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A Simplified Design Procedure for Electrical Bandpass Filters

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A Simplified Design Procedure for Electrical Bandpass Filters

Prepared by D. L. Decker, W. S. Elliott, and D. P. Viterisi

Cedar Rapids Division | Collins Radio Company, Cedar Rapids, Iowa

ERRATA TO CTR 202

A SIMPLIFIED DESIGN PROCEDURE

FOR ELECTRICAL BAND-PASS FILTERS

The following correction should be made to your copy of CTR 202, A Simplified Design Procedure for Electrical Band-Pass Filters.

Page 42 Change equation (19) to read as follows:

$$C_2 = \frac{1}{\pi f_m R \left(\frac{f_2}{f_1}\right)^{1/2} \left(\frac{f_2}{f_1} - 1\right)}$$

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A SIMPLIFIED DESIGN PROCEDURE FOR ELECTRICAL BANDPASS FILTERS

Section I

CTR-101, "Report on Development of Electrical Bandpass Filters" by R. N. Hargis, presents design information for the development of passive L-C filters. However, CTR-101 follows the pattern of most texts and modifies the results obtained for the case of dissipationless components to account for dissipative elements. It is the purpose of this report to present a simplified procedure whereby L-C bandpass filters containing dissipation may be developed directly from the design equations.

Section II

L-C bandpass filters consisting of cascaded three-element sections have been used successfully in several Collins equipments. These filters have been applied where the center frequency of the pass band or the bandwidth-to-center frequency ratio of the pass band has been greater than that obtainable with mechanical filters. The design problems of the L-C filter involve the determination of the number of sections and the component values required to obtain a specified over-all filter characteristic. Since considerable trial and error is involved when using the design procedure of CTR-101, curves and charts have been prepared in an attempt to simplify the effort. These curves show the variations in insertion loss, bandwidth, shape factor, etc., for variations in Q and the number of filter sections employed.

Section III

To promote a better understanding of the purpose and origin of the curves, the theory of the filter development will be reviewed. The schematic for a complete six-section filter is shown in figure 1(a). An equivalent circuit which contains six identical π -sections is illustrated in figure 1(b) to establish that the schematic is a six-section filter. An examination of the schematic will show that a three-element repetitive combination, illustrated in figure 2, also appears. Suffice it to say that this

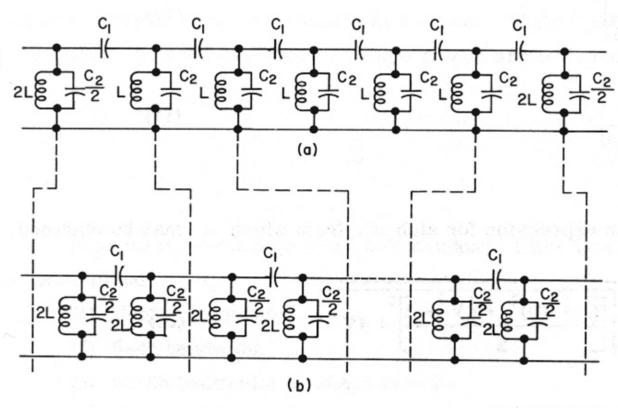

Figure 1. (a) Six-Section Filter Schematic, (b) Equivalent π -Section Diagram of Six-Section Filter

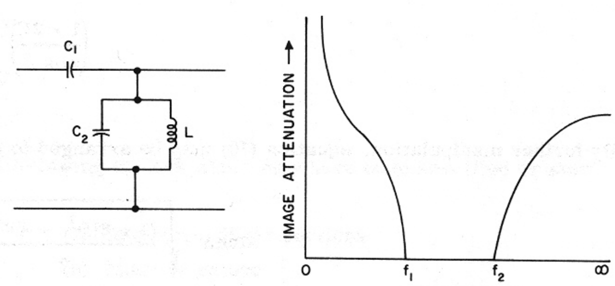

Figure 2. Dissipationless Three-Element Filter Section

three-element combination as shown in figure 2 is equivalent to the five element π-section of figure 1(b). The attenuation curve for the filter section has been drawn for the dissipationless case to establish the following definitions to be used later:

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}}, \quad \text{upper theoretical cutoff frequency} \qquad (1)$$

$$f_1 = \frac{1}{2\pi\sqrt{L(C_2 + 4C_1)}}, \quad \text{lower theoretical cutoff frequency} \qquad (2)$$

$$f_m = \sqrt{f_1 f_2}, \quad \text{filter center frequency} \qquad (3)$$

The mathematical function expressing the image attenuation characteristic of the filter section in terms of the circuit elements is derived from the basic formula for a ladder network*,

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2} \qquad (4)$$

where γ is the propagation constant, Z_1 is the impedance of the full series arm, and Z_2 is the impedance of the full shunt arm. Since γ and the circuit impedances are complex, they are usually written in the following manner:

$$\gamma = \alpha + j\beta \qquad (5)$$

$$\frac{Z_1}{4Z_2} = U + jV \qquad (6)$$

where α is the attenuation constant, β is the phase constant, and U and V are the respective real and imaginary component of the impedance function of equation (6). When these expressions are substituted into equation (4), we obtain

$$\cosh(\alpha + j\beta) = 1 + 2U + 2jV \qquad (7)$$

which can be expanded into its real and imaginary components;

$$\cosh\alpha \cos\beta = 1 + 2U \quad \text{(real)} \qquad (8)$$

$$\sinh\alpha \sin\beta = 2V \quad \text{(imaginary)} \qquad (9)$$

Following through, these equations may be solved simultaneously to obtain functions of α and β alone, giving

$$\left(\frac{1 + 2U}{\cosh\alpha}\right)^2 + \left(\frac{2V}{\sinh\alpha}\right)^2 = 1 \qquad (10)$$

$$\left(\frac{1 + 2U}{\cos\beta}\right)^2 - \left(\frac{2V}{\sin\beta}\right)^2 = 1 \qquad (11)$$

By further manipulation, equation (10) may be arranged to obtain an expression for sinh α, from which α may be obtained;

$$\sinh\alpha = \sqrt{\frac{(1 + 2U)^2 + 4V^2 - 1}{2} + \sqrt{\left[\frac{(1 + 2U)^2 + 4V^2 - 1}{2}\right]^2 + 4V^2}} \qquad (12)$$

$$\alpha = \ln\left(\sinh\alpha + \sqrt{1 + \sinh^2\alpha}\right) \qquad (13)$$

* J. D. Ryder, Networks, Lines and Fields, Chapter 4

Similarly,

$$\cos\beta = \sqrt{\frac{(1+2U)^2+4V^2+1}{2} - \sqrt{\frac{(1+2U)^2+4V^2+1}{2} - (1+2U)^2}} \tag{14}$$

from which β can be obtained. Equations (12) and (14) become invalid when V approaches zero.

The series and shunt arms of the filter section may be obtained by inspection from figure 3, which shows the three-element section with the dissipative factor added. By using these impedances, equations (1), (2) and (3), and the relationship, $d = \frac{R}{\omega L} = \frac{1}{Q}$, U and V may be expressed in terms of the characteristics of the three-element section:

$$U + jV = \frac{d + j\left[1 - \left(\frac{f_m}{f}\right)^2 \left(\frac{f_2}{f_1}\right)\right]}{(d + j)\left[\left(\frac{f_2}{f_1}\right)^2 - 1\right]} \tag{15}$$

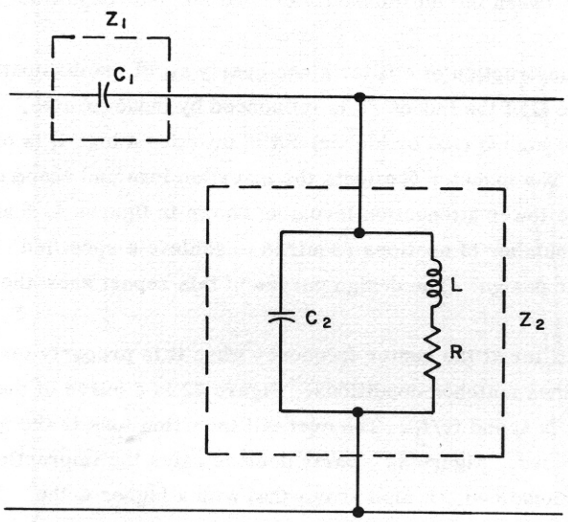

Figure 3. Three-Element Filter Section with Dissipation

It should be pointed out that f_1 and f_2 are the lower and upper theoretical cutoff frequencies, respectively, of the dissipationless filter and do not necessarily correspond to the 6-db frequencies of the filter with dissipation. Equation (15) ties together these dissipationless expressions and the Q of the filter inductor. U and V may be obtained by solving equation (15) for the real and imaginary parts, giving:

$$U = \frac{d^2 + 1 - \left(\frac{f_m}{f}\right)^2 \left(\frac{f_2}{f_1}\right)}{(d^2 + 1)\left[\left(\frac{f_2}{f_1}\right)^2 - 1\right]} \tag{16}$$

$$V = \frac{-d\left(\frac{f_m}{f}\right)^2 \left(\frac{f_2}{f_1}\right)}{(d^2 + 1)\left[\left(\frac{f_2}{f_1}\right)^2 - 1\right]} \tag{17}$$

The final step for obtaining values of α and β now involves substituting equations (16) and (17) into equations (12) and (14) and carrying them to a numerical solution. Without the aid of a computer, the time required for obtaining a solution is prohibitive. The various figures of this report are the result of several programs applied to the IBM 650 computer.

Section IV

In general, the design of an L-C bandpass filter involves the following factors which may have been specified or must be determined.

(1) Center frequency
(2) 6-db bandwidth
(3) 60-db bandwidth or shape factors
(4) Q of available inductors
(5) Insertion loss
(6) Image or characteristic impedance
(7) Number of filter sections
(8) Inductor values
(9) Capacitor values
(10) Physical size
(11) Environment
(12) Cost, etc.

The center frequency of the filter is normally specified, determined by spurious response considerations, or established by some arbitrary means. For passive filters, the center frequency is usually defined as the frequency midway between the frequencies at the 6-db attenuation levels of the over-all filter attenuation characteristic. However, for the purpose of this report, the center frequency referred to herein will be as defined by equation (3). It should be noted that this frequency will not necessarily equal the center frequency determined by the 6-db frequencies.

The 6-db bandwidth is the frequency difference between the frequencies established by the 6-db attenuation level of the over-all filter characteristic. Figures 4, 5 and 6 are attenuation characteristics of a single filter section having variations in Q and the design factor, f_2/f_1. Since the attenuation of each filter section in cascade is additive, the 6-db bandwidth of a filter based on any one of the curves will depend upon the number of filter sections employed. This becomes evident in figures 7 through 15, which are plots of 6-db bandwidths for filters with variations in Q, f_2/f_1 and the number of sections used.

The 60-db bandwidth is the frequency difference between the frequencies established by the 60-db attenuation level of the over-all filter characteristic. Figures 16 through 24 are plots of the shape factor or ratio of the 60-db bandwidth to the 6-db bandwidth for filters with variations in Q, f_2/f_1 and the number of sections. The shape factors expressed in these plots are the direct ratios of the two bandwidths and do not take into consideration any dissymmetry of the attenuation characteristic which may exist. Similar information is shown for the ratio between the 100-db bandwidth and the 6-db bandwidth in figures 25 through 31.

The Q of available inductors is a primary consideration in the construction of a filter since nearly all of the dissipation of a filter section is due to the series resistance of the inductors. The Q of the inductors is influenced by inductor size, filter center frequency, etc. Most filter theory deals with inductors of high Q (150 or higher) but in practice a high Q is often difficult to obtain, particularly if size is an important consideration. The inductor Q affects the insertion loss and shape of the filter attenuation characteristic. Its principal effect appears at the lower attenuation levels as shown in figures 4, 5 and 6. The shape of this portion of the characteristic largely determines the number of sections required to achieve a specified bandwidth or shape factor and thus places a practical limit on the filter design. The design curves of this report show the effect of variations of inductor Q between values of 50 and 150.

The insertion loss of the filter is the attenuation of the over-all filter at the center frequency when it is properly terminated. As a note of caution, all the design information herein assumes matched conditions. Figure 32 is a curve of the center frequency attenuation α_m, for a single section with variations in Q and f_2/f_1. The over-all insertion loss is the product of the single-section insertion loss times the number of sections used. Figure 32 clearly demonstrates the impractical use of an f_2/f_1 ratio less than approximately 1.04 for the inductor Q's assumed. It also shows that with a higher Q the practical limit of this ratio can be reduced.

The image or characteristic impedance of the filter is that impedance which will properly match the filter. This impedance depends solely upon the component values of the filter sections. Reference to figure 1 will show that the shunt components at the ends of the filter differ from the middle components to permit termination in a resistive element. It is interesting to note that these shunt components at the ends of the filter are identical to the shunt components of the π-section.

The number of filter sections will be determined by a compromise between the bandwidth, insertion loss and other factors of the filter design. The curves of this report have been specifically drawn for discrete steps in the number of filter sections to avoid difficult extrapolation.

The other design factors, i.e., inductor values, capacitor values, size, environment, etc., are determined by the filter application or during the course of the filter design. In most filter problems two major factors are fixed, the 6-db bandwidth and the center frequency. There may, or course, be other requirements or restrictions. Knowing the fixed requirements it is then the engineer's responsibility to design a filter which includes the fixed factors and optimizes the remaining variables.

The design procedure which follows has been devoted exclusively to attenuation characteristics. To present as much information in this report as possible, phase characteristics have also been calculated for a single-section filter and the information has been plotted in figures 33 through 37. Each figure is a plot of β versus f/f_m for three values of the f_2/f_1 ratio and a single value of inductor Q. The total phase shift of a filter is the product of the number of filter sections and the phase shift of the single-section filter at a particular frequency.

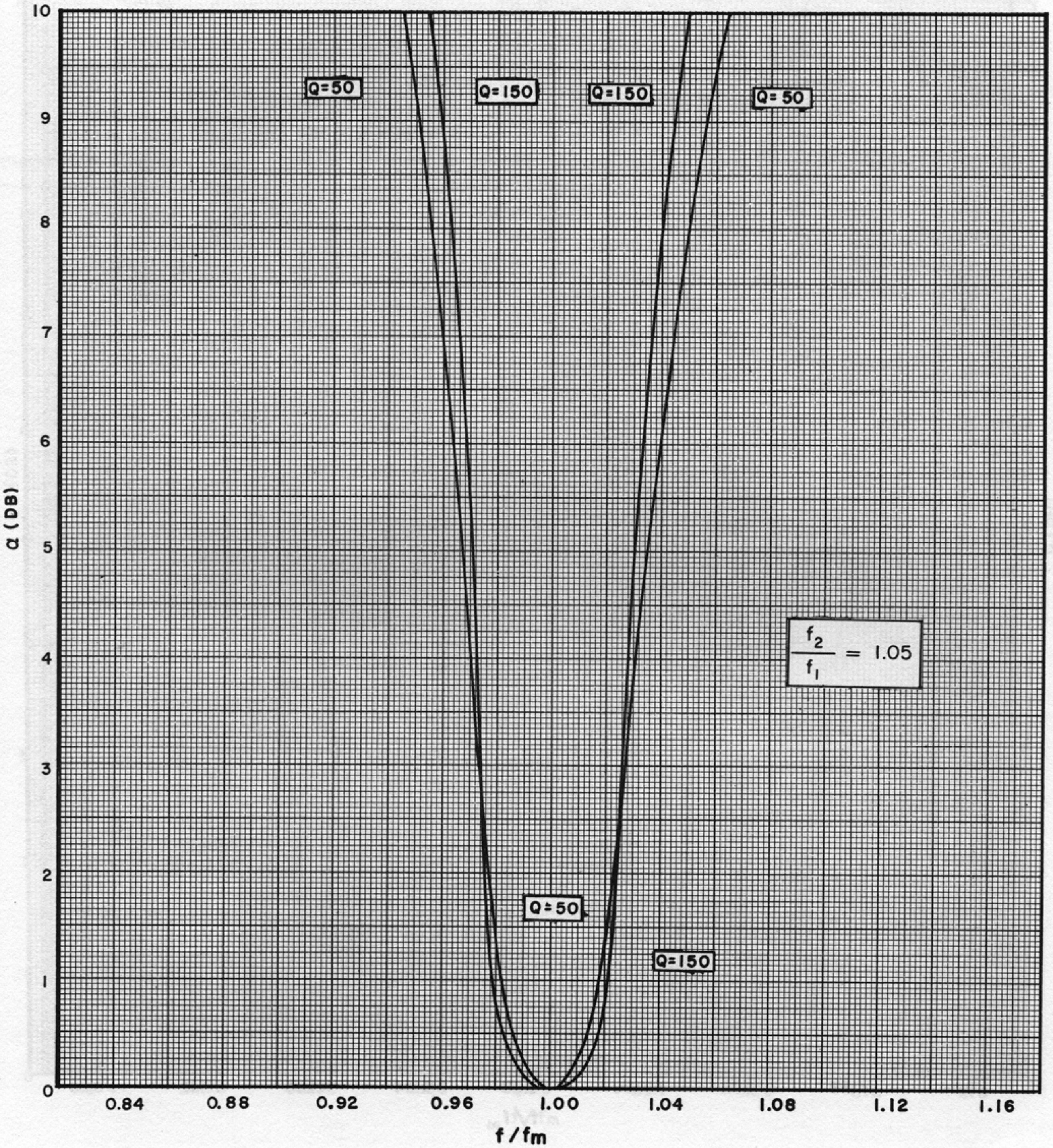

Figure 4. Attenuation Characteristic for a Three-Element Filter Section ($f_2/f_1 = 1.05$)

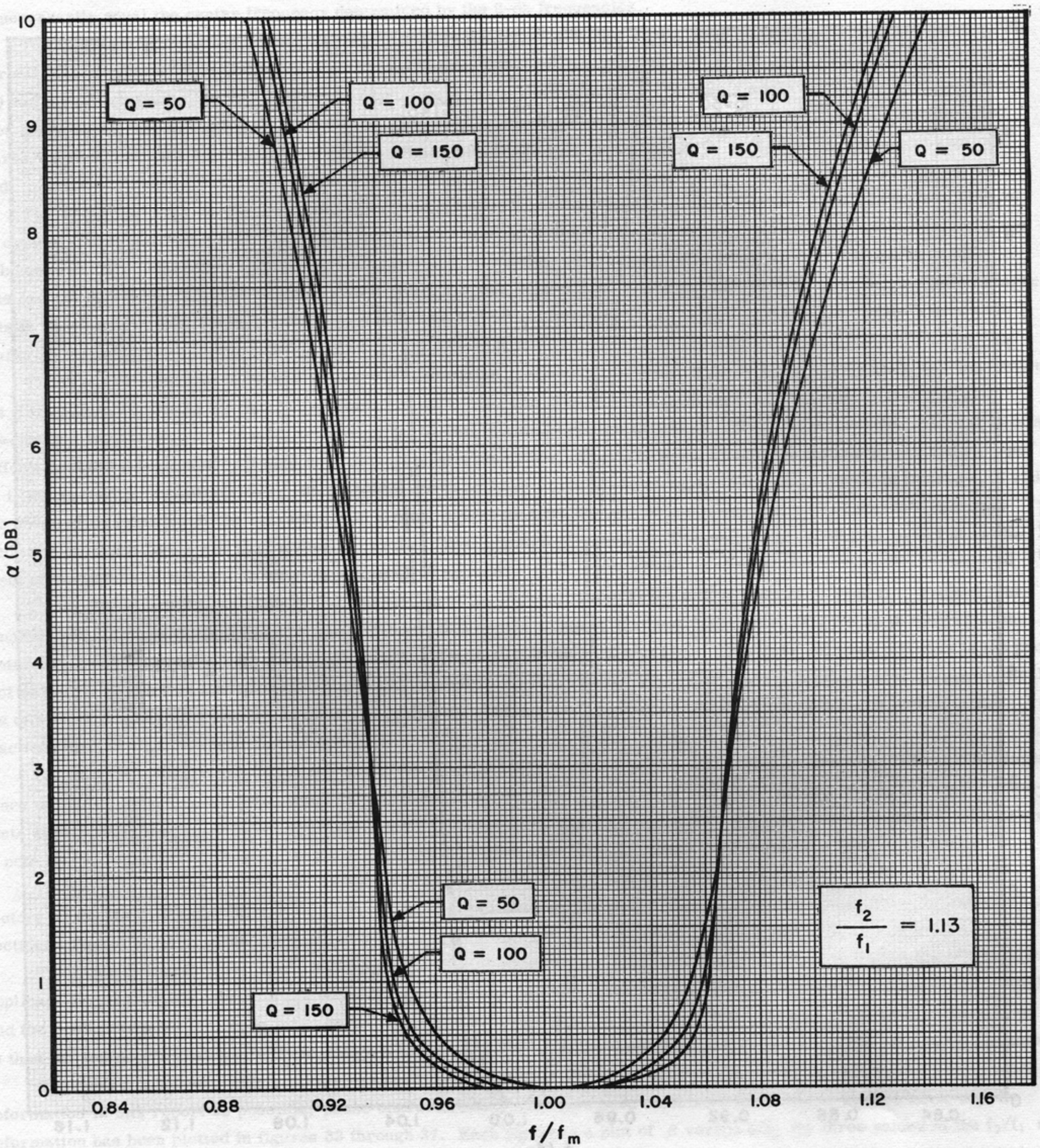

Figure 5. Attenuation Characteristic for a Three-Element Filter Section (f_2/f_1 = 1.13)

