CHAPTER 6
PRINCIPLES OF SERVOMECHANISMS

1. DEFINITIONS

A servomechanism is most commonly defined as a feedback control system of which at least one element is mechanical in nature. Voltage regulators for power supplies, automatic volume control and automatic frequency control circuits used in radio equipment and thermostats used to regulate temperature in home heating equipment and in various electrical appliances are examples of feedback control systems. Power steering, gun turret positioning devices, and airplane autopilots are examples of servomechanisms.

In all feedback control systems, the quantity to be controlled is measured in some manner. This measured value is compared to a desired, or reference, value to form an error signal, and the controlling action is governed by some function of the error signal. Feedback control devices may contain electrical, mechanical, pneumatic, hydraulic, and other types of elements. Frequently, a human operator is included in the feedback loop.

2. A TYPICAL POSITION SERVO SYSTEM

Figure 6-1 illustrates a typical position follow-up type of servomechanism. This type of device can be used for repeating the position of a shaft at a remote point. For example, in an airborne radio transmitter, it could make the shaft of a precision oscillator follow a dial in the pilot's control box. Follow-up servos are also used to repeat the shaft position of a delicate instrument at a shaft where a large amount of torque is needed. In this case, the purpose of the servo is to provide torque amplification.

In figure 6-1 the reference input R is a shaft position. A voltage proportional to the shaft position is obtained from a linear potentiometer connected across a battery. This voltage is mixed with a voltage proportional to the controlled variable to form the error signal E which is then amplified and applied to the winding of a motor. The motor shaft is coupled through a gear train to the load, which in most cases is a friction device, although it sometimes contains a

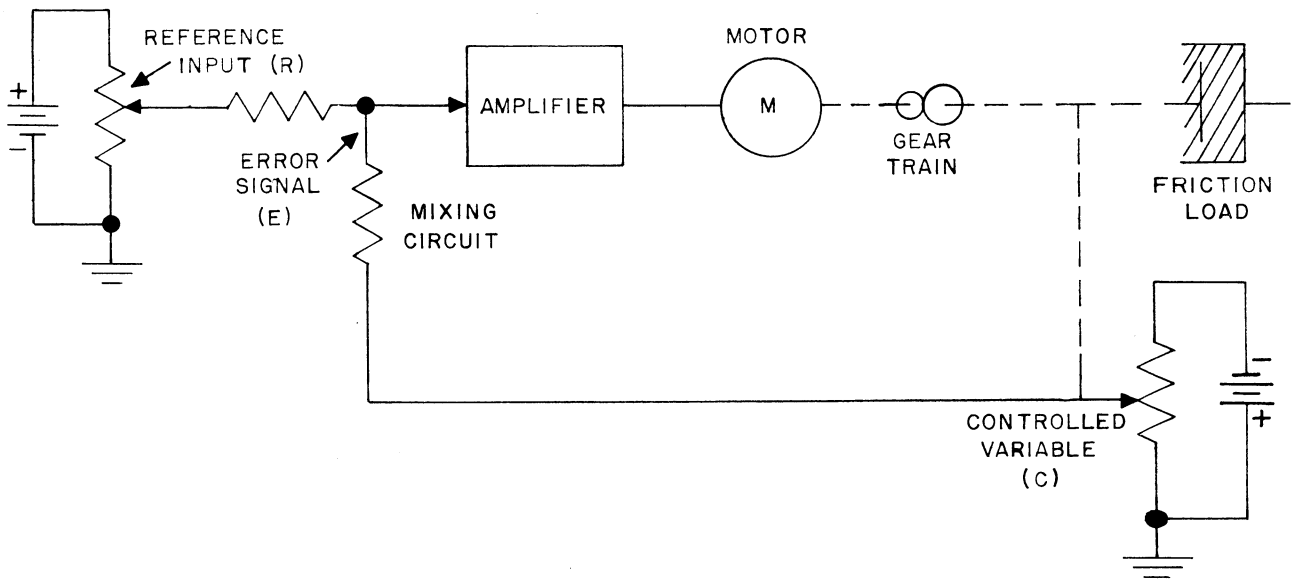

Figure 6-1. Position Follow-up Servomechanism

significant amount of inertia. Coupled to the load shaft is another potentiometer which produces a voltage C, proportional to the position of the output shaft. The mixing circuit subtracts the controlled voltage C from the reference voltage R to obtain error signal E. The amplifier drives the motor in such a way that if E is positive the motor turns in one direction, and if E is negative the motor reverses. When E is zero, the motor stops. Below the saturation level of the motor and amplifier, the motor speed is proportional to the error signal. A device of this kind is called a proportional controller.

