW = 10 \log \left( \frac{P}{1W} \right)$$

**Competition in satellite-based navigation systems:**

In 2004, the European navigation system EGNOS was started. News forecasted the European system Galileo for 2008. GLONASS is a Russian Navigation System.

**Comparison of the front-ends used in a GPS and in a GLONASS receiver:**

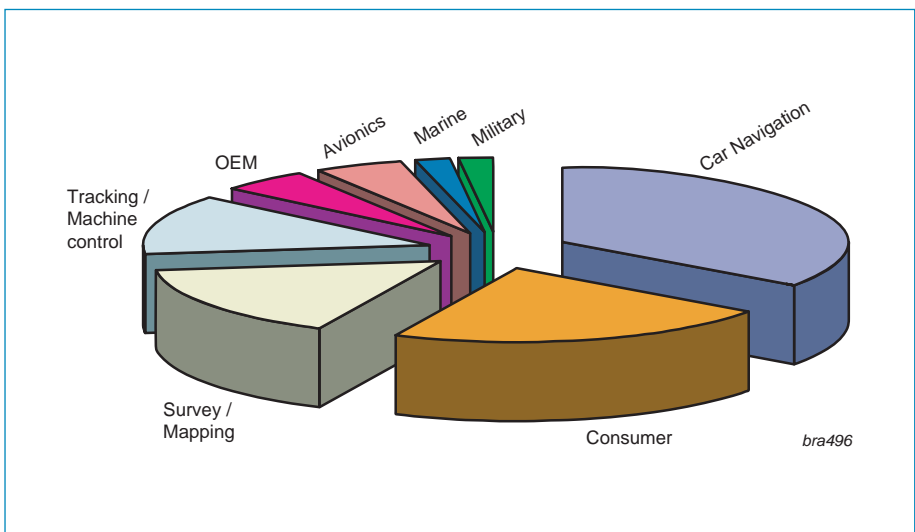
All GPS satellites use the same L1 frequency of 1575.42MHz, but different C/A codes, so a single front-end may be used. To achieve better sky coverage and accelerated operation, more than one antenna can be used. In this case, separate front-ends can be used. Using switches based on Philips' PIN-diodes makes it possible to select the antenna with the best signal in e.g. automotive applications, for operation in a city.

Each GLONASS satellite will use a different carrier frequency in the range of 1602.5625 MHz to 1615.5 MHz, with 562.5 kHz spacing, but all with the same spread code. The normal method for receiving these signals uses several parallel working front-ends, perhaps with a common first LNA and mixer, but certainly with different final local oscillators and IF mixers.

**Application examples:**

- Personal Navigations
- Railroads
- Recreation, walking-tour
- Off shore drilling
- Satellite ops. ephemeris timing
- Surveying & mapping
- Network timing, synchronization
- Fishing & boating
- Arm clocks <Alarm clocks?>
- Laptops and palms
- Mobiles
- Child safety
- Car navigation systems
- Fleet management systems
- Telecom time reference
- Highway toll systems
- First-aid calls via mobiles

**GPS Market & Applications**

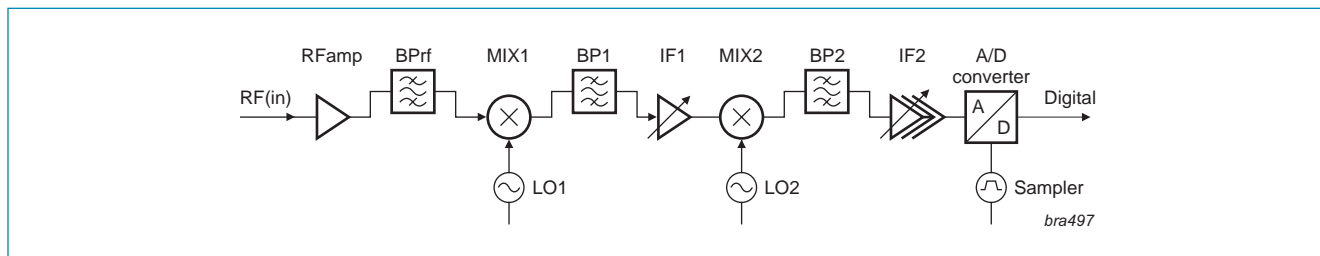




**References:**

- Office of Space Commercialization, United States Department of Commerce
- U.S. Coast Guard Navigation Center of Excellence
- NAVSTAR Global Positioning System
- NAVSTAR GPS USER EQUIPMENT INTRODUCTION
- Royal school of Artillery, Basic science & technology section, BST, gunnery careers courses, the NAVSTAR Global Positioning System