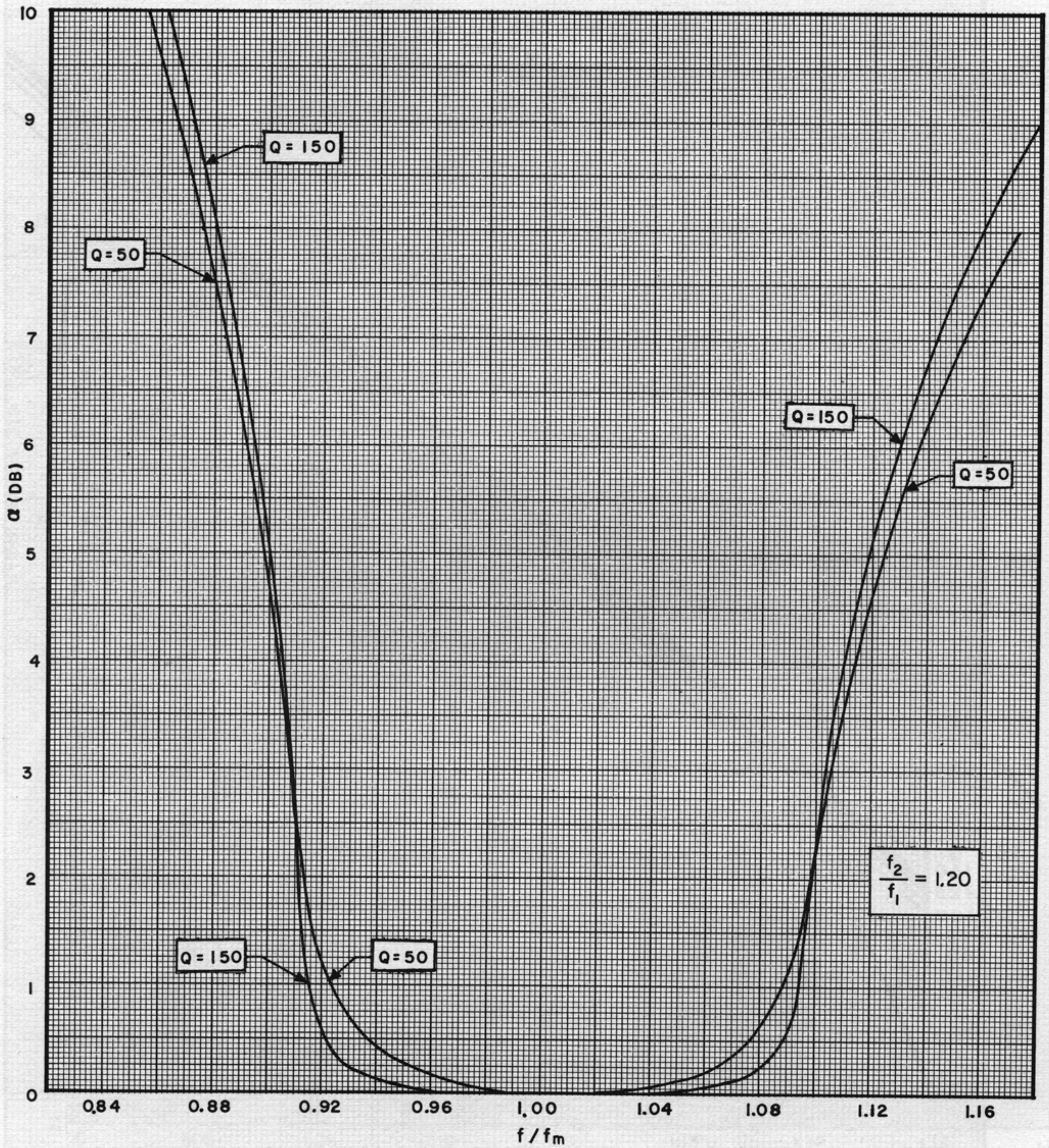

Figure 6. Attenuation Characteristic for a Three-Element Filter Section (f_2/f_1 = 1.20)