Suppose that when power is applied to the circuit, the reference input R is greater than the controlled variable C so that E is positive. If the motor is connected so that a positive E causes the motor to turn in such a direction as to increase C, then as the motor turns, E will decrease, and the motor will slow down until R and C become equal. Then E is zero, and the motor stops. If the battery voltages feeding the reference input and the control variable potentiometers are equal, and the electrical angles of the two potentiometers are equal, this null condition will occur only when the load shaft is at the same angular position as the reference input shaft. If some external force, such as vibration, displaces the output shaft so as to increase C, E will become negative, and the motor will apply torque to the load in a direction to decrease C. This torque is proportional to the displacement, and the result is similar to the effect of a spring. In a feedback device of the type described above, the motor will run as long as there is an error signal sufficient to overcome the load friction. Consequently, the amount of residual error in this type of servo depends only on the amount of friction in the load, and not upon the value of c.

In contrast to this type of behavior is a class of feedback control devices typified by the voltage regulator circuit of figure 6-2. In this case, the controller is a tube whose plate to cathode resistance varies in proportion to its grid voltage. The feedback signal C, obtained from a voltage divider across the regulated output, is compared in a mixer circuit with a reference voltage obtained from a voltage reference tube. The resulting error signal E is amplified and applied to the grid of the controller. If the supply voltage suddenly drops, C will be reduced and E increased. The resulting change in the grid voltage of the controller will reduce its effective resistance and raise the output voltage. However, the output voltage can not become quite as high as it was before the supply voltage dropped, for if it did, E would return to its original value, and the resistance of the controller would become the same as it was before the decrease in the supply voltage. This condition could be corrected by replacing the controller with a motor driven rheostat and connecting the motor to the amplifier. In this case, as soon as the supply voltage dropped, the resulting error voltage would be amplified and applied to the motor causing it to drive the rheostat to reduce the resistance and increase the output voltage. The motor would continue to run until the error signal went to zero, at which time the output voltage must be up to its original value.

The accuracy with which a positioning servo can repeat the shaft position R is in most cases limited by the amount of torque required to move the friction load. Figure 6-3 shows the torque versus error signal characteristic of a typical servo with the motor stalled. The figure shows that a certain error voltage must exist to produce enough torque to move the load shaft against the starting friction. The battery feeding the controlled variable potentiometer determines how many volts of error signal E will be produced per degree displacement of the controlled variable shaft. Increasing this battery voltage will increase the stiffness of the system at the load shaft.

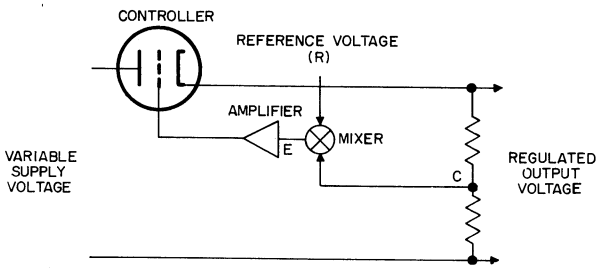

Figure 6-2. Voltage Regulator

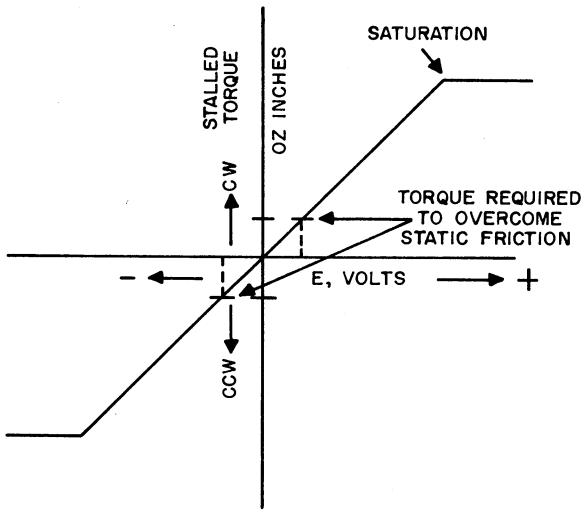

Figure 6-3. Idealized Torque versus Voltage Curve

Stiffness is defined as the reaction torque at the load shaft divided by displacement of the shaft. The greater the stiffness of the system, the smaller will be the residual error.

