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Microwave Power Amplifier Fundamentals

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Published: October 2008
Revision: A

Introduction

The Need to Amplify Signals

An amplifier is one of the most common electrical elements in any system. The requirements for amplification are as varied as the systems where they are used. Amplifiers are available in a large number of form factors ranging from miniscule ICs to the largest high-power transmitter amplifiers. In the following discussion the focus will be on solid state power amplifiers used at microwave frequencies, particularly in test and measurement applications.

Microwave power amplifiers may be used for applications ranging from testing passive elements, such as antennas, to active devices such as limiter diodes or MMIC based power amplifiers. Furthermore, other applications include testing requirements where a relatively large amount of RF power is necessary for overcoming system losses to a radiating element, such as may be found at a compact range, or where there is a system requirement to radiate a device-under-test (DUT) with an intense electromagnetic field, as may be found in EMI/EMC applications.

As varied as the system requirements may be, the specific requirements of a given amplifier can also vary considerably. Nevertheless, there are common requirements for nearly all amplifiers, including frequency range, gain/gain flatness, power output, linearity, noise figure/noise power, matching, and stability. Often there are design trade-offs required to optimize any one parameter over another, and performance compromises are usually necessary for an amplifier that may be used in a general purpose testing application.

The following discourse includes a description of amplifier topologies introducing the basics of spatially combined distributed amplifiers, a discussion of typical amplifier specifications and a review of performance verification measurements.

Broadband Microwave Power Amplifiers

There are numerous techniques for designing microwave power amplifiers. These may be broadly split between tube and solid state technologies. For high power requirements (> 100 Watts), typically these are satisfied with tube based designs. Tube amplifiers, such as Traveling Wave Tube Amplifiers (TWTAs), require a high voltage power supply, typically require warm-up time, and have significant aging related issues. For solid state amplifiers to achieve similar performance often requires switching between narrow-band amplifiers, with deleterious effects to the overall linearity and gain/power flatness. The switches themselves embody performance compromises. Mechanical switches, while quite linear and relatively low loss, have switching speed limitations, and are subject to failure after repeated switching cycles. Solid state switches may overcome the speed issue, but are not nearly so linear or low loss. Both the signal fidelity and loss issues limit the usefulness of solid state switches for high power microwave amplifiers. Furthermore, switching between narrowband amplifiers requires external stimulus with the software control complication that entails.

A topology often favored for generating modest amounts of microwave power output is to combine the outputs of several relatively low output power amplifiers. The individual amplifiers usually have a "distributed" or "traveling wave" topology[1]. The distributed amplifier topology achieves a large frequency range by arraying individual transistors; each representing shunt capacitances between series inductances, to create a semi-lumped representation of a transmission line (see Figure 1). This amplifier topology is often fabricated using MMIC techniques, and has been optimized to the point where single amplifiers can provide up to nearly 1 Watt of saturated power output. Nevertheless, it is no trivial task, using conventional planar circuit techniques, to combine the power output of even a small number of these distributed amplifiers over a full decade frequency range, without incurring unacceptable losses or poor flatness characteristics.

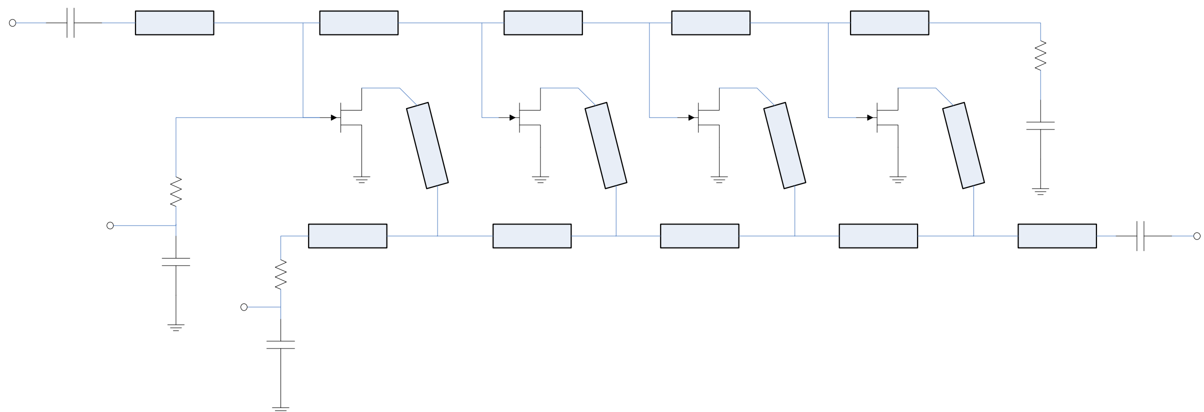

Figure 1: Four Cell Distributed Amplifier

New techniques, employing spatial combining, enable high power and flat gain over a broad bandwidth. Spatial combining can be used to sum a much larger number of amplifiers over a decade-wide frequency range, than would be practical using conventional planar circuit techniques.