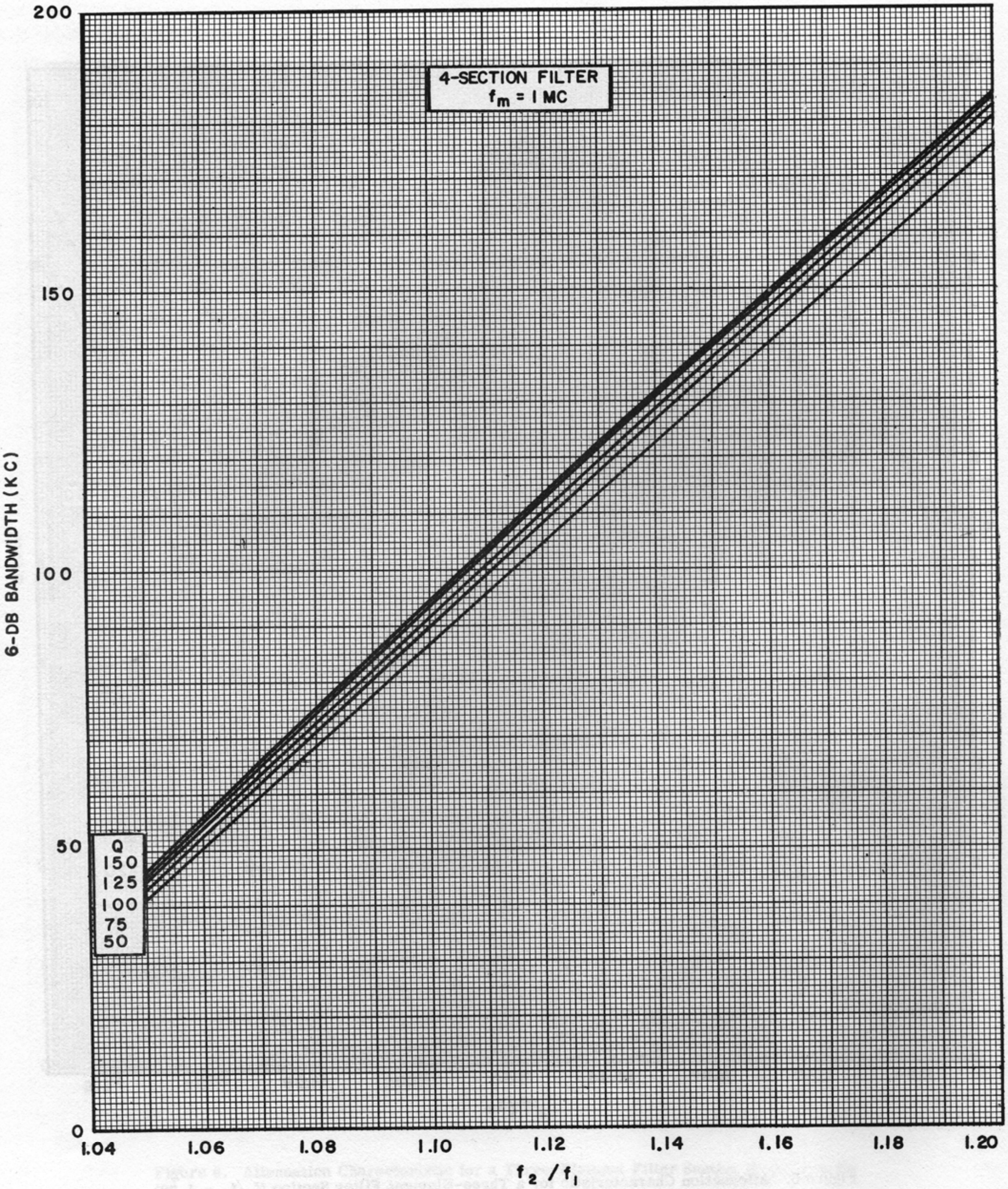

Figure 7. 6-Db Bandwidth of Four-Section Filter

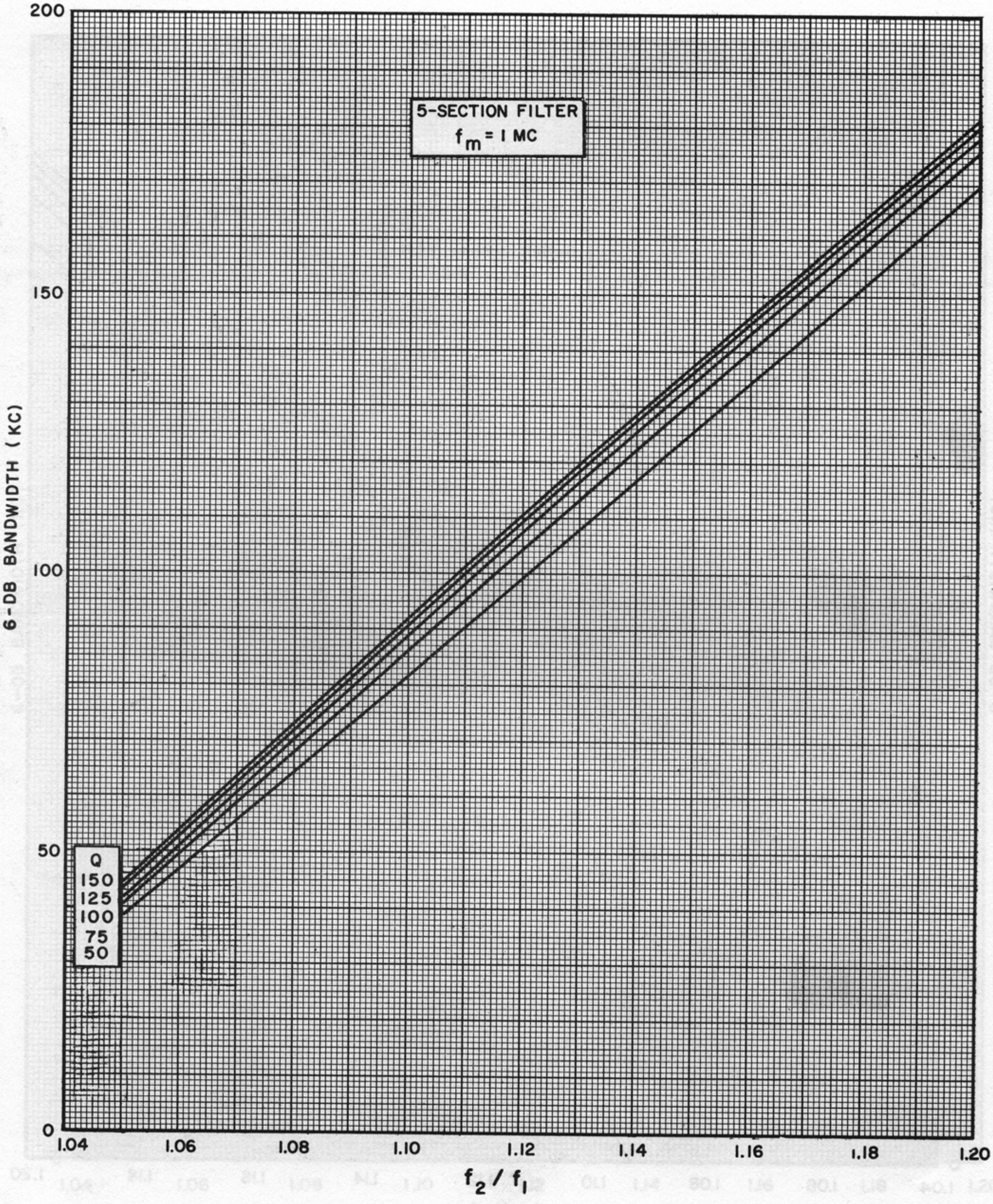

Figure 8. 6-Db Bandwidth of Five-Section Filter

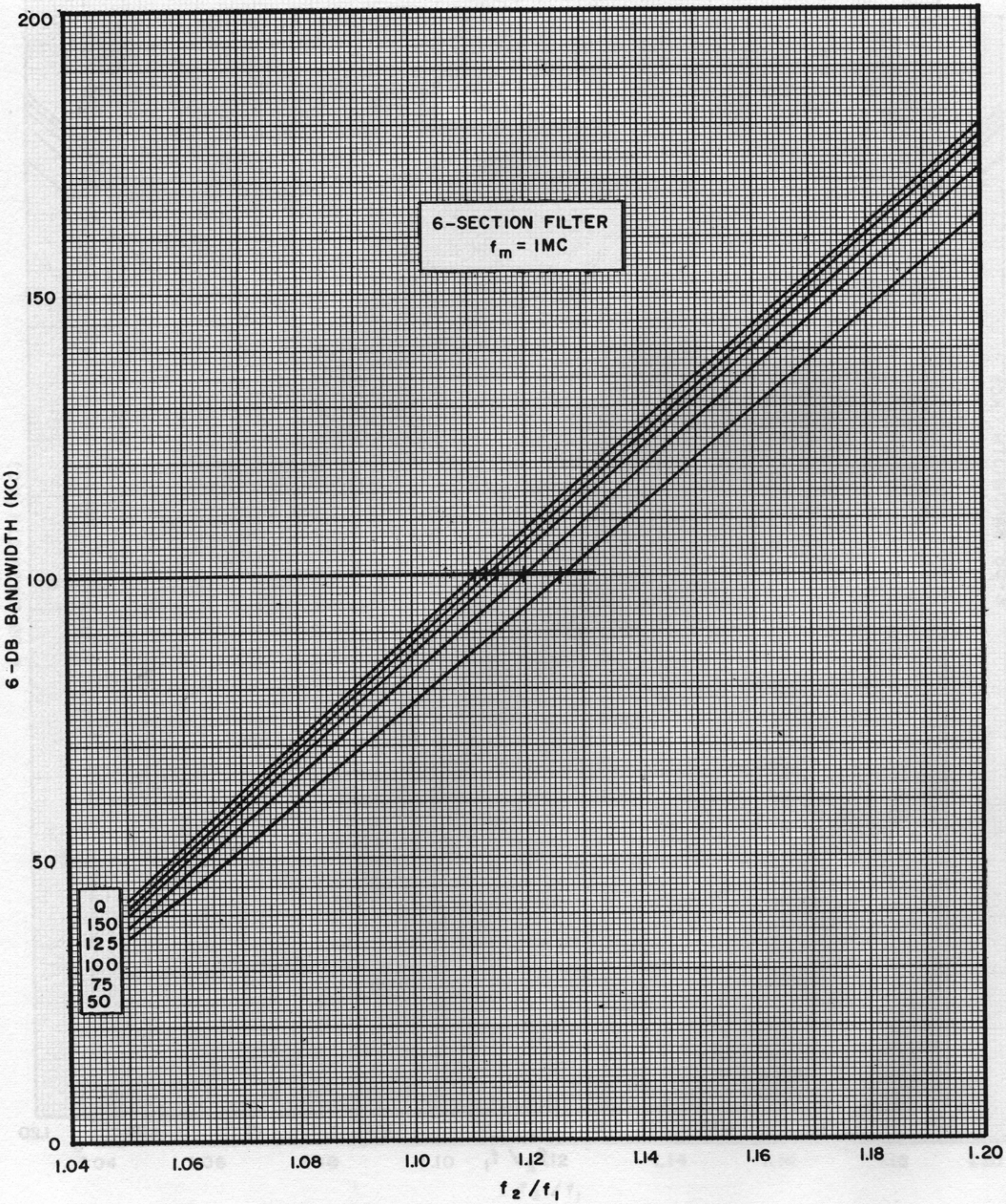

Figure 9. 6-Db Bandwidth of Six-Section Filter

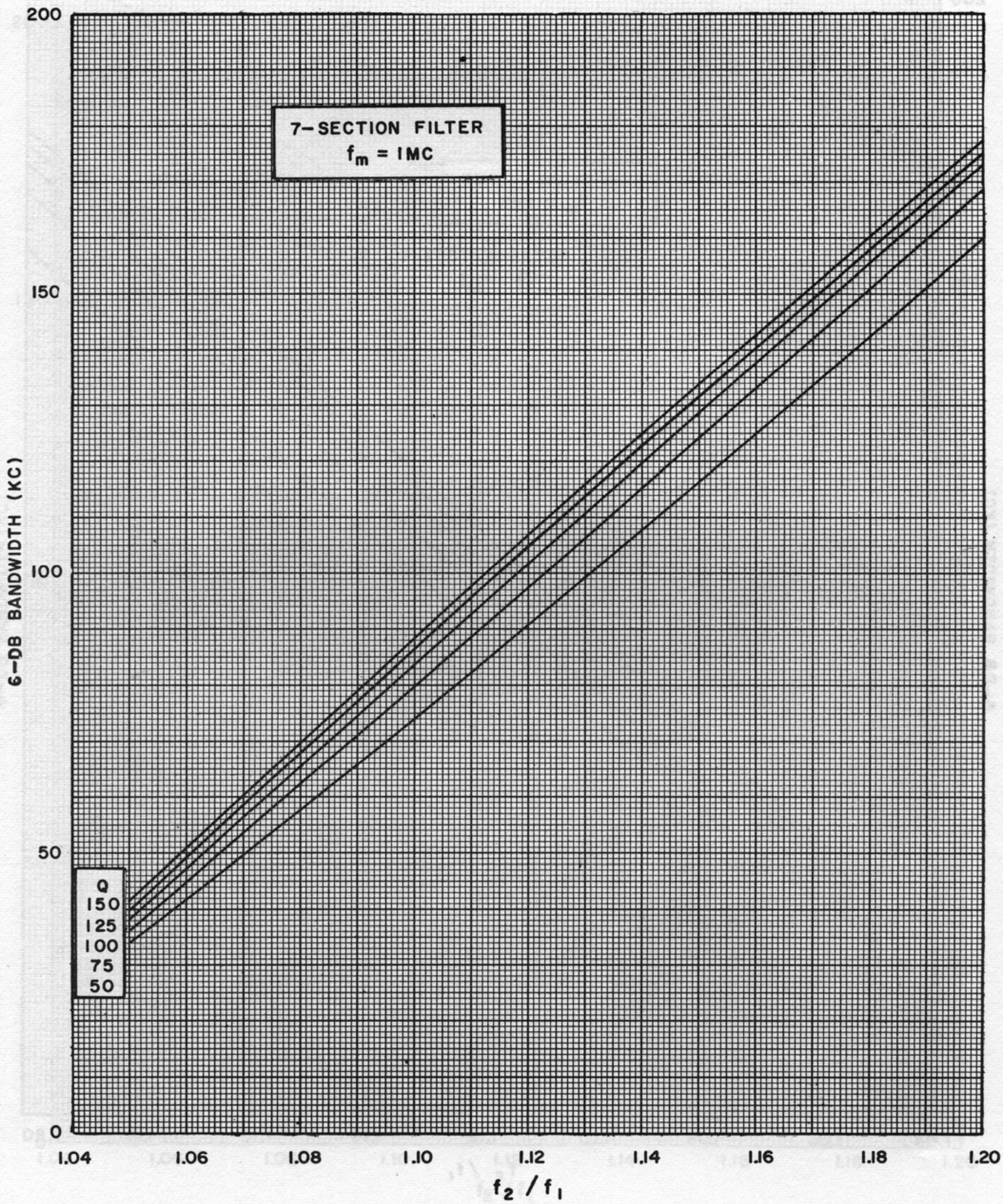

Figure 10. 6-Db Bandwidth of Seven-Section Filter

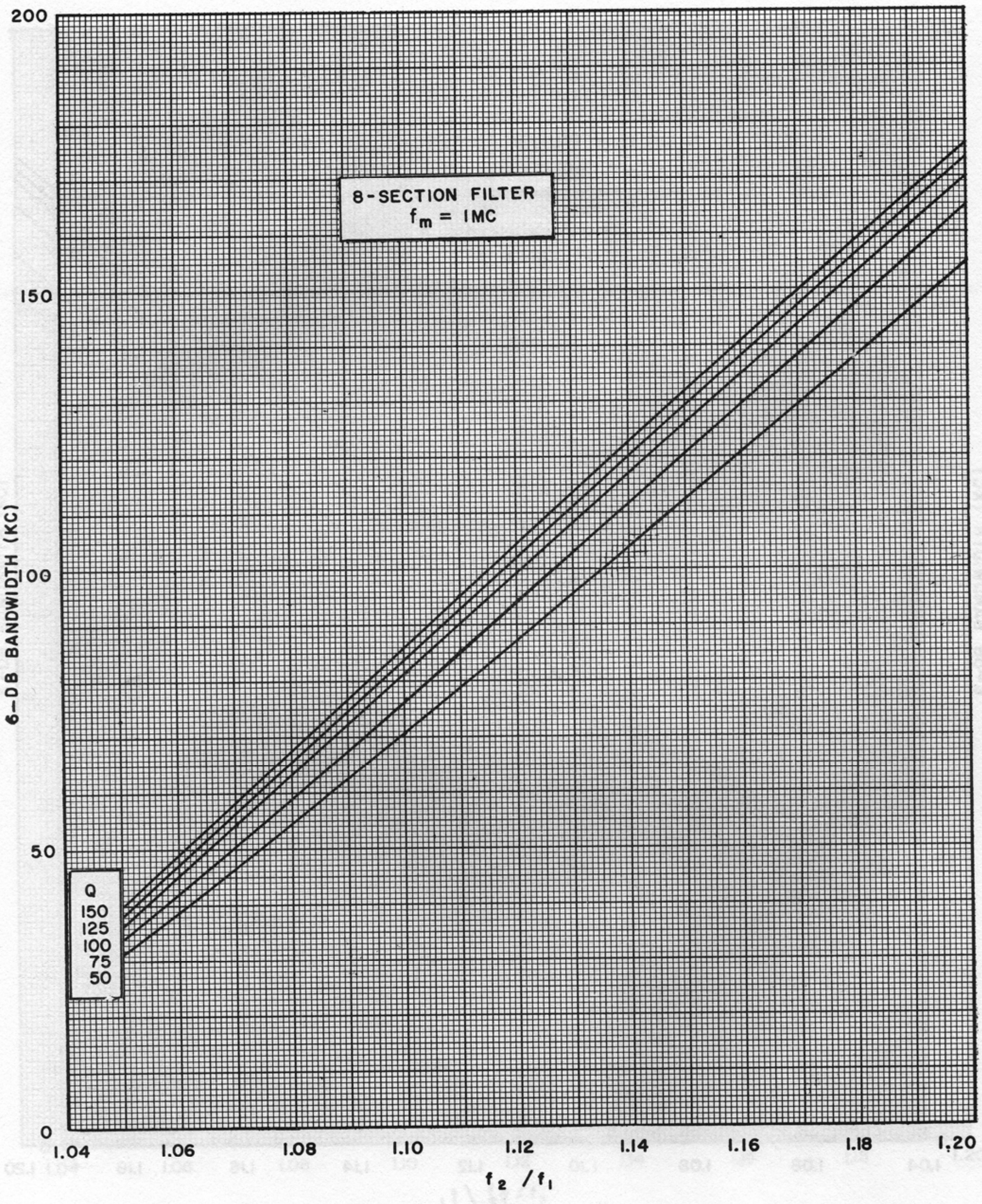

Figure 11. 6-Db Bandwidth of Eight-Section Filter

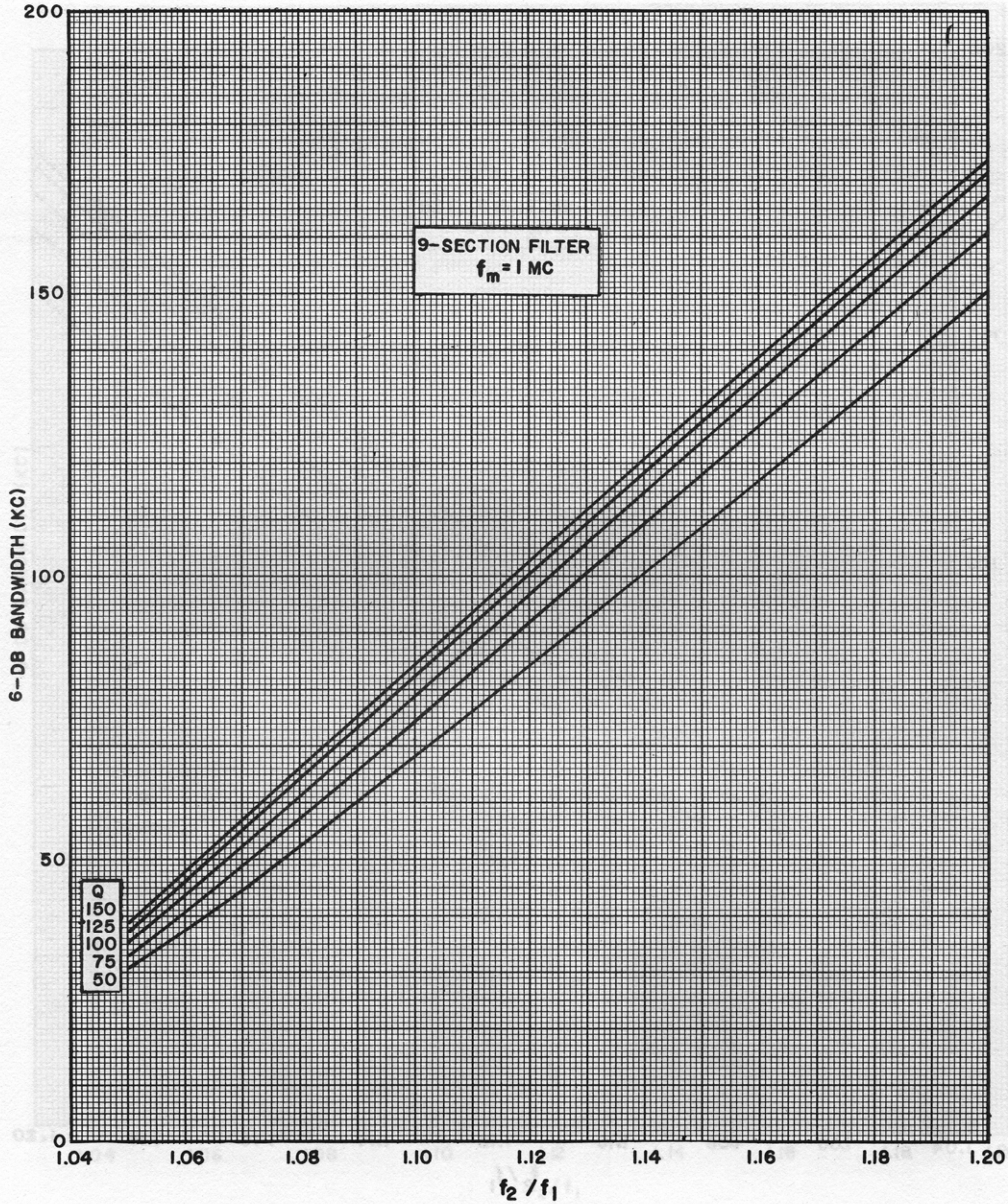

Figure 12. 6-Db Bandwidth of Nine-Section Filter

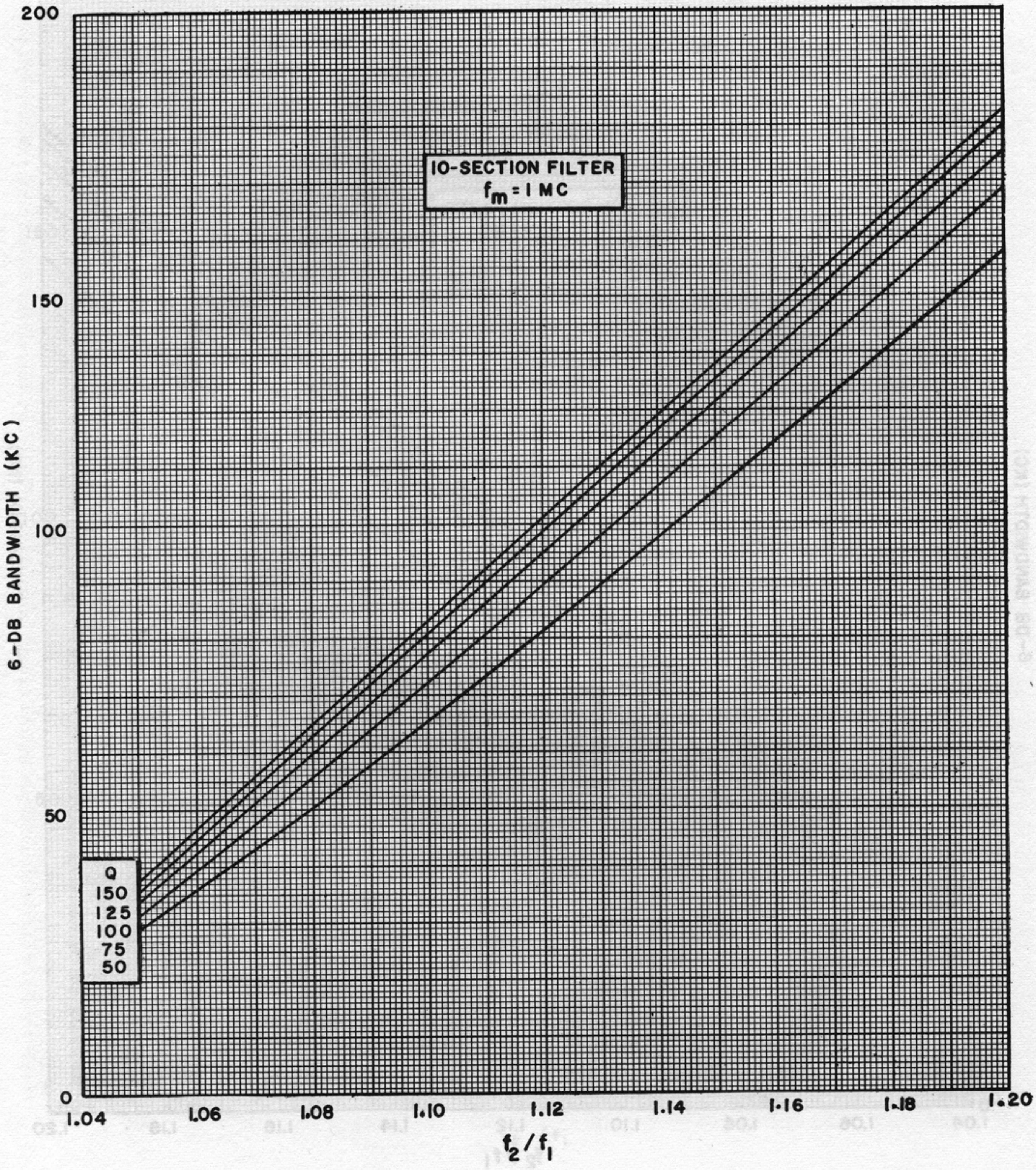