If the controlled variable shaft is required to follow the reference shaft when it is moving at a constant speed, the motor must turn the output shaft at the same speed as the reference input shaft. Since a certain amount of power from the motor is required to overcome the running friction of the load, there must be a fixed difference between R and C sufficient to produce an error signal capable of driving the motor at the required speed. This difference between R and C indicates that the controlled variable shaft lags behind the reference input shaft by a certain number of degrees, although both are traveling at the same speed. This type of error, called a dynamic tracking error, has a magnitude at any speed of reference input determined by the velocity constant of the servo. To obtain the velocity constant, the shaft of the controlled variable potentiometer is uncoupled from the gear train and displaced one degree from the null position, producing an error signal. The speed of the output shaft is measured, and the ratio of this speed in degrees per second to the error in degrees required to produce it is the velocity constant of the servo system. In figure 6-1, increasing either the gain of the amplifier or the voltage of the battery across the controlled variable potentiometer will increase the velocity constant as well as the stiffness of the servo, so that increasing the over-all gain of the system will decrease both the static friction error and the dynamic tracking error of the system.

3. SERVO STABILITY REQUIREMENTS

A servo system such as that of figure 6-1 may be represented by the block diagram shown in figure 6-4.

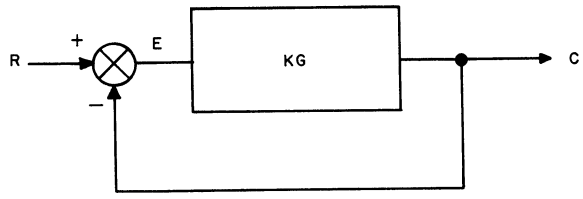

Figure 6-4. Servo Loop Block Diagram

The box labeled KG represents the amplifier, the motor, and the gear train. K is the gain constant of the system, in this case the velocity constant of the entire loop. It includes the amplifier gain, the motor velocity constant, and the gain of the reference input and controlled variable potentiometers. The constant K is independent of the frequency of the applied signals, but G is an expression that describes the frequency response or time response of the amplifier and motor to error signals. From figure 6-4,

$$E = R - C, \text{ and} \quad (1)$$

$$C = KGE \quad (2)$$

Solving (2) for E:

$$E = \frac{C}{KG} \quad (3)$$

Substituting (3) into (1):

$$\frac{C}{KG} = R - C \quad (4)$$

$$C = RKG - CKG \quad (5)$$

$$C(1 + KG) = RKG \quad (6)$$

Hence, the effective gain of the closed loop is:

$$\frac{C}{R} = \frac{KG}{1 + KG} \quad (7)$$

Since the amplifier and motor must be built with physically realizable components, the function G represents a certain finite bandwidth. The inertia of the motor will tend to slow down its response to high-frequency error signals, and since a roll-off in frequency response is accompanied by a phase shift, there will be some frequency at which the controlled variable C will lag the error signal E by 180 degrees. At this frequency the quantity KG becomes negative, and if KG approaches -1, the denominator of equation 7 approaches 0 so that $\frac{C}{R}$ approaches infinity. Physically, this can be interpreted to mean that an output C is obtained with no input R. This is the condition under which the loop will oscillate and is known as the Nyquist stability criterion. Because of the finite bandwidth of G, this condition for stability imposes a limitation on the value of K that may be used.

Now that the condition required for stability has been developed mathematically, the system of figure 6-1 may be examined to see what happens when a servomechanism is unstable. The higher the velocity constant, which includes amplifier gain and the voltage of the battery driving the controlled variable potentiometer, the greater will be the motor speed at any given value of error signal. Because the motor, gear train, and load possess inertia, the system in motion has kinetic energy equal to $J\omega^2$. In order to stop the motor, this energy must be dissipated. Because an inertia cannot be made to move instantaneously, the response of motor speed to a step of error signal