**Simplified block diagram of a typical GPS receiver analog front-end IC**



Typically, an integrated double-superhet receiver technology is used in the analog rail. The under sampling analog to digital converter (ADC) is integrated in the analog front-end IC with a resolution of 1 to 2 bits. Due to under sampling, it acts as the third mixer for down-converting into the digital stream IF band. After this ADC, comes the digital baseband processor. Up till this point, the SNR of the received satellite signals is negative. In the baseband processor, the digital IF signal is parallel processed in several C/A correlators and NAV-data code discriminators. During this processing, the effective Nyquist bandwidth is shrunk down to few Hertz. De-spreading and decoding of the GPS signal then creates a positive SNR. Because typically, front-end ICs are produced in a highly integrated, low-power, relatively noisy semiconductor technology, there is a need for an external Low-Noise-Amplifier (LNA) combined with band-pass filters. Because the available GPS IC chipsets on the market differ in their electrical performances like, gain, Noise Figure (NF), linearity and sensitivity, one and two-stage discrete front-end amplifiers are used. The numbers of filters in the front-end vary with the needs of the application’s target environment, costs and sizes. The processed number of GPS carriers, as well as the navigation accuracy, determines the min. allowed bandwidth of the analog front-end rail.

Philips Semiconductors offers MMICs with internal 50Ω matches at the input and output (I/O) and without internal matching. The internal matched broadband MMICs typically need an output inductor for DC biasing and DC decoupling capacitors at the amplifier I/O. The internal non-matched devices need an I/O matching network typically made by lumped LC circuits in an L-arrangement. This gives additional selectivity. Another advantage of this MMIC is the integrated temperature compensation in contrast to a transistor. In a system, typically the first amplifier’s noise figure is very important. For example, the **BGU2003** SiGe MMIC offers both (NF+IP3) with a good quality. Its silicon brother **BGA2003** comes with a lower IP3 and NF. IC chipsets that need high front-end gain made by one MMIC may be able to use the **BGM1011** or **BGM1013**. A two-stage design e.g. will use **BGA2001**, **BGA2011** eventually combined with **BGA2748** or **BGA2715** or **BGA2717**. Some examples of configuration for an L1-carrier LNA are shown in the next two tables.

**Single front-end amplifier:**

Amplifier	BFG 325W	BFU 540	BGU 2003	BGM 1013	BGM 1011	BFG 410W	BGA 2011	BGA 2001	BGA 2003	BGA 2715	BGA 2748
Gain	14dB	20dB	14dB	34dB	35dB	18dB	12dB	14dB	14dB	23.2dB	21dB
NF	1dB	0.9dB	1.1dB	4.7dB	4.7dB	1.1dB	1.5dB	1.3dB	1.8dB	2.7dB	2dB
IP3o(out)	+24dBm	+21dBm	+21dBm	+21dBm	+20dBm	+15dBm	+10dBm	+9.5dBm	+9.2dBm	+1dBm	-1.6dBm
Matching	External	External	External	Internal	Internal	External	External	External	External	Internal	Internal

**Two-stage cascaded circuit front-end amplifier:**

1 <sup>st</sup> Stage	BFG325W	BFG410W	BFU540	BFG325W	BGA2011	BGU2003	BGA2011	BGA2003	BGA2011	
2 <sup>nd</sup> Stage	BFU540	BFU540	BGU2003	BFG410W	BGA2011	BGA2001	BGA2715	BGA2715	BGA2748	
Cascaded Gain	31dB	35dB	29dB	35dB	29dB	21dB	25dB	32.2dB	34dB	30dB
Cascaded NF	1.19dB	1.25dB	1.32dB	1.11dB	1.28dB	2dB	1.5dB	2.5dB	2.6dB	2.2dB
Cascaded IP3o	+21dBm	+21dBm	+21dBm	+15dBm	+15dBm	+10dBm	+9.5dBm	+1dBm	+1dBm	-1.6dBm

- Note:**
- [1] Gain=|S21|<sup>2</sup>; data @ 1.8GHz or the next one / approximated, found in the data sheet / diagrams
  - [2] For cascaded amplifier equations refer to e.g. 4<sup>th</sup> edition RF Manual appendix, 2.4GHz Generic Front-End reference design
  - [3] The evaluated cascaded amplifier includes an example interstage filter with 3dB insertion loss (NF=+3dB; IP3=+40dBm).
  - [4] MMICs: BGAxxxx, BGMxxxx, BGUxxxx Transistors: BFGxxx, BFUxxx

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