Figure 13. 6-Db Bandwidth of Ten-Section Filter

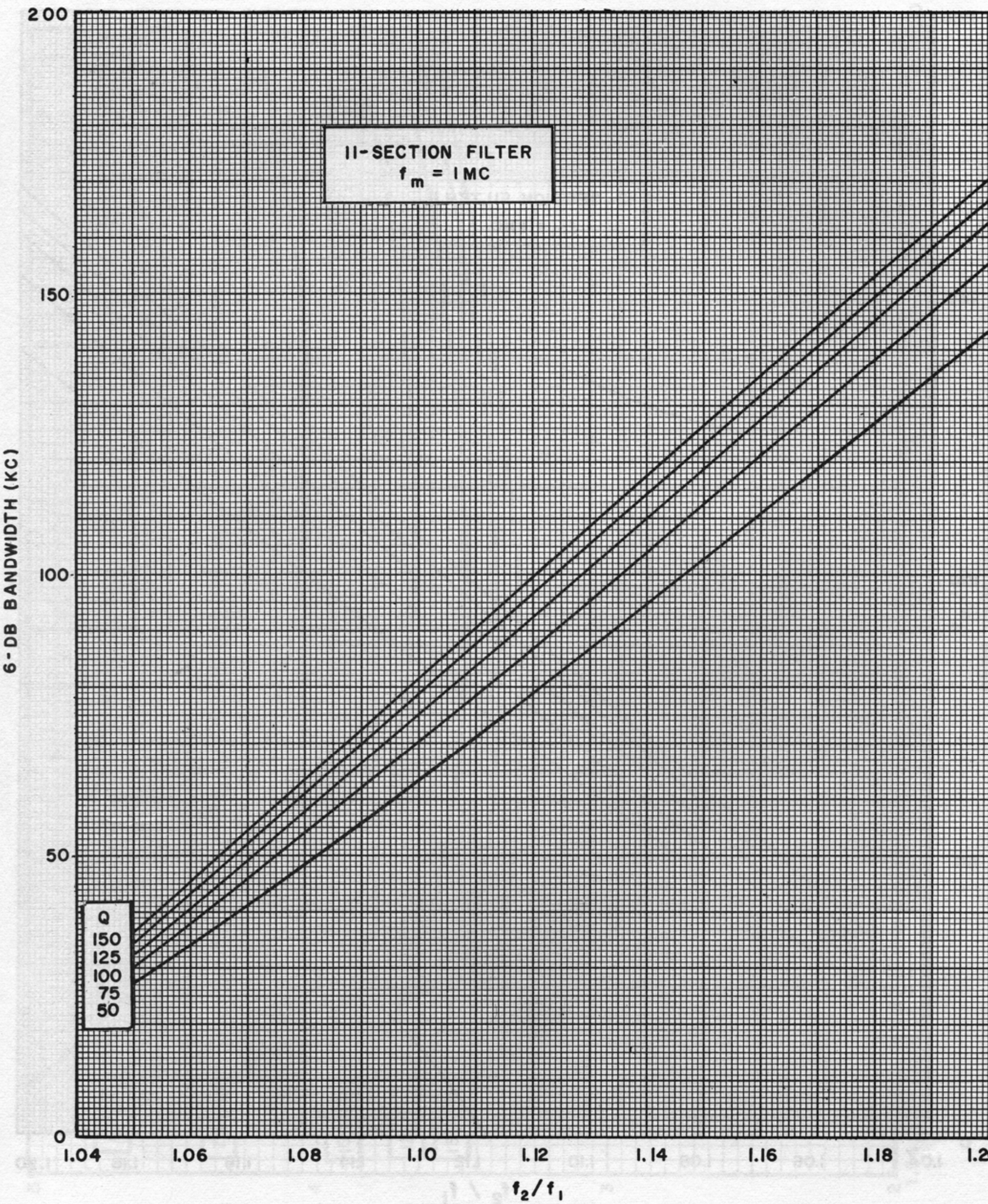

Figure 14. 6-Db Bandwidth of Eleven-Section Filter

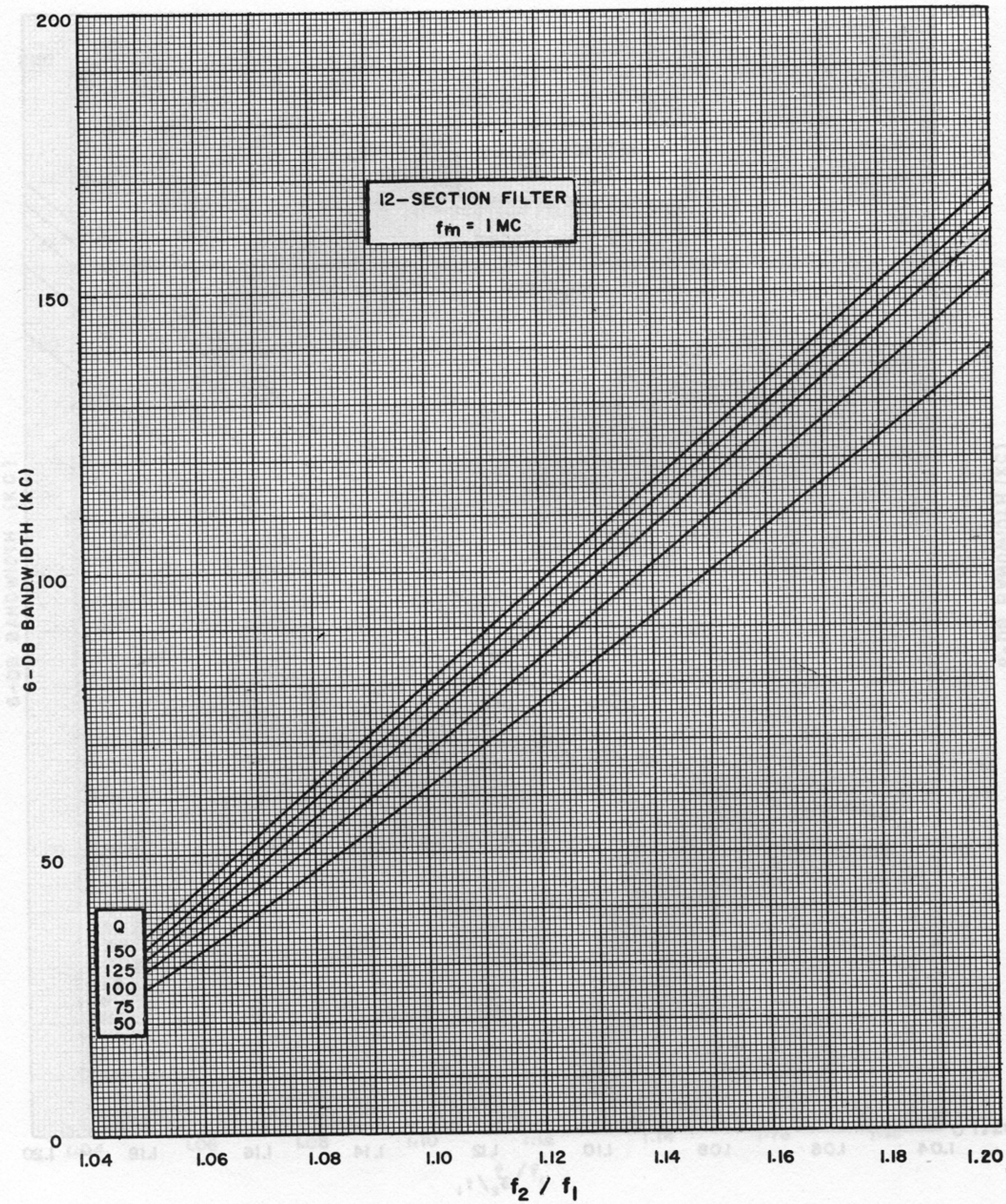

Figure 15. 6-Db Bandwidth of Twelve-Section Filter

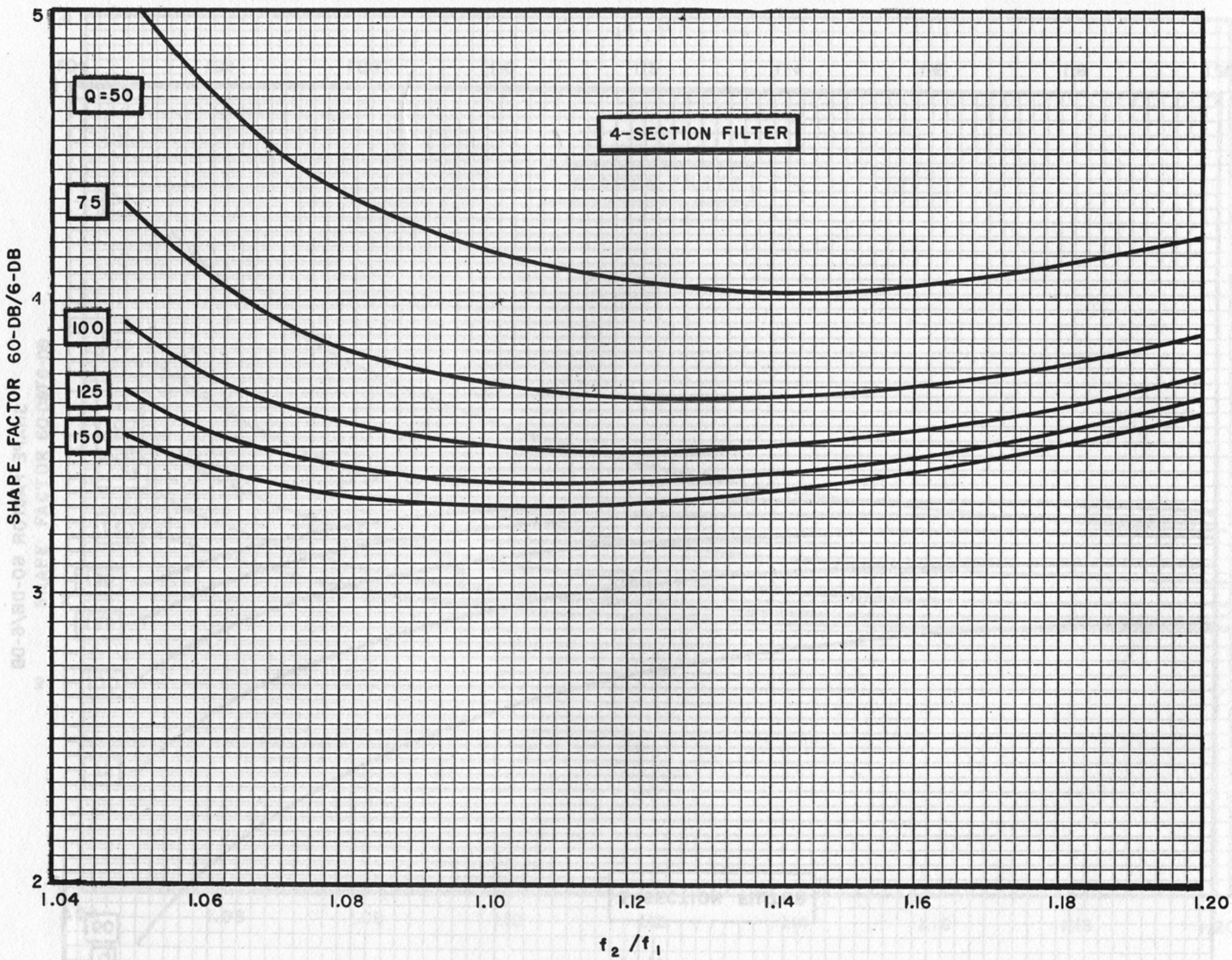

Figure 16. 60/6-Db Shape Factor of Four-Section Filter

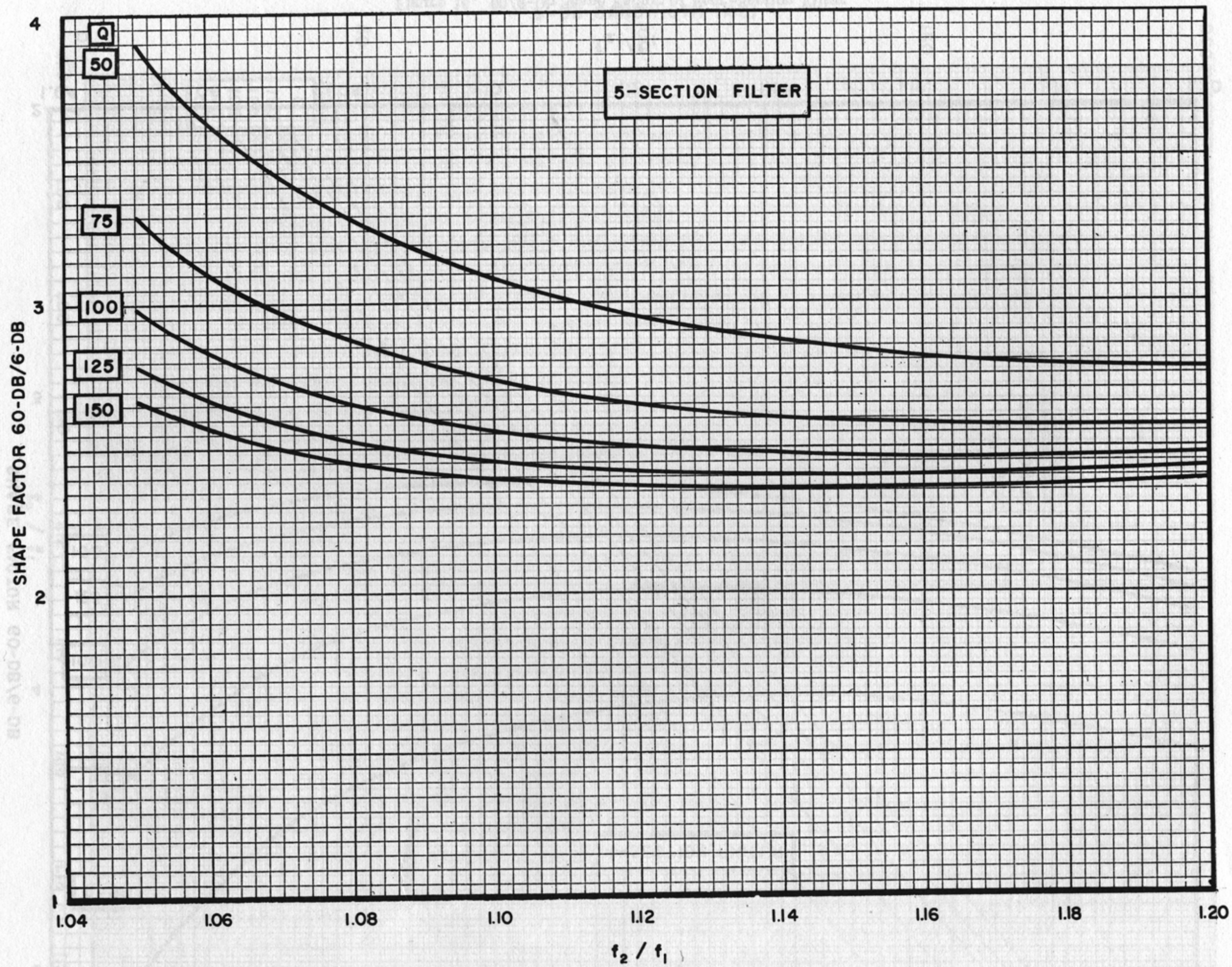

Figure 17. 60/6-Db Shape Factor of Five-Section Filter

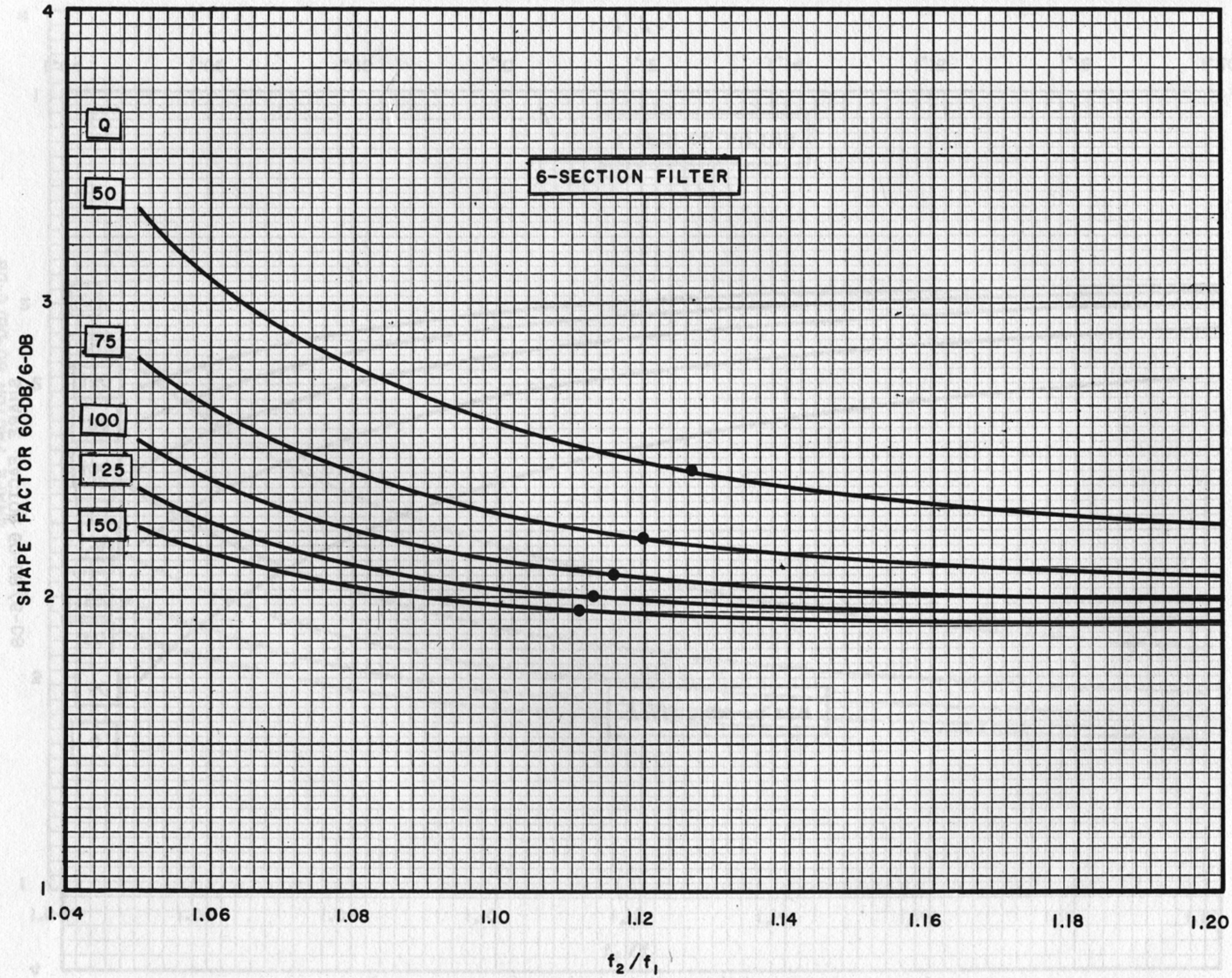

Figure 18. 60/6-Db Shape Factor of Six-Section Filter

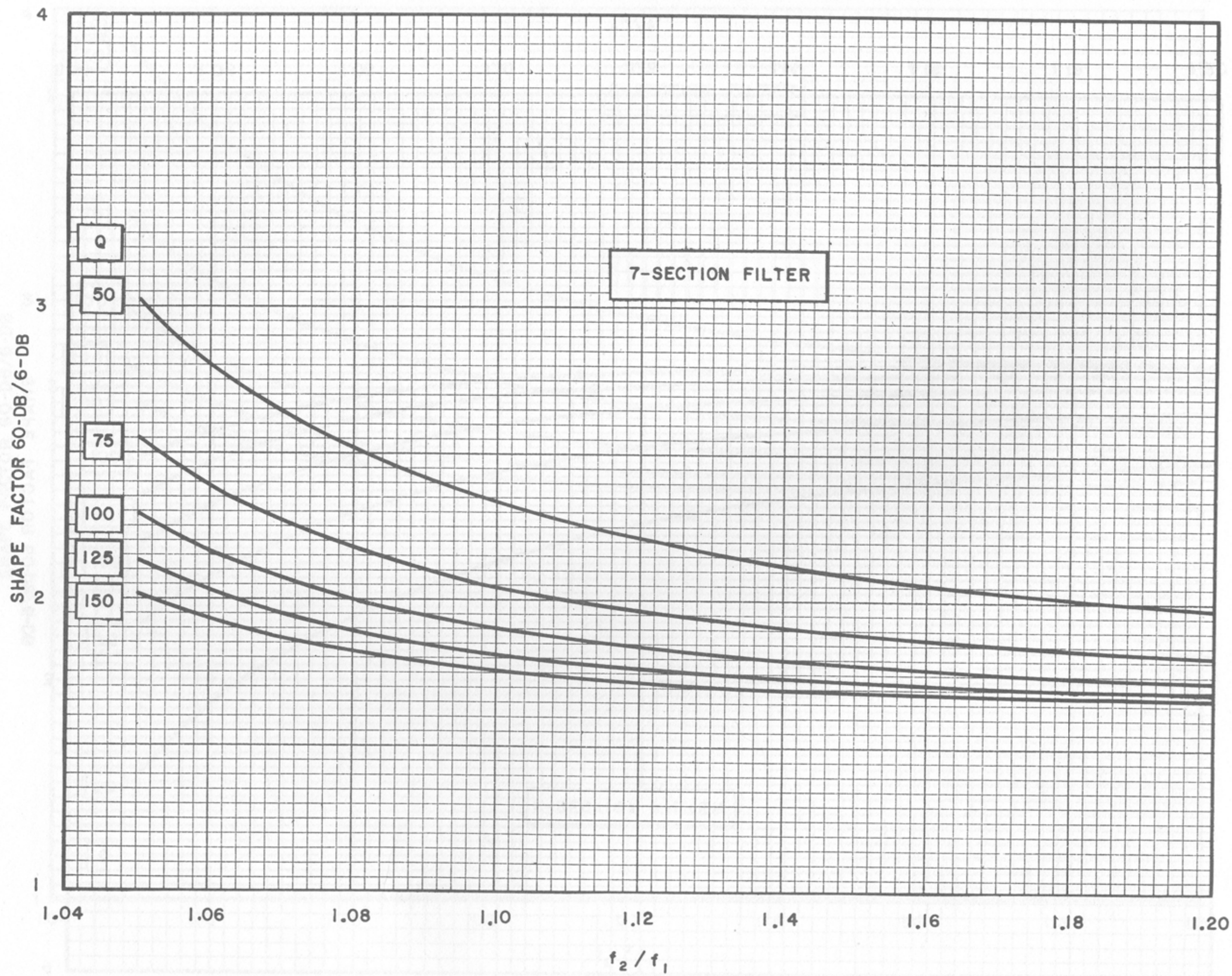