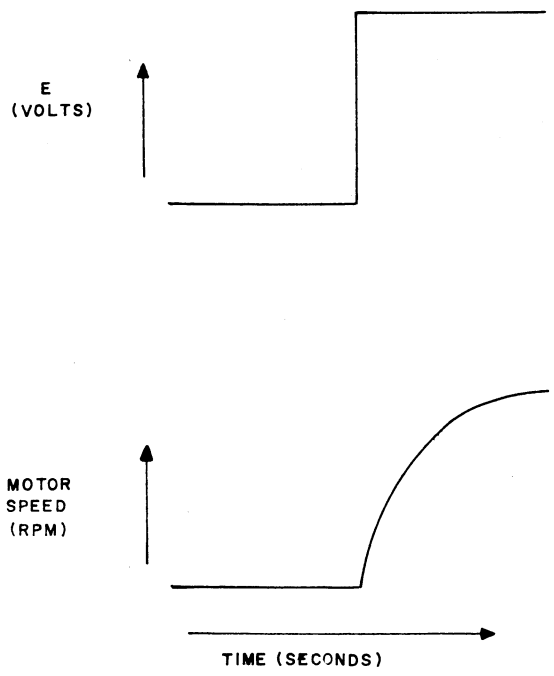

Figure 6-5. Response of Motor and Amplifier to Step Input

voltage into the amplifier is as given in figure 6-5. At the beginning of the step, a step of torque is applied to the rotor, producing acceleration. However, as the motor builds up speed the friction in the load, motor bearings, and gear train dissipates an increasing amount of energy. Eventually, the motor reaches a speed at which the amount of power supplied to the motor winding by the amplifier equals the total amount of power dissipated in the friction load, the motor and gear train bearing friction, and in the copper loss in the motor winding. At this point, the motor speed remains constant. If the system gain is low, the stiffness at the load will be quite small. As the output shaft approaches the null position, the motor torque drops off rapidly enough that the friction can dissipate all the kinetic energy in the motor. Consequently, the response of the closed loop system to a step input will be as shown in figure 6-6b. This condition of the servo loop is referred to as overdamped. If the gain is increased, increasing the stiffness, an oscillatory condition is reached in which the friction load cannot dissipate the energy stored in the inertia by the time the error signal gets to zero. In this case, the motor will overshoot the null position, and the position feedback signal will produce a reverse torque, causing the motor to overshoot in the opposite direction. This oscillation will continue with less energy imparted to the system on each oscillation, until a point is reached where the total system energy at the null is zero. This system is underdamped and has the response to a step of R as shown in figure 6-6c. If the system gain is made sufficiently large, the stiffness is so great that the amplifier is able to add more kinetic energy to the motor in each successive cycle of oscillation than the friction load can dissipate. This condition results in divergent oscillations, as shown in figure 6-6d.

4. STABILIZING METHODS

In many cases a servo may be satisfactorily damped by merely adding some sort of velocity-proportional friction device to absorb the energy stored in the inertia. Automobile shock absorbers are an example of this type of damper, and small instrument servomotors are frequently equipped with a drag cup fastened to the motor shaft and turning in a fixed magnetic field. Such dampers produce a torque proportional to velocity. Friction dampers reduce the velocity constant of a system because the motor requires more voltage to run at a given speed. However, the damping effect allows the gain to be made up in the amplifier, so that for a given velocity constant a greater stiffness may be realized, and this reduces the static error in the system.