For example, an amplifier cell can be created using tapered-gap antipodal finline baluns[2] to transition microstrip to a balanced finline structure suitable for launching into a coaxial waveguide. Antipodal finline is a balanced planar transmission line structure where two conducting strips of metal, separated by a dielectric substrate, are offset by a typically small gap (see Figure 2). Antipodal finline can provide a wide range of characteristic impedances. Closed form design equations for this transmission line media are not available; however, if the dielectric media is thin, and the gap is large, the design equations for unilateral finline can be used as an approximation.

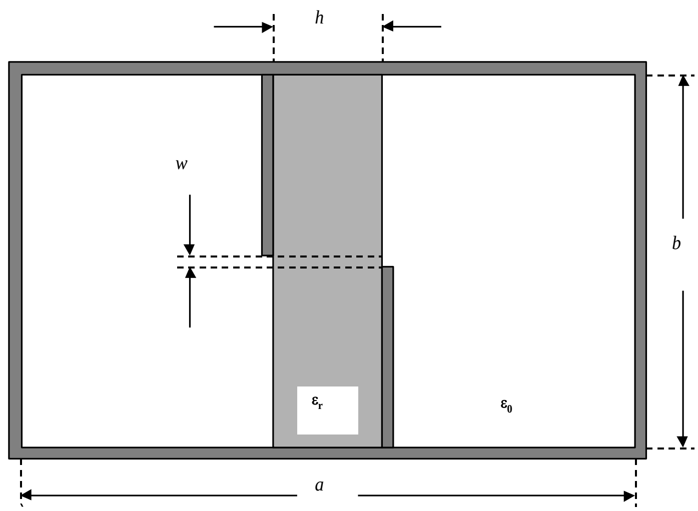

Figure 2: Antipodal Finline

Both microstrip and antipodal finline transmission lines support quasi-TEM propagation; i.e., the electric and magnetic fields are mutually perpendicular, and perpendicular to the direction of propagation. TEM mode propagation is naturally a very low dispersion propagation mode. By tapering the transition from microstrip to antipodal finline, the electro-magnetic fields are gracefully transitioned from an unbalanced structure (microstrip), to a balanced structure (antipodal finline), without introducing excessive dispersion. Innately, this balun is a very broadband, low dispersion transducer. A cylindrical array of amplifier cells, each with the previously described transitions, can launch into (or from) a coaxial waveguide. With the input coaxial waveguide being an n-way power splitter, and the output being an n-way power combiner, the antipodal finline excites or receives a TEM wave in each coaxial waveguide. Unlike common rectangular waveguide, the principle propagation mode in a coaxial waveguide is TEM, just like an ordinary coaxial cable, thus is low dispersion, and yet low loss.

Using a tapered coaxial waveguide allows many relatively high impedance sources to be efficiently combined (or split), whereas this would be very difficult and inefficient to reproduce using purely conventional planar circuit techniques over a decade of frequency range. Thus the entire spatially combined propagation structure from beginning to end is quasi-TEM with low dispersion. Furthermore, the microstrip-antipodal finline baluns create a very broadband impedance transformation, allowing efficient combining/splitting of a large number of parallel amplifiers.

Specification Parameters

There exist a very large number of potential electrical specifications that can be applied to a microwave power amplifier. Nonetheless, there are a number of specifications that would be nearly universal, and deserve further discussion.

Gain/Gain Flatness

Gain usually is specified within the context of power output. Often, if no context for power output is given, then this is assumed to be small signal gain. Conditions for small signals at the input and output are usually easy to reproduce and verify, whereas gain and gain flatness can vary significantly when an amplifier approaches compression. Gain flatness for an amplifier with a significant frequency range is often specified over subsets of the entire frequency range, as well as over the entire frequency range. Gain and Gain Flatness typically include an implicit assumption that the reverse gain from the output to the input is negligible; i.e., the amplifier is unilateral.