Figure 19. 60/6-Db Shape Factor of Seven-Section Filter

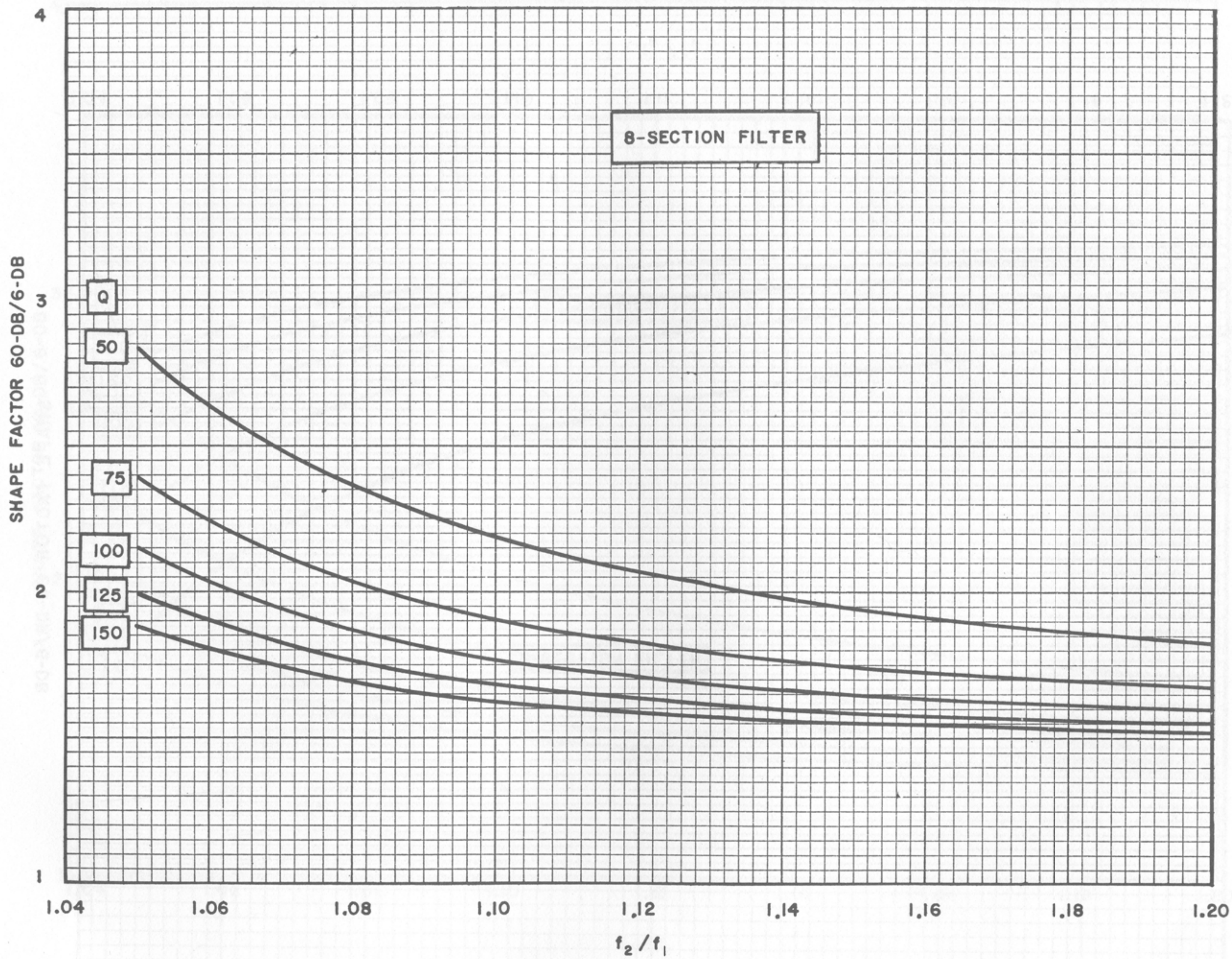

Figure 20. 60/6-Db Shape Factor of Eight-Section Filter

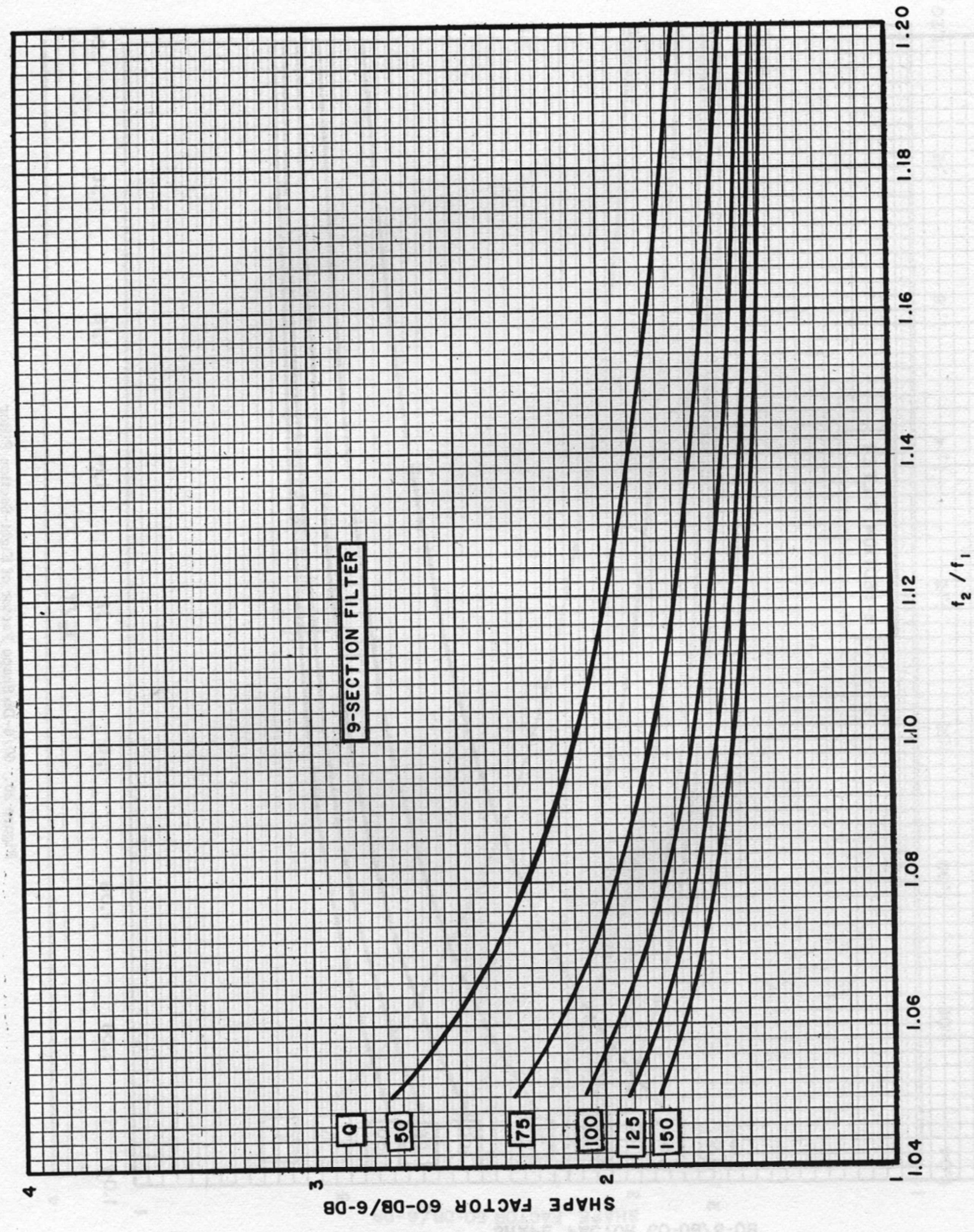

Figure 21. 60/6-Db Shape Factor of Nine-Section Filter

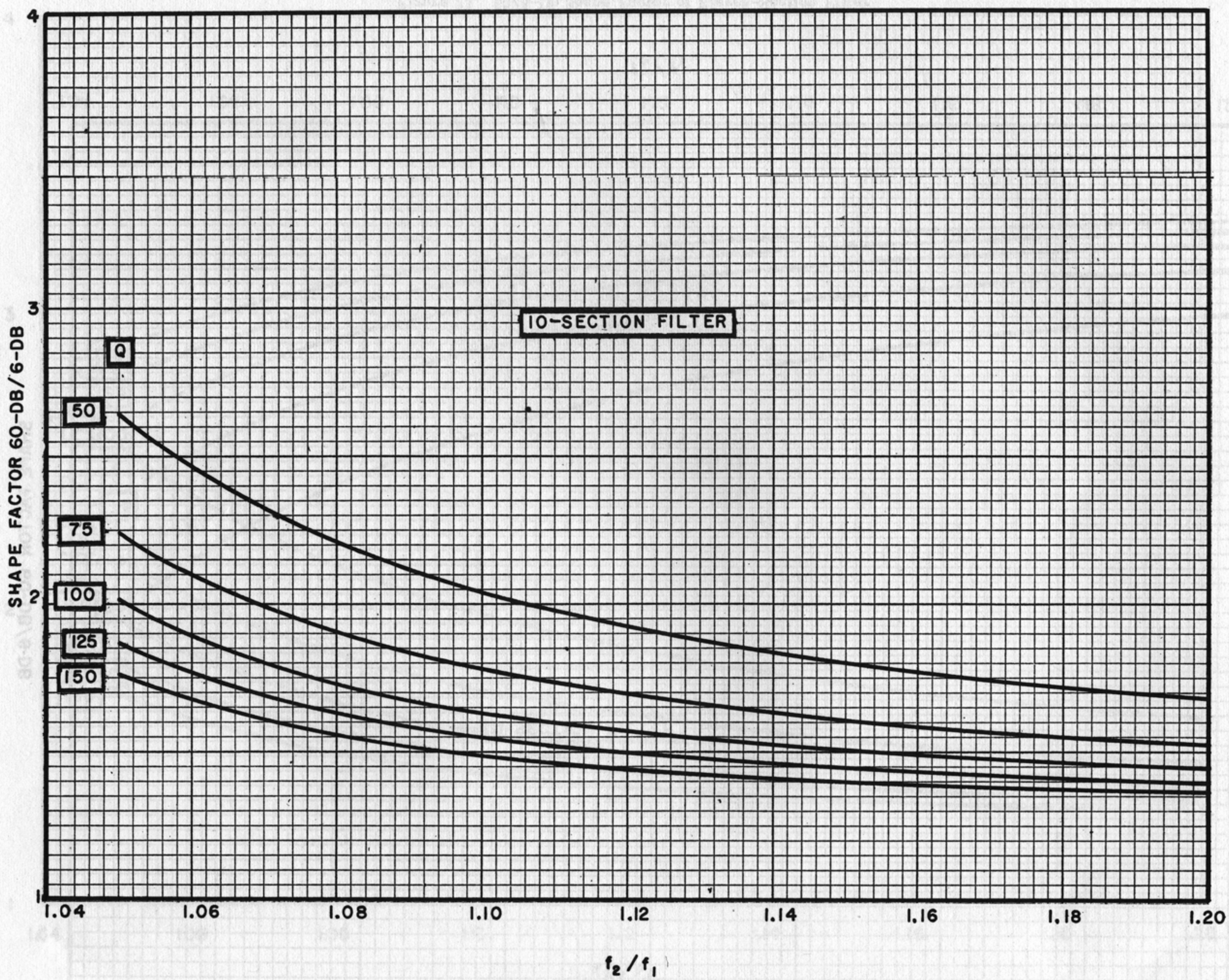

Figure 22. 60/6-Db Shape Factor of Ten-Section Filter

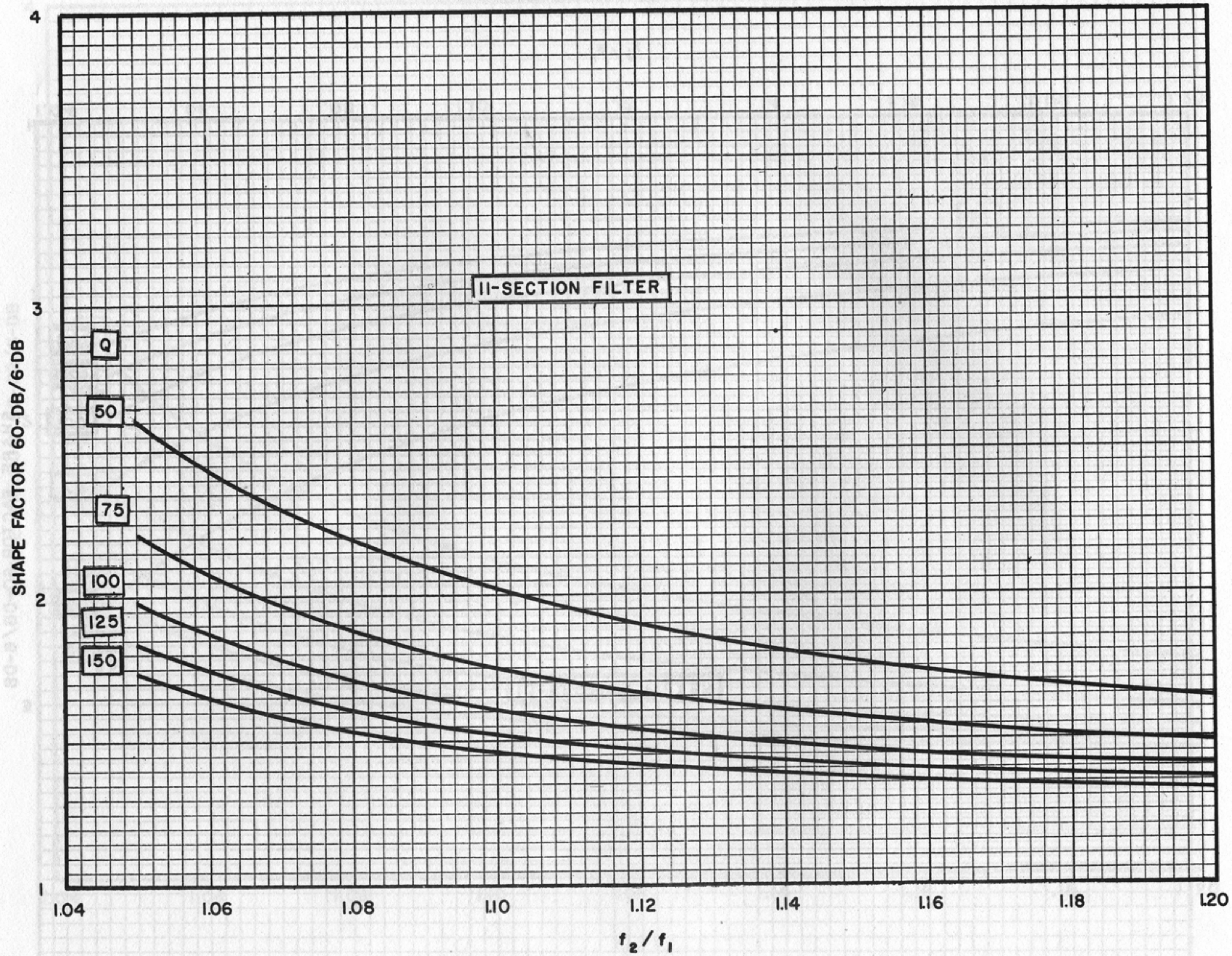

Figure 23. 60/6-Db Shape Factor of Eleven-Section Filter

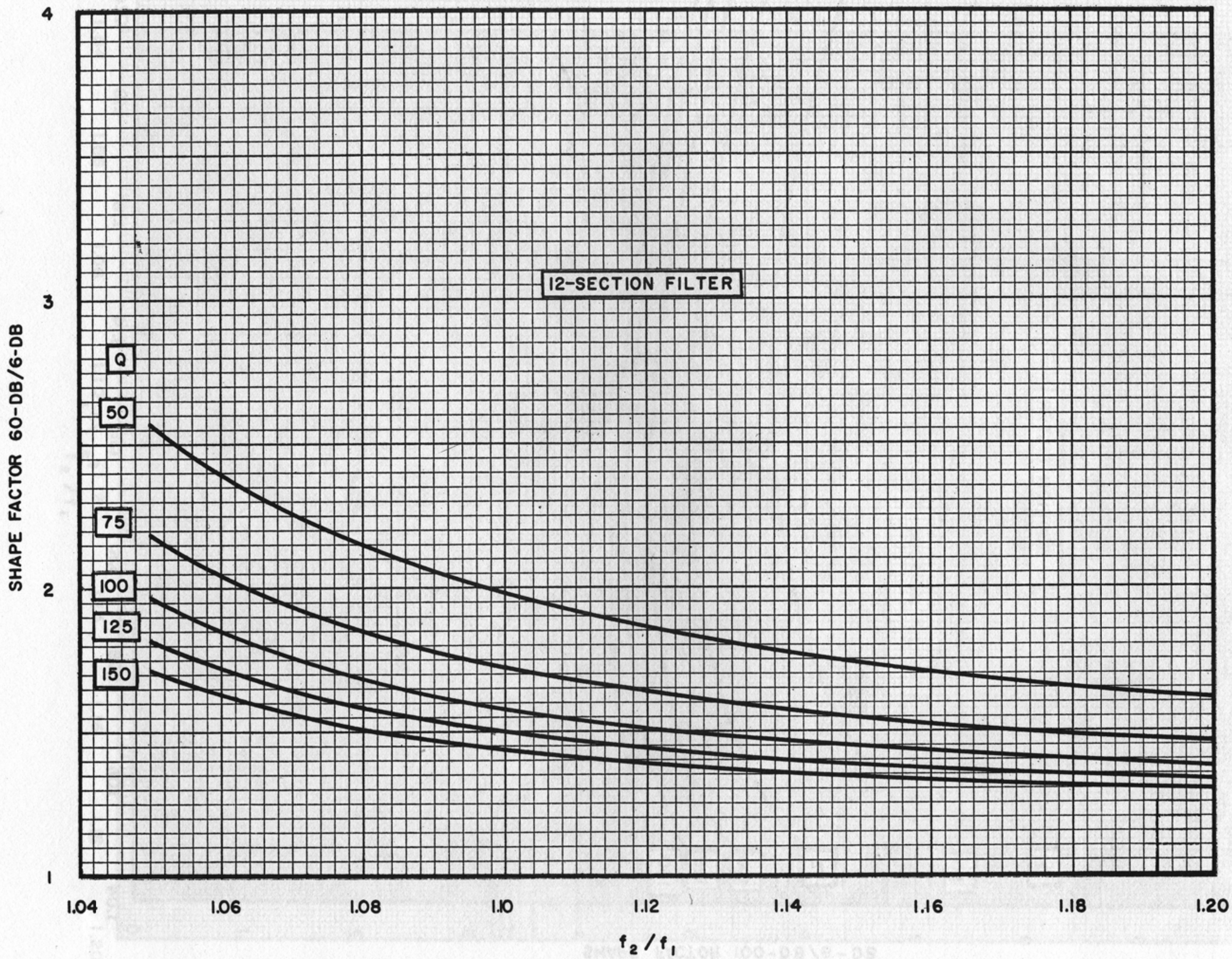

Figure 24. 60/6-Db Shape Factor of Twelve-Section Filter

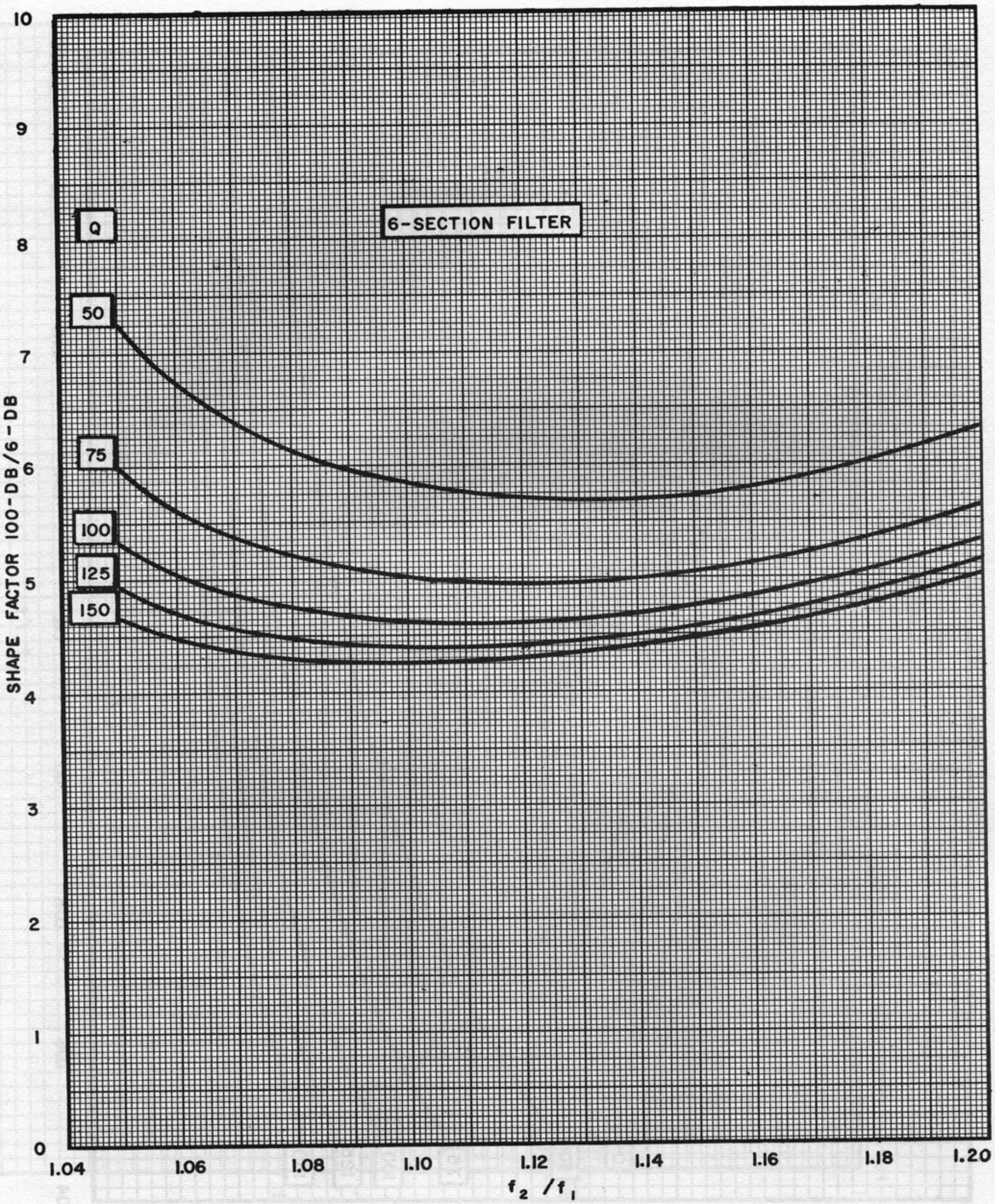