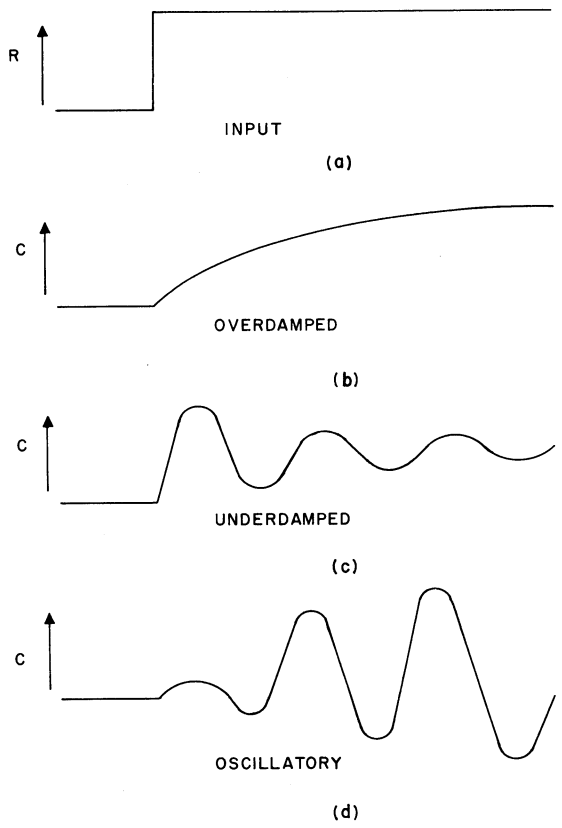

Figure 6-6. Response of Follow-Up Servo to Step Input

Another method of damping a servo is to use rate feedback, as shown in figure 6-7b. A generator is coupled directly to the motor shaft, and the output of the generator is a voltage directly proportional to the motor speed. If this voltage is fed back inversely to the amplifier, it results in a torque proportional to speed, the same as would be obtained with a friction or drag cup damper. However, since the subtracting is done at a low signal level, the amplifier is not required to supply any more power than when it is driving a motor with no load, and the motor does not have to be so large. Also, there is no requirement for dissipating the motor's energy when it is running. Figure 6-7a shows the response of motor speed to a step of error voltage, similar to figure 6-5. Let us assume for illustration that in the steady state condition .1 volt of E is required to make the motor turn at 1000 rpm. Very little torque is available to accelerate the motor initially because of the small error signal, and hence the rise time is rather large. In figure 6-7b, the generator fastened to the motor shaft puts out 1 volt per 1000 rpm. When excited with a step function E of 1.1 volts, the error signal ϵ will at first be 1.1 volts, since the motor starts at zero speed and the generator output is initially zero. As the motor picks up speed, the generator voltage which is an exponential will subtract from the 1.1 volts initial value, and in the steady state condition, when the motor reaches 1000 rpm, ϵ will be 1.1 volts minus the 1 volt generator output, which leaves .1 volt. The voltage going into the amplifier for the steady state condition is the same as in figure 6-7a, but the 1.1 volt spike present in the amplifier input in figure 6-7b produces eleven times more acceleration torque at the motor, reducing the rise time. Because the amplifier gain K is the same in both cases and because the rate generator output is 0 when the motor is stalled, the stalled torque for a given error signal is the same in both cases. Because of the much larger value of E required to get 1000 rpm with a rate generator, the velocity constant for this case will be 1/11 that of the motor alone. If in figure 6-7b, the amplifier gain K were multiplied by 11, the two systems would then have the same velocity constant but the system with the rate generator would have 11 times the stiffness, and the positioning accuracy would be 11 times as great.

Another method of stabilizing a servo and allowing an increase in its stiffness is to use a lead network. Figure 6-8 shows a lead network and its transient response to a step input. If 1 volt is suddenly applied

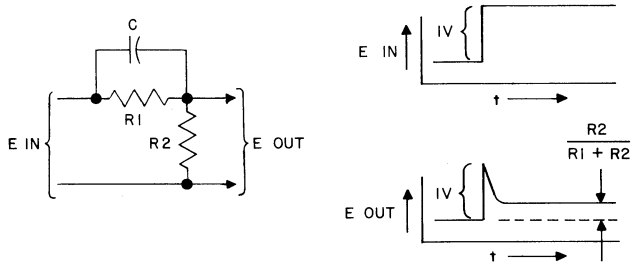

Figure 6-8. Lead Network

as E_{in}, the entire 1 volt will appear as E_{out} because the voltage across C cannot be changed instantaneously. As the capacitor charges up, E_{out} will drop exponentially and approach the value it would have if C were not present, which is $E \frac{R_2}{R_1 + R_2}$. This transient response is similar to that of ϵ in figure 6-7b. Figure 6-9 shows how a lead network is used to accomplish a result similar to that obtained with a rate generator.

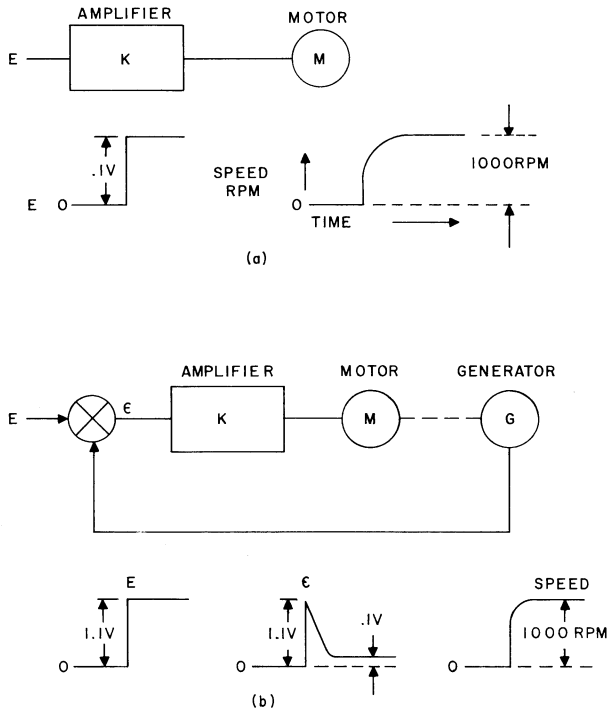

Figure 6-7. Effect of Rate Generator on Response Time

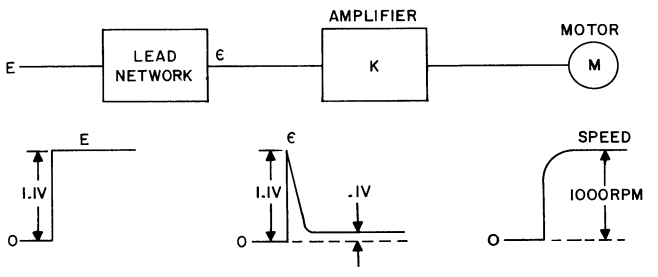

Figure 6-9. Effect of Lead Network on Response Time

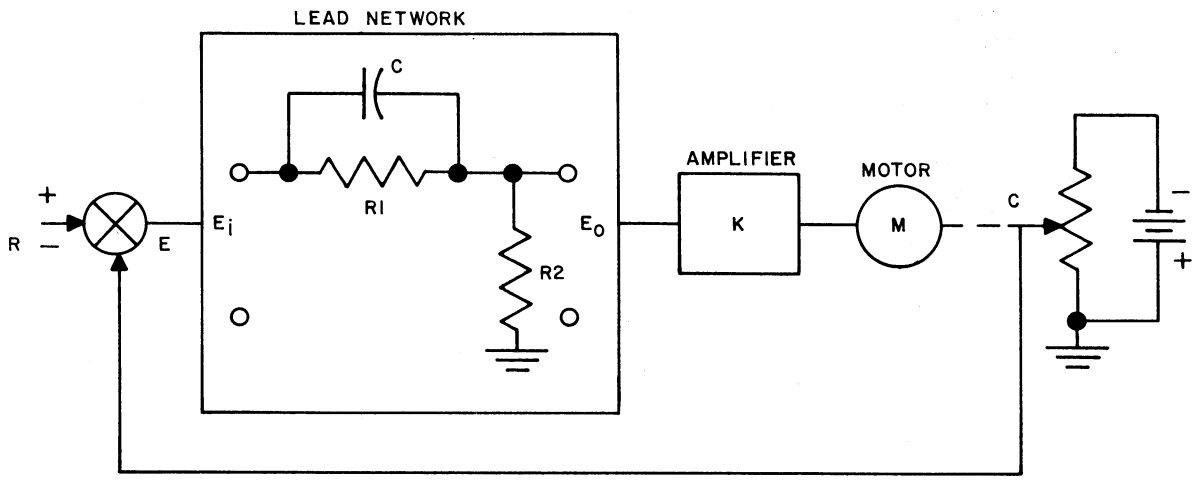

Figure 6-10. Position Follow-up Servo with Lead Network

If $\frac{R_2}{R_1 + R_2}$ is made equal to 1/11, a voltage of 1.1 volt applied at E will produce the wave form shown at ϵ and this will produce a rapid acceleration of the motor to a speed of 1000 rpm. The capacitor in the lead network must be chosen to produce the same time constant in the signal at ϵ as that of figure 6-7b.

The transfer function of the lead network of figure 6-8 may be derived by considering it a voltage divider with the parallel combination of R_1 and C in the top leg and R_2 in the bottom leg. Thus the output impedance is

$$Z_o = R_2$$

The parallel impedance of R_1C is

$$\frac{R_1 \frac{1}{pC}}{R_1 + \frac{1}{pC}} = \frac{R_1}{1+pR_1C}$$

and the total series impedance of the divider is

$$Z_t = R_2 + \frac{R_1}{1+pR_1C} \quad \text{Hence}$$

$$\frac{E_o}{E_i} = \frac{Z_o}{Z_t} = \frac{R_2}{R_2 + \frac{R_1}{1+pR_1C}} = \frac{R_2(1+pR_1C)}{R_1+R_2+pR_1R_2C}$$

$$= \frac{R_2}{R_1+R_2} \cdot \frac{1+pR_1C}{1+p\frac{R_1R_2}{R_1+R_2}C}$$

This expression may be rewritten as follows:

$$\frac{E_o}{E_i} = \frac{R_2}{R_1+R_2} + \frac{R_2\left(R_1 - \frac{R_1R_2}{R_1+R_2}\right)Cp}{R_1+R_2 \quad 1+\frac{R_1R_2}{R_1+R_2}Cp}$$