Figure 25. 100/6-Db Bandwidth of Six-Section Filter

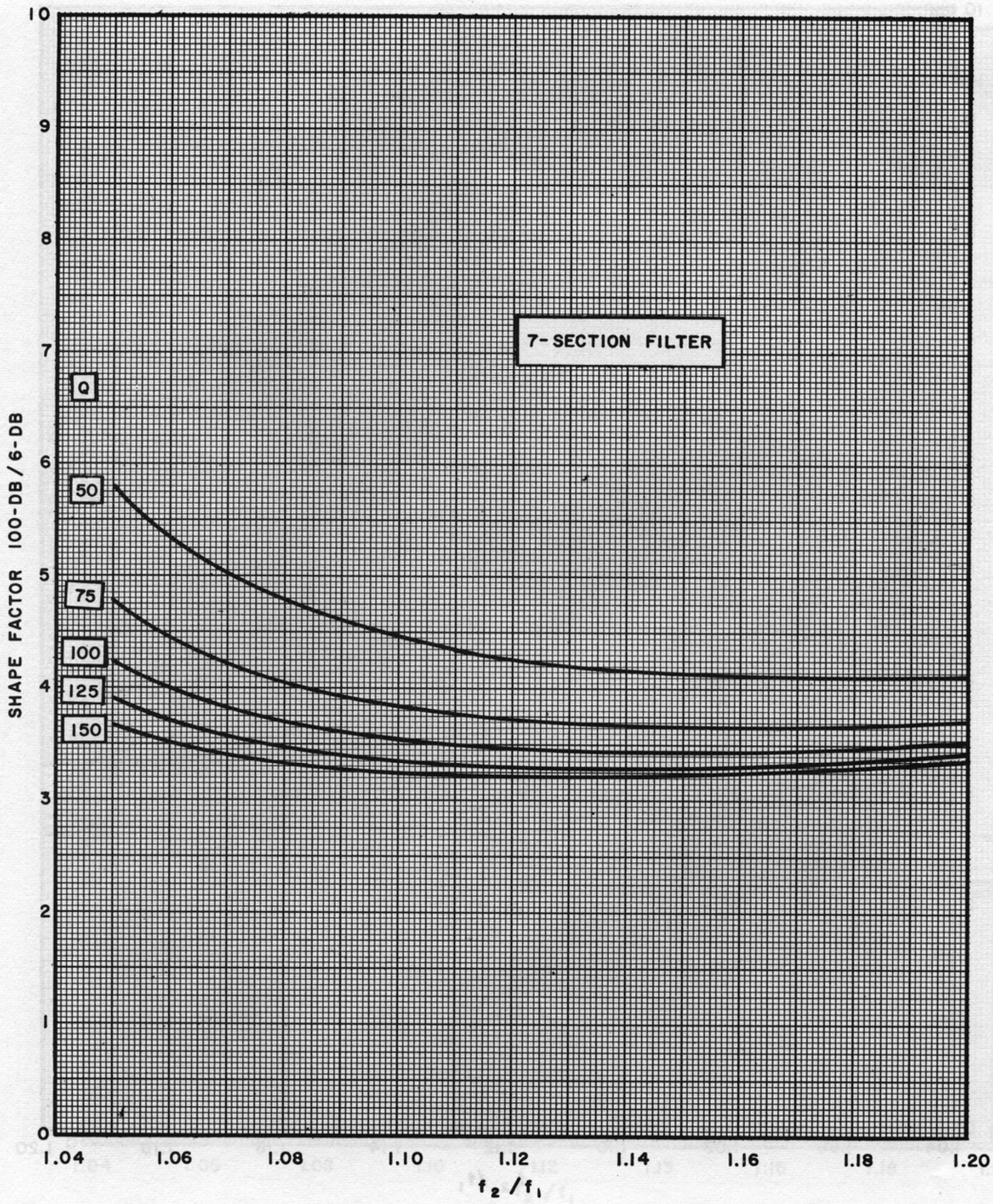

Figure 26. 100/6-Db Bandwidth of Seven-Section Filter

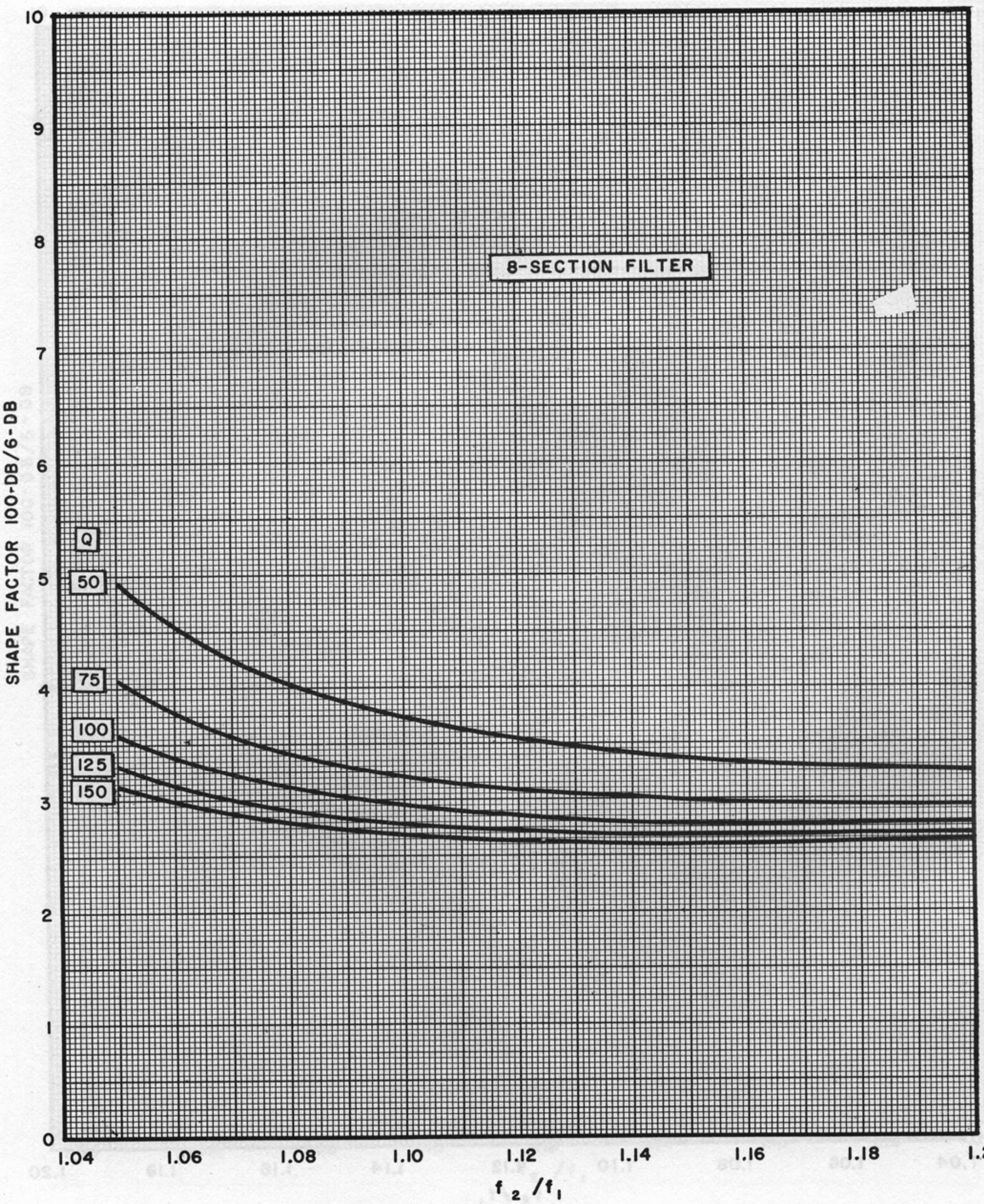

Figure 27. 100/6-Db Bandwidth of Eight-Section Filter

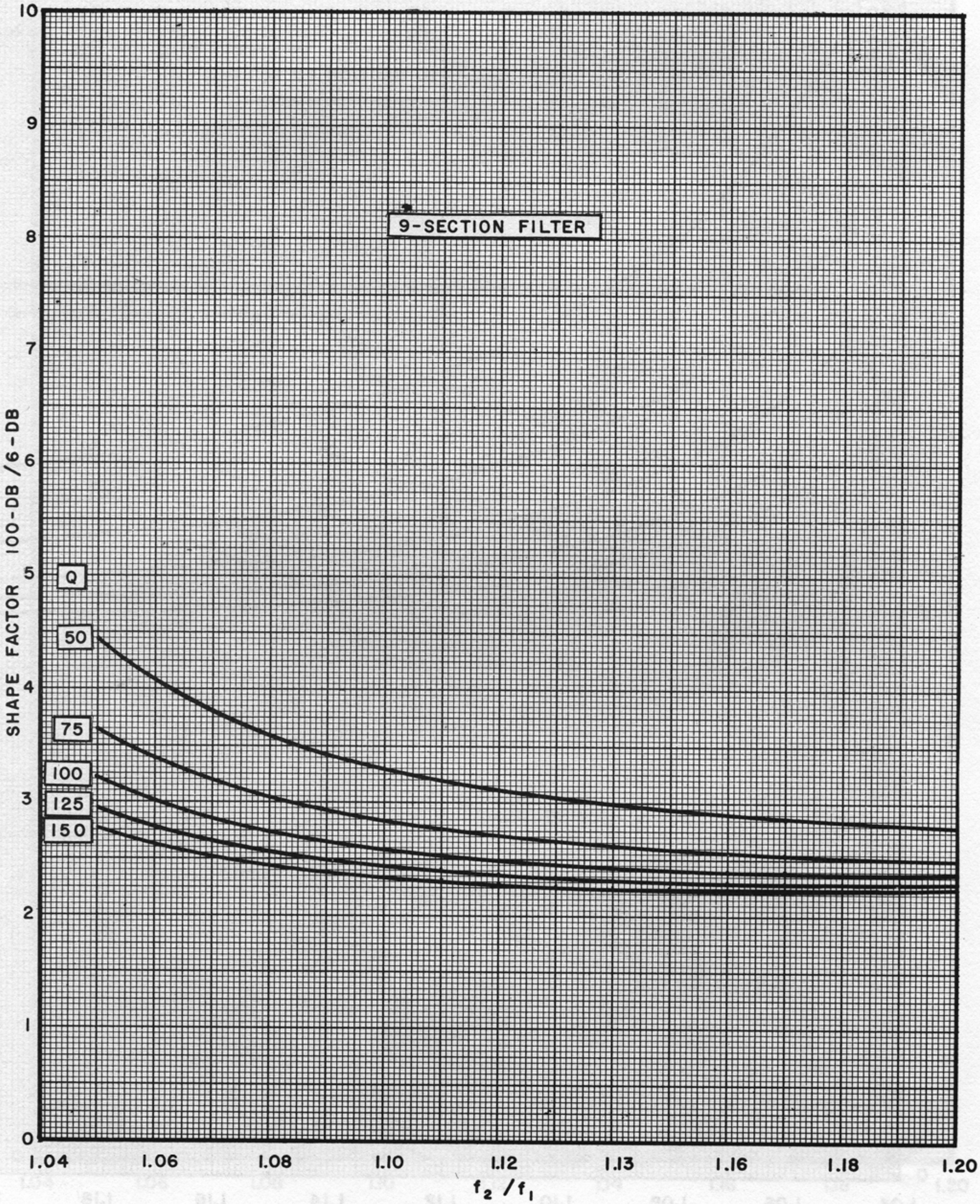

Figure 28. 100/6-Db Bandwidth of Nine-Section Filter

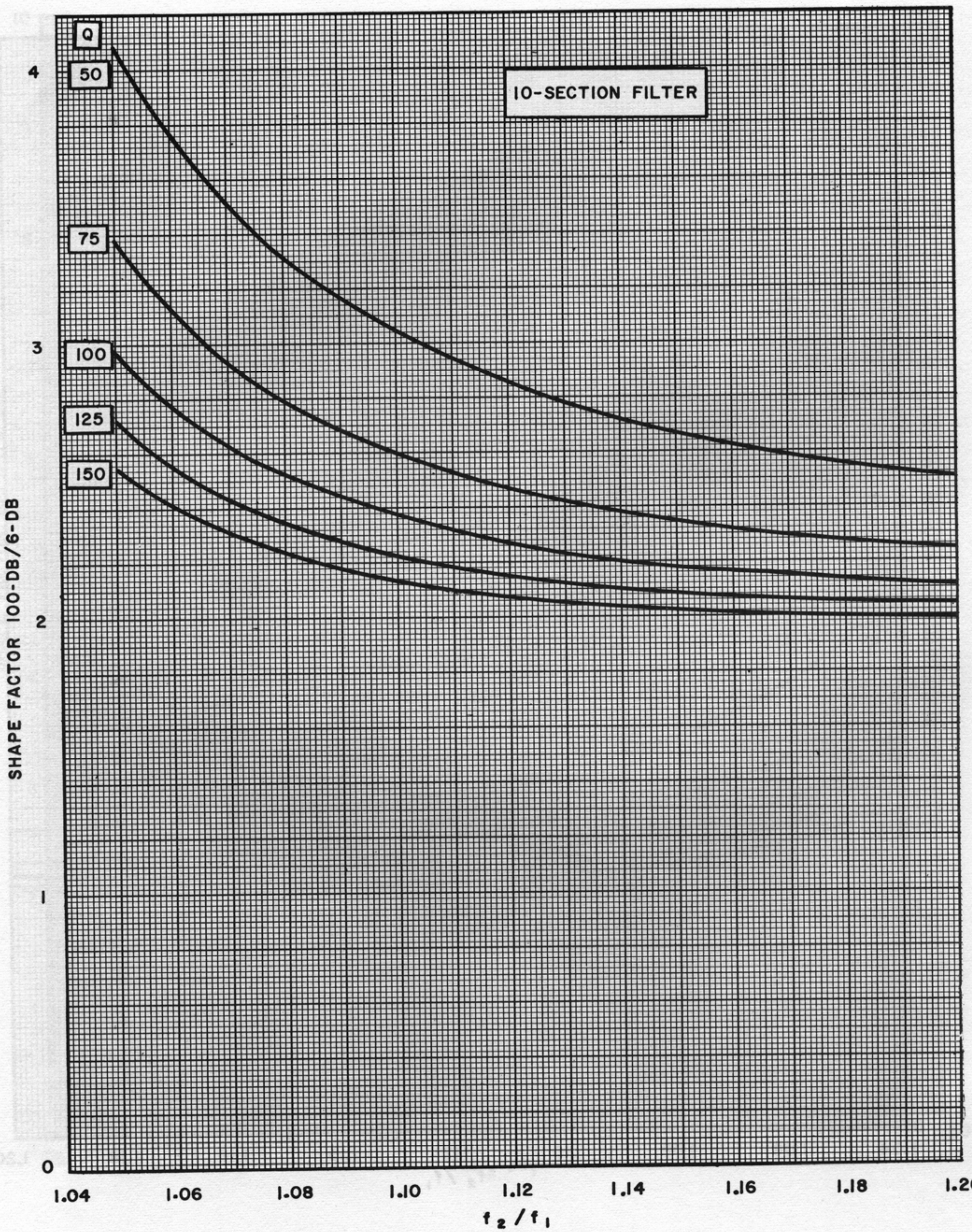

Figure 29. 100/6-Db Bandwidth of Ten-Section Filter

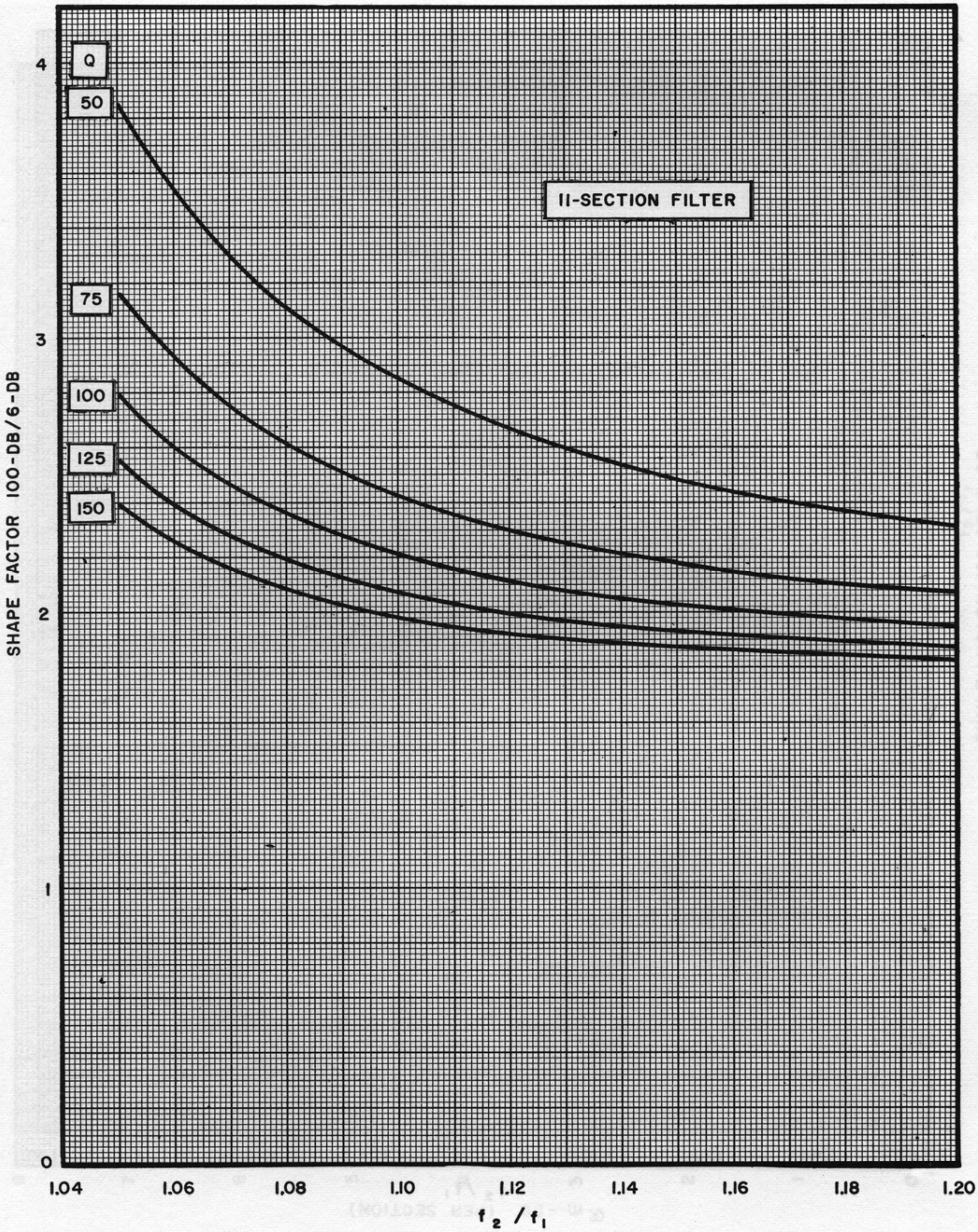

Figure 30. 100/6-Db Bandwidth of Eleven-Section Filter

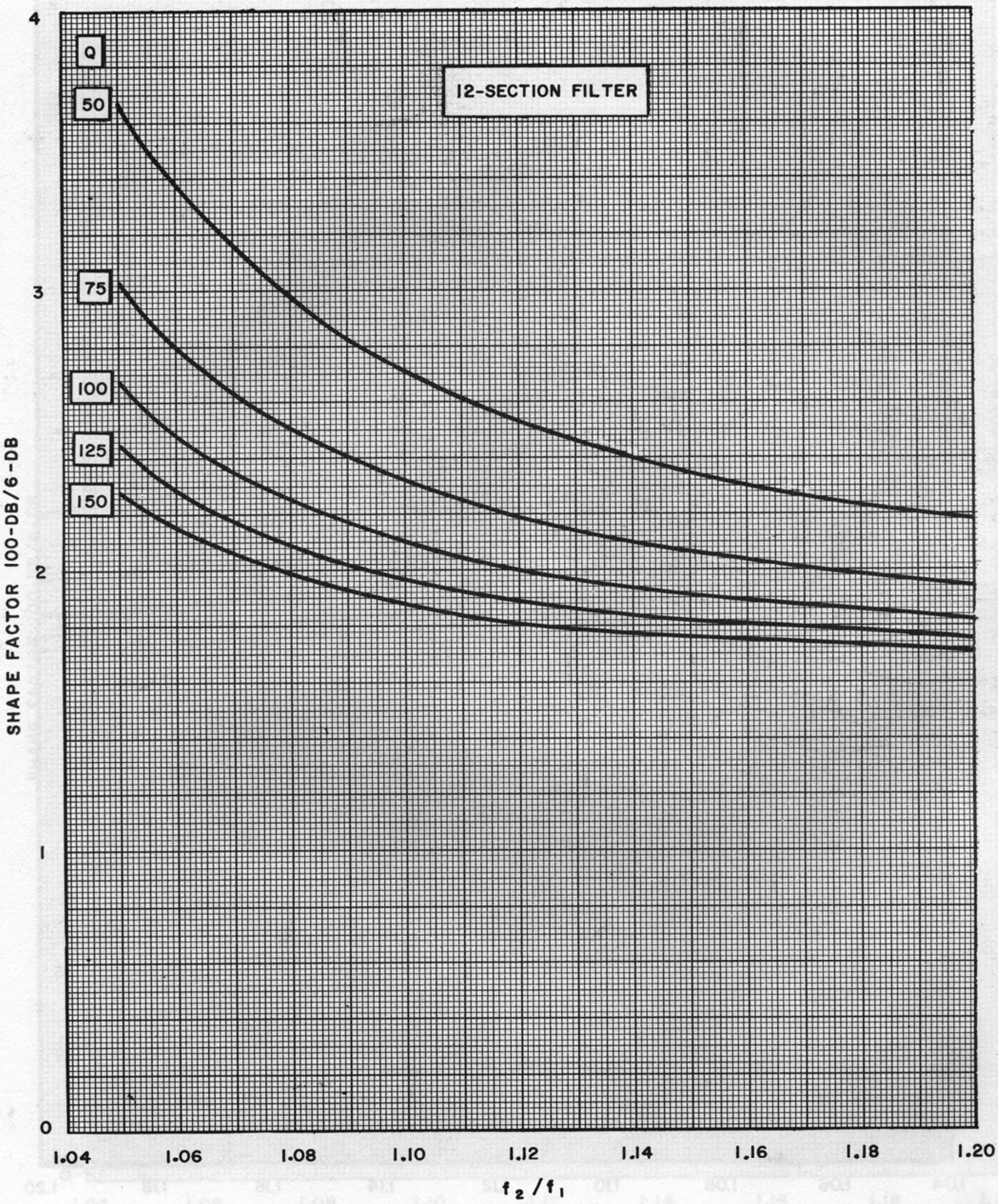