$$\frac{E_o}{E_i} = \frac{R_2}{R_1+R_2} + \frac{R_1^2R_2C}{(R_1+R_2)^2} \cdot \frac{p}{1+\frac{R_1R_2}{R_1+R_2}p}$$

$$\frac{E_o}{E_i} = K_1+K_2\frac{Tp}{1+Tp}$$

therefore $E_o = K_1E_i + K_2\frac{Tp}{1+Tp}E_i$

Thus, the lead network behaves like a straight feed with a gain of K_1 plus a high-pass filter with a gain of K_2. The network differentiates low frequency signals. When connected in a closed position loop, as shown in figure 6-10, a lead network provides the sum of a position signal and a differentiated position signal, so that the rate of change of ϵ is used, producing an effect similar to that of a rate generator except that R is differentiated as well as C.

5. COMPONENTS AND CIRCUITS

In the preceding discussion of the position follow-up servomechanism of figure 6-1, it was assumed that

the reference input and controlled variable voltages, the amplifier, and the motor were all direct-current components. In practice, 400 cycle or 60 cycle carrier systems are more commonly used for small, low power servo systems in radio communication equipment. The use of an a-c carrier system simplifies the design of the amplifier, and since an amplifier bandwidth of 40 or 50 cycles usually suffices, the d-c operating point of the individual stages is of no consequence. Because of the low-frequency requirement, the junction transistor is well suited to servo work. Where 20 watts or more of amplifier output is required, a magnetic amplifier or saturable reactor driven by a transistor preamplifier may be used. Some servo amplifiers employ high performance magnetic amplifiers for all stages.

The most commonly used type of servomotor is the two phase induction motor, and frequently a two phase induction generator is built into the same case and coupled to the motor shaft. The schematic diagram for a typical motor generator appears in figure 6-11. The reference phase of the motor must be driven with a voltage 90° out of phase with the voltage on the control phase in order to obtain torque. If the control phase voltage leads the reference phase voltage, the motor will turn in one direction, and if the control voltage lags the reference voltage, the motor will turn in the opposite direction. Thus if the error signal source is a 400 cps signal, a phase reversal of the error signal produces a reversal of motor rotation, as required.

In some cases where the servo amplifier output is either in phase or 180 degrees out of phase with the line voltage depending on the sense, the required quadrature relationship between control and reference winding is obtained by means of a phase shift capacitor C which produces a quadrature voltage on the reference winding. Sometimes however, it is more convenient to produce the required 90° phase shift inside the servo amplifier, in which case the reference winding is connected directly to the 400 cps line. When its excitation winding is driven from the 400 cps line, the rate generator produces across its output terminals a voltage proportional to the speed of rotation and either in phase or out of phase with the excitation voltage, depending upon the direction of rotation.

If the purpose of the servomechanism is merely to repeat the position of a shaft or to produce an output shaft position proportional to a voltage used as a reference input, a-c line voltages may be used across reference and controlled variable potentiometers in place of the d-c voltages used in the illustration of figure 6-1. In this case, as the controlled variable voltage increases and becomes larger than the reference input, the phase of the error signal voltage reverses and reverses the direction of rotation of the two-phase servomotor. In this way a carrier servo system may be constructed in which all variables are represented by 400 cps a-c voltages.

In some transmitter tuning servos and antenna matching networks, it is desired to have the servomotor and gear train position a mechanical tuning element such as a variable capacitor to a position such that the phase shift imposed upon an r-f signal by the tuned circuit is zero. An r-f phase discriminator circuit of the type used for detection of FM signals may be used to obtain a d-c voltage proportional to the magnitude of the phase shift through the r-f circuit, and of polarity determined by whether the output leads or lags the input. If this d-c error information is to be fed into a carrier type servomechanism, it may be converted to a-c by means of an electromechanical chopper connected as in figure 6-12. The chopper consists of a vibrating reed and a pair of contacts which form a single-pole double-throw switch. The reed is excited from a magnetic coil and is driven from the 400 cps line. In most cases the action of the reed contact is not in phase with the excitation voltage fed to the coil, so that a phase shift network must be used on the coil