Figure 31. 100/6-Db Bandwidth of Twelve-Section Filter

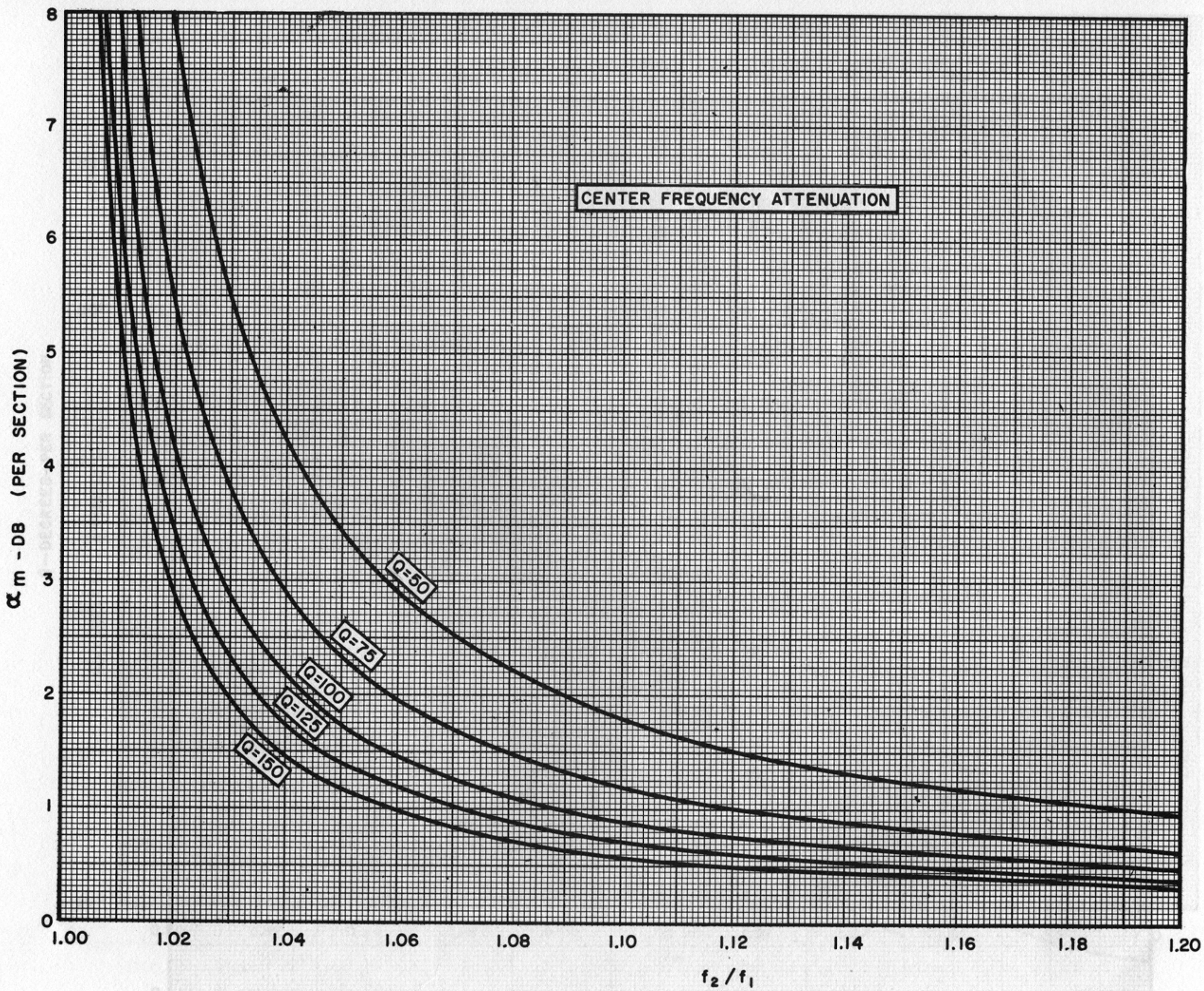

Figure 32. Center Frequency Attenuation for Three-Element Filter Section

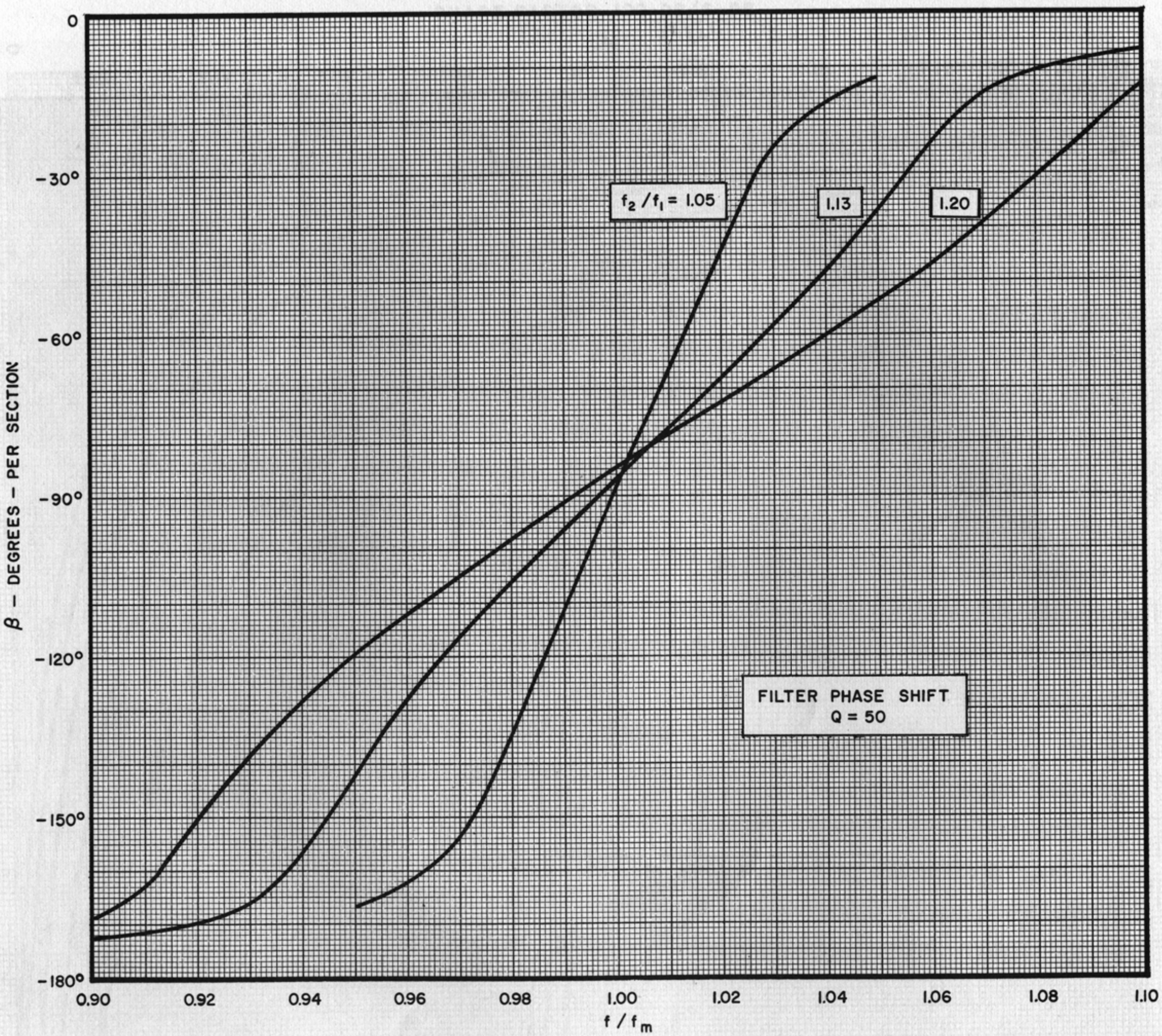

Figure 33. Phase Shift of Three-Element Filter Section (Q = 50)

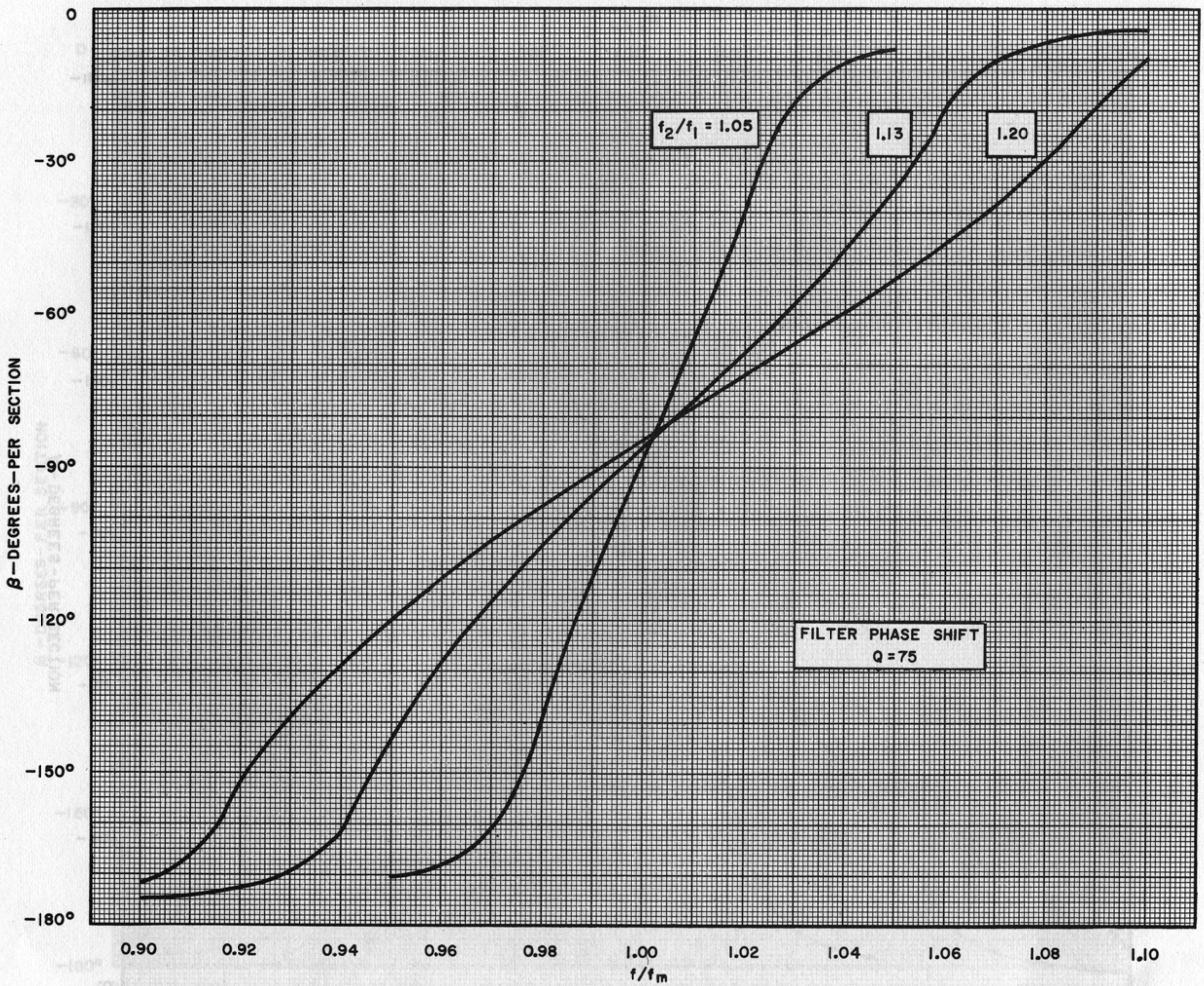

Figure 34. Phase Shift of Three-Element Filter Section (Q = 75)

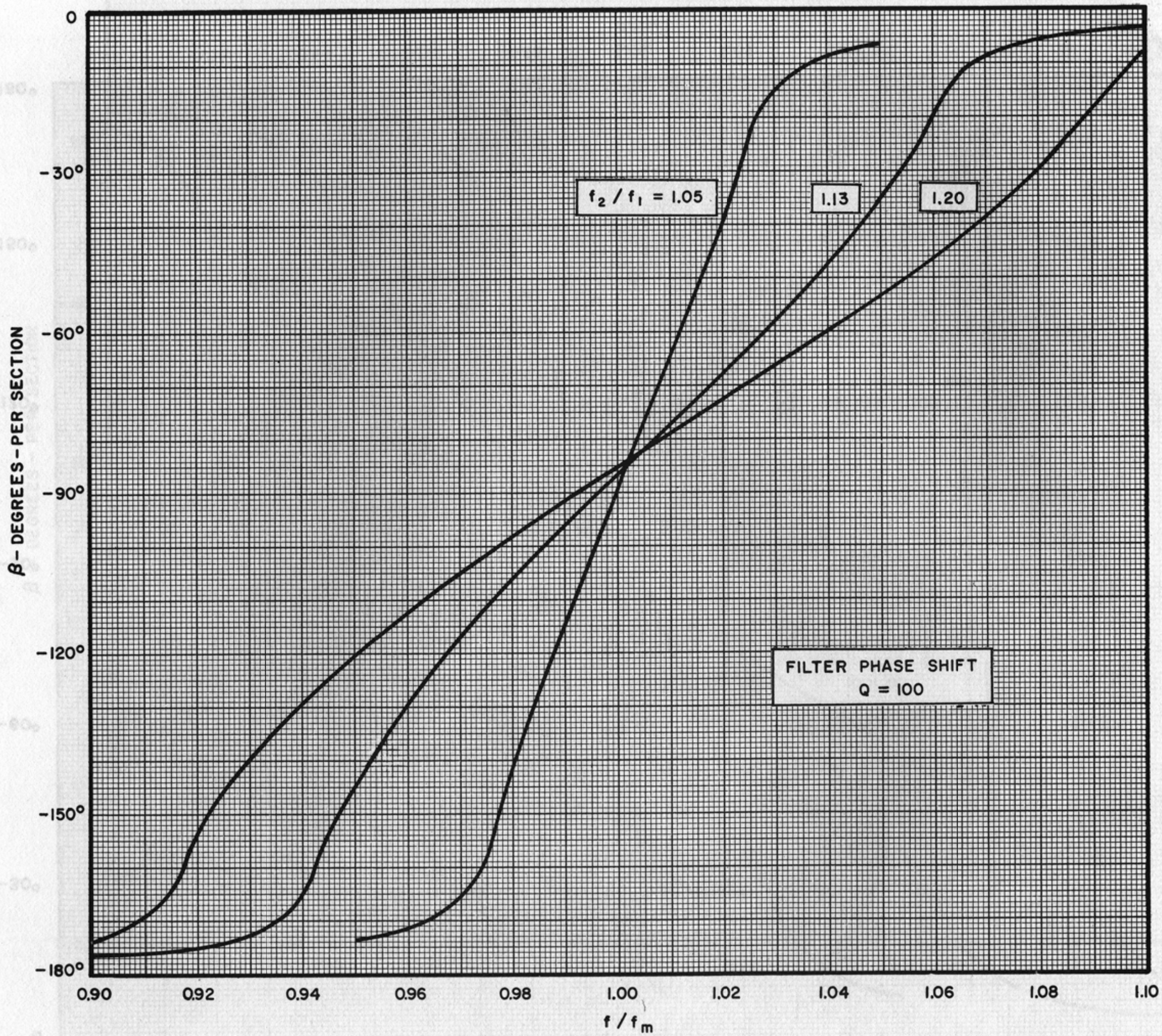

Figure 35. Phase Shift of Three-Element Filter Section (Q = 100)

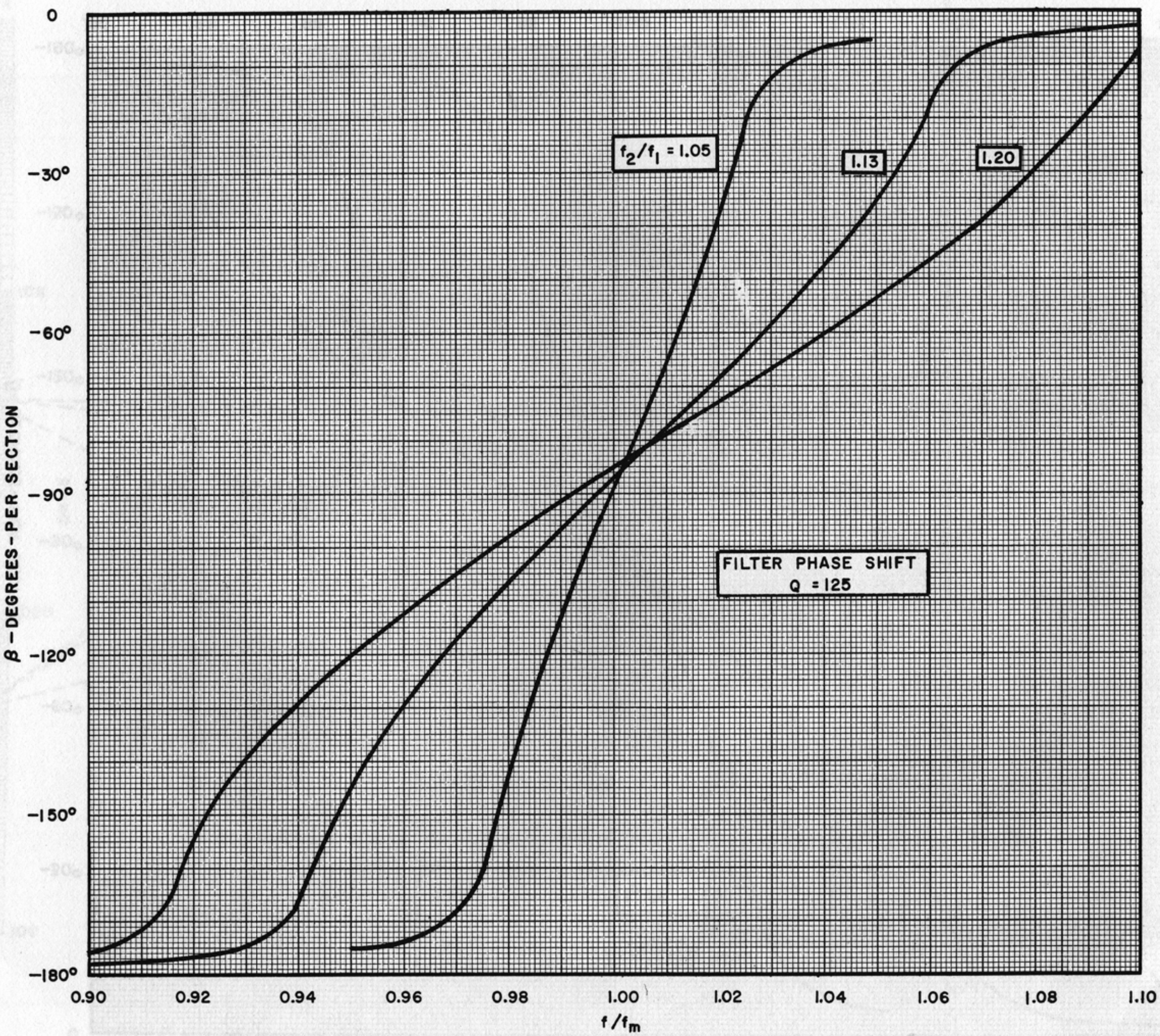

Figure 36. Phase Shift of Three-Element Filter Section (Q = 125)

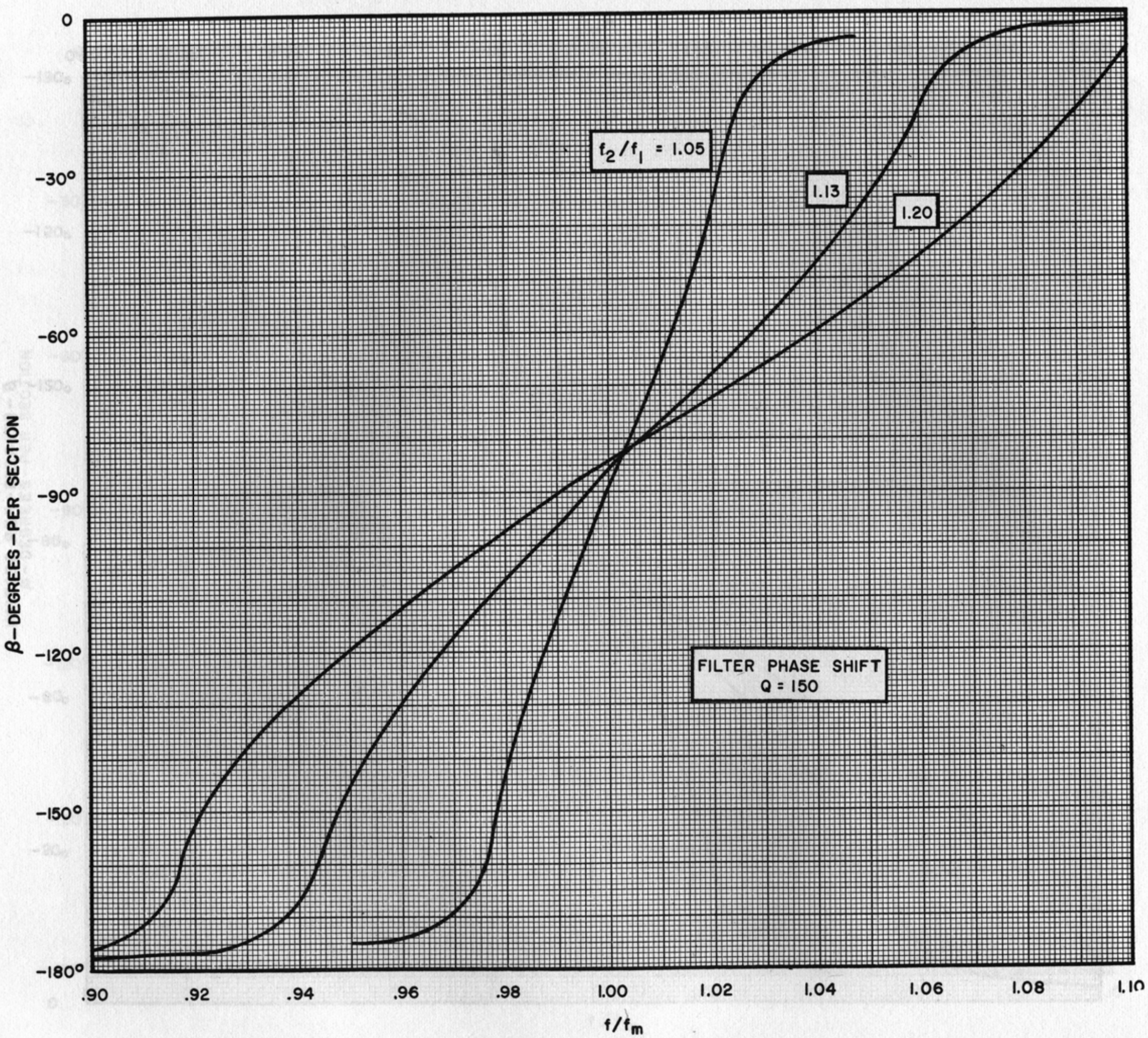

Figure 37. Phase Shift of Three-Element Filter Section (Q = 150)

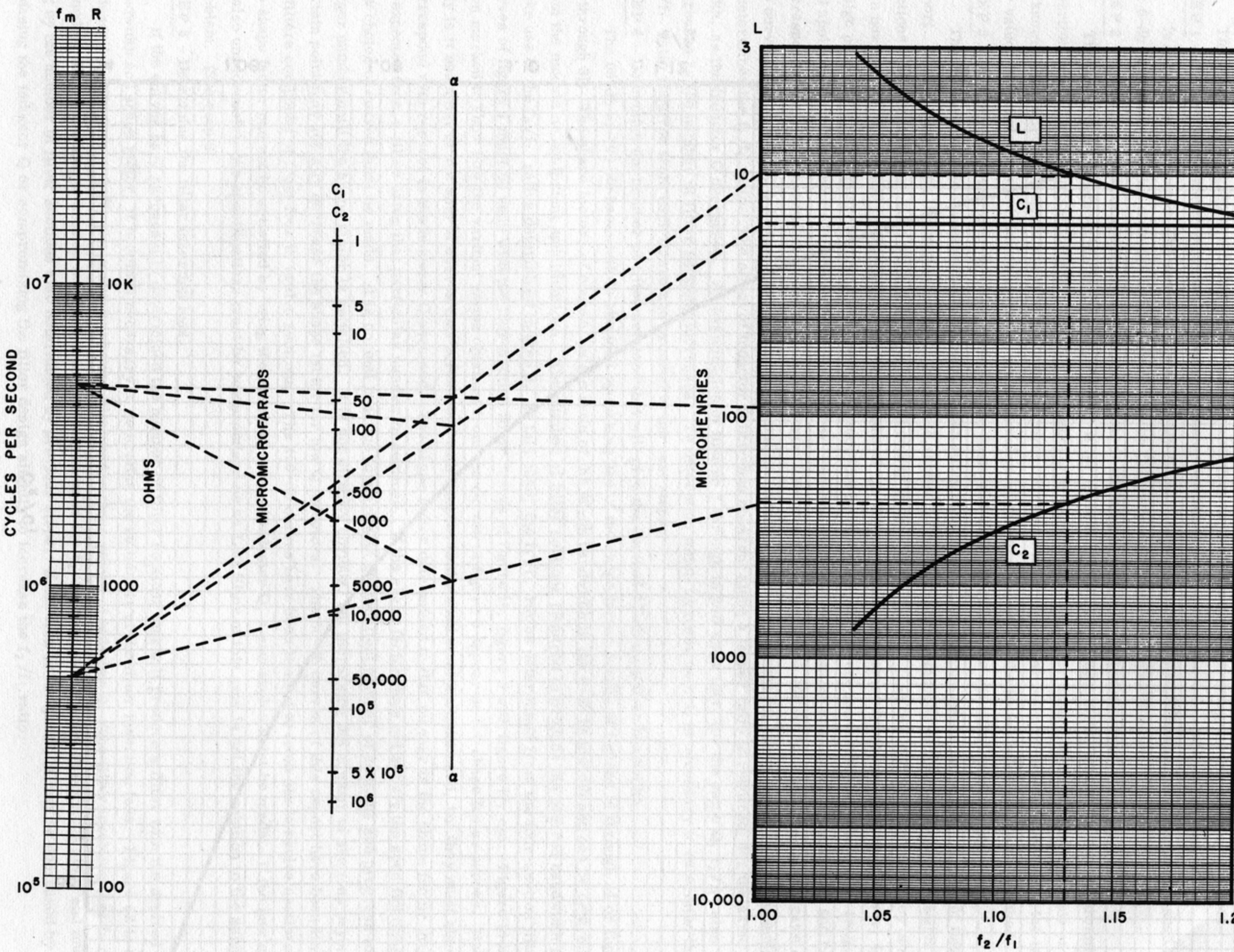

Figure 38. L–C Filter Component Nomograph

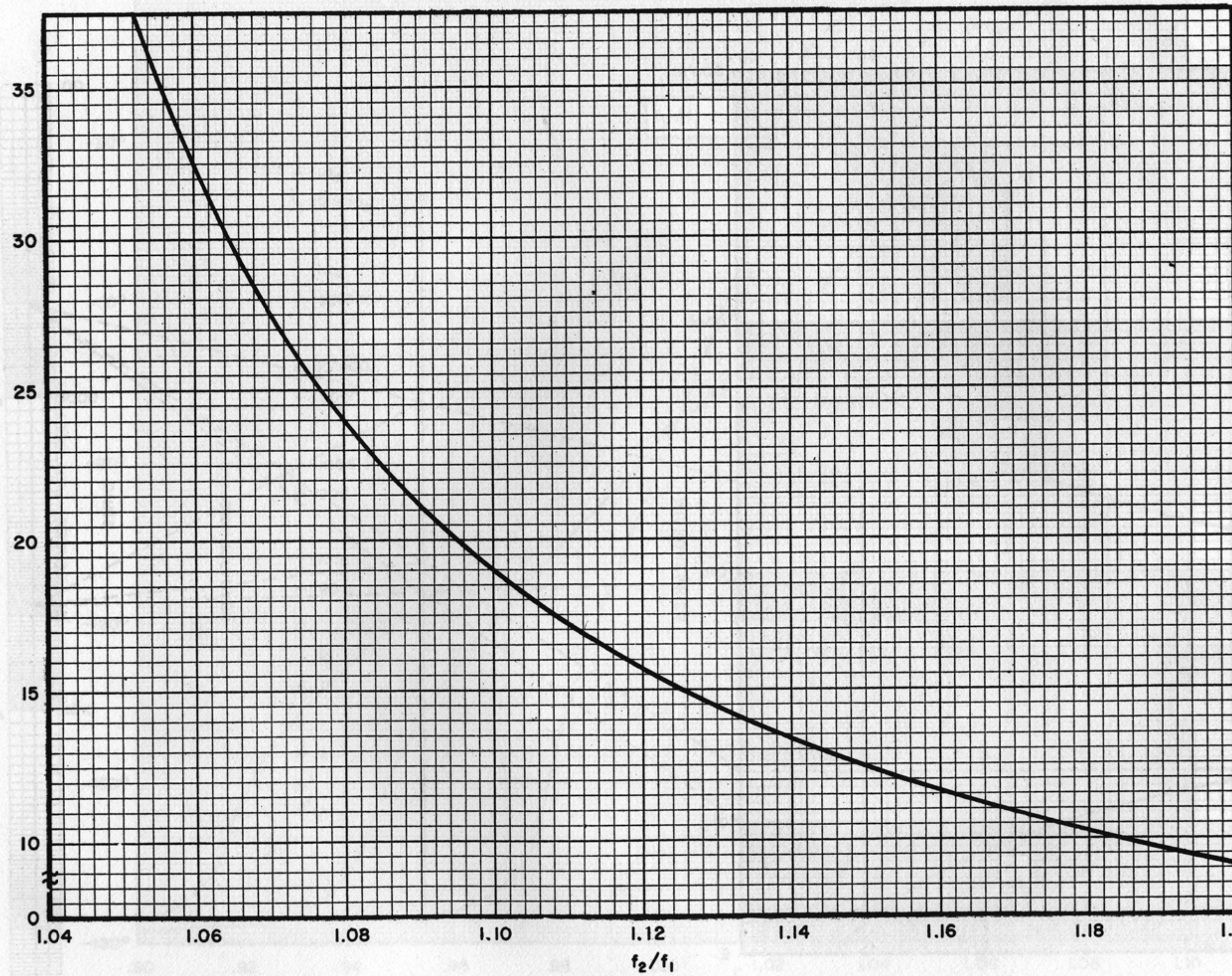

Figure 39. C_2/C_1 Versus f_2/f_1 for Three-Element Filter Section

Section V

The following paragraphs will outline the design procedure for a bandpass filter using the curves of this report:

STEP 1 - Determine the known factors and requirements of the filter.

In the majority of cases, the center frequency of the filter is specified and allowable limits have been established for the 6-db bandwidth, 60-db bandwidth and insertion loss by the equipment or module specification.

STEP 2 - Determine the Q or quality factor of the inductors to be employed in the filter construction.

The inductor Q influences the performance of the filter more than any other single item, since it affects bandwidth and insertion loss. The important thing is not that a very high Q is essential for the filter design, but that the magnitude of the Q be known. The effect of the inductor Q will become apparent in later steps. Since the inductor Q will depend upon the inductor value, frequency, space limitations, etc., an assumed value must be used until the actual value can be determined.

STEP 3 - Determine the ratio, f_2/f_1, that will give the desired 6-db bandwidth at the specified center frequency.

The ratio, f_2/f_1, is a controlling parameter of the filter design and is used to correlate the various curves of this report. Figures 7 through 15 are plots of 6-db bandwidth versus f_2/f_1 for filters composed of from four through twelve sections. The curves have been plotted using a defined center frequency of one mc. The bandwidth at any frequency other than one mc can be obtained by multiplying the bandwidths of the curves by the new center frequency in mc. However, for design purposes it is simpler to multiply the desired bandwidth by the ratio between one mc and the desired center frequency and apply this computed bandwidth directly to the curves. For example: 50-kc bandwidth at a center frequency of 500 kc corresponds to a 100 kc bandwidth at one mc. A horizontal line corresponding to this 100-kc bandwidth has been drawn on the curves of figure 9. The intersection of this horizontal line and the bandwidth curves for the various values of inductor Q determines the f_2/f_1 ratio. The effect of inductor Q becomes apparent since a larger ratio is required, for the same bandwidth, as the inductor Q decreases. An examination of figures 7 through 15 will show that, for the same Q, f_2/f_1 also increases as the number of filter sections increases. To determine which figure to use, it is necessary to determine the f_2/f_1 ratio required from each figure and proceed to the next steps.

STEP 4 - Determine the number of filter sections that will give the desired 60-db or 100-db bandwidth.

The 60-db or 100-db bandwidth is obtained by making use of the shape factor curves of figures 16 through 24 or figures 25 through 31. The shape factor, of course, is the ratio of the 60-db bandwidth to the 6-db bandwidth and can be determined from the module or equipment specifications. The number of filter sections required to produce the desired bandwidth must make use of the f_2/f_1 ratios determined in step 3. In this case, the f_2/f_1 ratios determined from the four-section filter curves of figure 7 must be used with the four-section filter curves of figure 16, figure 8 with figure 17, etc. Figure 18 has been marked to show how the various ratios obtained from the curves of figure 9 appear on the shape factor curves. Not only is it essential to use sets of curves that correspond with respect to the number of sections used, but they must also correspond with respect to the inductor Q assumed. The shape factor determined in this manner is the shape factor that can be expected from a filter using that particular number of sections. If the shape factor does not satisfy the specifications, a new choice of curves must be made. It is normal practice to start with the smallest number of sections and progress to a larger number until the shape factor is satisfied. It is not directly obvious, but increasing the number of sections beyond a certain point may actually decrease the shape factor for low Q inductors. The 60-db curves of this report have been limited to filters composed of from four to twelve sections. The 100-db curves have been limited to from six to twelve sections. If the shape factor can not be satisfied by using the assumed value of inductor Q, a new choice must be made. If the new choice is also unsuccessful, the specifications must be compromised or an L-C filter of this type is not the solution to the selectivity problem. (See step 9)

STEP 5 - Determine the filter insertion loss.

If the curves have successfully determined the number of filter sections that will produce a filter with the desired 6-db bandwidth and shape factor, it becomes necessary to determine the suitability of the filter insertion loss. Figure 32 is a plot of insertion loss versus f_2/f_1 for a single-section filter and various values of inductor Q. The filter insertion loss is determined by finding the single-section insertion loss corresponding to the f_2/f_1 ratio determined in previous steps and multiplying by the number of filter sections also determined by previous steps. If the loss is too great, it may be decreased by increasing the inductor Q or compromising the filter design slightly to increase the f_2/f_1 ratio.

STEP 6 - Determine the value of C_1, C_2 and L.

The component values required for the construction of the filter may be obtained from the following equations which make use of the f_2/f_1 ratio determined in previous steps:

$$C_1 = \frac{\frac{f_2}{f_1} + 1}{4\pi f_m R \left(\frac{f_2}{f_1}\right)^{1/2}} \tag{18}$$

$$C_2 = \frac{1}{4\pi f_m R \left(\frac{f_2}{f_1}\right)^{1/2} \left(\frac{f_2}{f_1} - 1\right)} \tag{19}$$

$$L = \frac{R\left(\frac{f_2}{f_1} - 1\right)}{4\pi f_m \left(\frac{f_2}{f_1}\right)^{1/2}} \tag{20}$$

The value of C_2 found by equation (19) includes stray capacities that exist due to physical placement of parts in the filter. Therefore, the actual component value must be reduced to take the stray capacity into consideration.

The characteristic impedance of the filter, R, appears in each of the component equations. This value may be specified by the circuit application, but it is more desirable if this impedance can be adjusted to permit the selection of standard component values. Figure 38 is a reasonably accurate nomograph which has been constructed to simplify the determination of component values.

STEP 6a - Determine the value of C_1 from nomograph. (See example plotted on nomograph figure 38.)

Using the f_2/f_1 ratio determined in previous steps, locate this point on the C_1 curve plotted on the logarithmic chart on the right-hand side of the nomograph. Extend this line horizontally to the left to intersect with the vertical line corresponding to an f_2/f_1 ratio equal to 1.00. Locate the center frequency of the filter on the vertical logarithmic scale labeled f_m. Connect the point of intersection on the chart with the point on the f_m scale by means of a straight-edge and mark the intersection of this line with the vertical line labeled a-a. Locate the assumed value of characteristic impedance on the vertical logarithmic scale labeled R. Connect this point with the intersection previously marked on the a-a scale by means of a straight-edge and mark the intersection of this second line with the vertical scale marked C_1. This intersection determines the value of C_1. However, the value may be obtained more accurately by connecting this point with either the 100, 1000, 10,000 or 100,000 point on the R scale by means of a straight-edge and reading the integers at the point of intersection on the L scale. The decimal point will be determined by the location on the C_1 scale.

STEP 6b - Determine the value of C_2 from nomograph.

The value of C_2 is determined in the same manner as the value of C_1, determined in step 6a, except the C_2 curve and scale are used. Figure 39 is a plot of C_2/C_1 versus f_2/f_1 which may be useful in establishing the capacity ratio.

STEP 6c - Determine the value of L from the nomograph.

The value of L is determined in the same manner as the value of C_1, determined in step 6a, except the L curve and scale are used and only two intersections are required to obtain reasonable accuracy.

STEP 7 - Construction of filter.

The previous steps have determined the component values and general characteristics that may be expected from the filter design. The construction of the filter will depend upon the inductor configurations, space limitations, etc., and will be left to the engineer's discretion. The design of the filter to this point has been based on the assumption of an inductor Q. Now, the Q must be determined more precisely so that the design steps may be retraced to give more accurate design information. Component tolerances and other circuit variations should also be taken into consideration while using the charts of this report so that proper compensation can be assigned. As previously stated, the center frequency of the filter as defined in this report may not equal the center frequency determined by the 6-db attenuation frequencies, so the filter must actually be designed slightly off-center to correct this situation. It should also be remembered that the shunt elements at the ends of the filter differ from the center elements (Refer to figure 1-a). The filter should be terminated in its characteristic impedance to obtain the predicted attenuation characteristic.

STEP 8 - Filter alignment.

The alignment of the filter is relatively simple. The filter is fed from a generator set at the corrected center frequency and having a source impedance equal to the characteristic impedance of the filter. The impedance may be obtained by connecting a "make-up" resistor in series with the generator lead. An a-c voltmeter is connected directly across the input terminals of the filter. Then, the second set of shunt components (from the input end of the filter) is shorted while the first set of shunt components is tuned for a maximum reading on the voltmeter. Next, the short is removed from the second set of shunt components and placed across the third set of shunt components while the second set is tuned for a minimum reading on the voltmeter. This procedure is continued by progressively shorting shunt components and alternately tuning for a maximum or minimum reading on the voltmeter. The final set of shunt components is tuned by loading the filter with its characteristic impedance, placing the voltmeter across the load, and tuning for a maximum output reading. This procedure is applicable for all filters without regard to the number of sections involved.

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