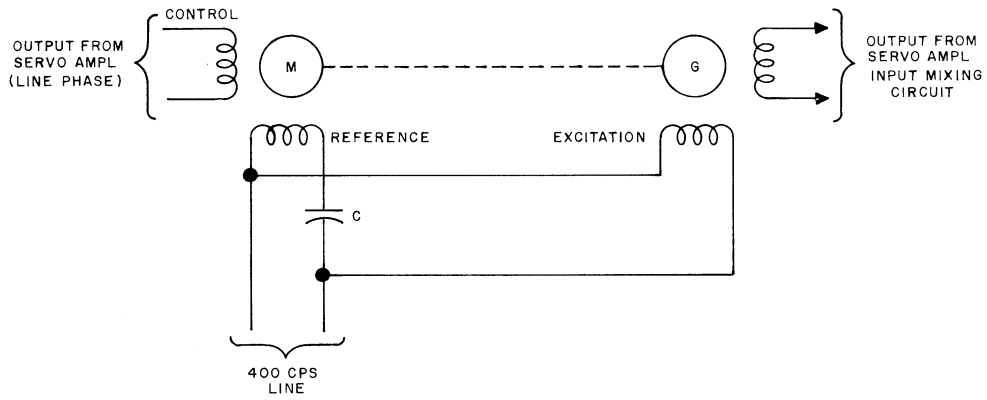

Figure 6-11. Motor-Generator Schematic Diagram

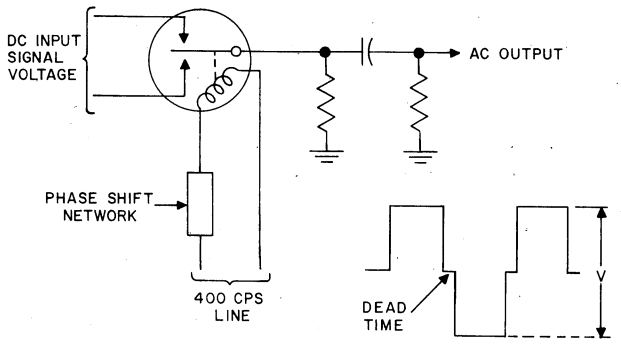

Figure 6-12. Electromechanical Chopper Circuit

to make the reed contact action either in phase with or in quadrature with the line, as required. In the chopper output wave form shown in figure 6-12, the peak to peak voltage of the square wave is the amplitude of the d-c voltage connected across the contacts.

Another type of position transmitter frequently encountered is the synchro. Figure 6-13 illustrates a typical synchro error circuit. The synchro transmitter may be thought of as being a transformer with a single primary, the rotor winding, and three secondaries, the three stator windings. The stator windings are placed with their axes 120° apart. The rotor winding induces an a-c voltage in each of the stator windings proportional to the cosine of the angle between the rotor winding axis and the respective stator winding axes. The stator windings of the control transformer are connected directly across the stator windings of the transmitter. Therefore, the same voltages will exist across each of the control transformer stator windings as are induced in the corresponding stator windings of the transmitter, and the field pattern set up inside the core of the control transformer will be a replica of the field pattern in the transmitter. If the control transformer rotor winding axis is lined up with this field, the error signal voltage developed across the rotor terminals will be maximum. As the control transformer rotor is turned, the error signal voltage will decrease and become zero when the rotor axis makes an angle of 90° with the field pattern set up by the stator windings. Further rotation of the rotor will produce an increasing error voltage of reversed phase. Thus, if the rotor of the transmitter is actuated by the reference input shaft of a position follow-up system (figure 6-1) and the control transformer rotor is coupled to the controlled variable shaft, the voltage developed across the rotor of the control transformer may be used as an error signal E to be fed into the servo amplifier, and the rotor of the control transformer will follow the transmitter rotor.

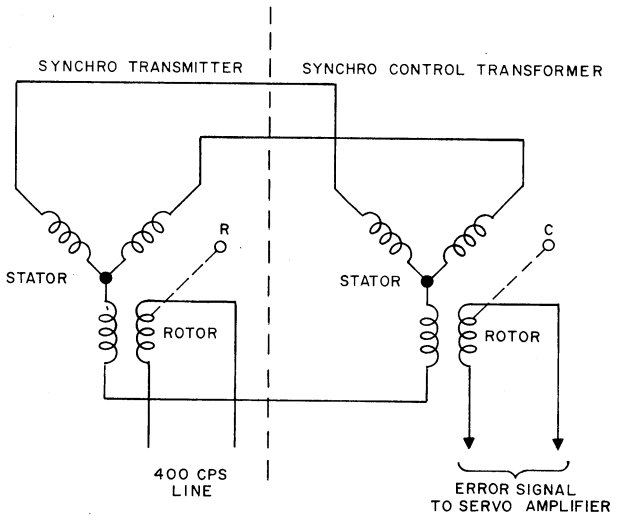

Figure 6-13. Synchro Error Circuit