Since voltages and currents can be difficult to directly measure at microwave frequencies, network scattering parameters, or s-parameters, were defined in terms of incident and reflected waves. The formal definition for scattering parameters for a two port network can be derived as[3]:

$$a_1 = \frac{V_1 + Z_0 I_1}{2\sqrt{Z_0}} \quad \text{Incident Wave - Port 1} \quad (1)$$

$$a_2 = \frac{V_2 + Z_0 I_2}{2\sqrt{Z_0}} \quad \text{Incident Wave - Port 2} \quad (2)$$

$$b_1 = \frac{V_1 - Z_0 I_1}{2\sqrt{Z_0}} \quad \text{Reflected Wave - Port 1} \quad (3)$$

$$b_2 = \frac{V_2 - Z_0 I_2}{2\sqrt{Z_0}} \quad \text{Reflected Wave - Port 2} \quad (4)$$

Voltage V_1 is the voltage drop across Port 1 of the two port network, and I_1 is the current into Port 1. The quantities V_2 and I_2 are the corresponding values at Port 2. Impedance Z_0 is the characteristic impedance of the network, and is usually 50 Ω. The factor of ½ applies where the voltage and current are peak values. The factor is dropped for rms voltages and currents. The voltages and currents are vector (real and imaginary) quantities. The s-parameters can then be defined as:

$$S_{11} \equiv \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad \text{Port 1 Reflection Coefficient} \quad (5)$$

$$S_{12} \equiv \left. \frac{b_1}{a_2} \right|_{a_1=0} \quad \text{Port 2 to 1 Transmission Coefficient} \quad (6)$$

$$S_{21} \equiv \left. \frac{b_2}{a_1} \right|_{a_2=0} \quad \text{Port 1 to 2 Transmission Coefficient} \quad (7)$$

$$S_{22} \equiv \left. \frac{b_2}{a_2} \right|_{a_1=0} \quad \text{Port 2 Reflection Coefficient} \quad (8)$$

The defined scattering parameters are also vector quantities. Nominally, it is assumed that port one is the input port, and port two is the output port. Using this assumption, S_{11} is the input reflection coefficient, and S_{22} is the output reflection coefficient of the two port network. The transducer power gain is defined as the power delivered to the load divided by the power available from the source. Using network analysis flow graphs, the expression for the transducer gain is:

$$G_T = \frac{P_{del}}{P_{src}} = \frac{|b_2|^2\left(1-|\Gamma_L|^2\right)}{|b_g|^2 / \left(1-|\Gamma_g|^2\right)} \quad (9)$$

Where:

Γ_g = Generator Reflection Coefficient
Γ_L = Load Reflection Coefficient
b_g = Wave incident from Generator

This expression can be re-written after some algebraic manipulation into terms of s-parameters as:

$$G_T = \frac{|S_{21}|^2\left(1-|\Gamma_g|^2\right)\left(1-|\Gamma_L|^2\right)}{\left|\left(1-S_{11}\Gamma_g\right)\left(1-S_{22}\Gamma_L\right)-S_{21}S_{12}\Gamma_L\Gamma_g\right|^2} \quad (10)$$

The wave incident from the Generator can be seen empirically as the factors of the difference of the wave reflected between the generator and port 1 ($1-\Gamma_g$), times the factor of the difference of the wave reflected between the load and port 2 ($1-\Gamma_L$), minus the term of the forward and reverse gains multiplied by the generator and load reflection coefficients. Again, all of these variables have vector (complex) values.

The point of this analysis is to demonstrate that while conceptually simple, gain at microwave frequencies can be somewhat complicated to determine precisely. Nevertheless, with a few assumptions, the expression for the transducer gain can be greatly simplified.

Assume that the generator and load are perfectly matched to the system characteristic impedance, Z_0. This implies:

$\Gamma_g = \Gamma_L = 0$, therefore equation (10) simplifies to:

$$G_T = |S_{21}|^2 \quad (11)$$

This can be expressed in decibels as:

$$G_T(dB) = 20\log|S_{21}| \quad (12)$$

Implicit in all of these expressions is the assumption that the system is *linear*, or put another way, that signals are small signals. Later, in the section on measurement techniques, more will be discussed about the above simplifying assumptions.

MMIC amplifiers with a distributed topology, have a very broadband, flat gain response. Since the transitions from antipodal finline to coaxial waveguide have low dispersion, the inherent gain flatness of the MMIC devices is undisturbed in the spatially combined amplifier topology. Typically, this could only be achieved over narrow bandwidths with classic reactive matching techniques, such as those used for internally matched devices. Attempts to broaden the gain-bandwidth of a high power microwave amplifier required trade-offs with resistive matching, or feedback techniques that rob power output. The spatially combined topology overcomes these limitations.

Power Output: P_{-1dB} and P_{sat}

Among the key specifications for microwave amplifiers are their power output specifications. Depending on the application, power output is typically specified at one decibel of compression and/or at saturation. Unlike the gain specification, implicitly it is assumed that the specification is at an operating point where the amplifier is exhibiting some degree of non-linear behavior. Depending on the degree of non-linear behavior, and the type of results desired, deriving a model for power output as a function of input power may require very sophisticated CAE tools. Nevertheless, a useful formula for estimating power output as a function of power input that only requires relatively simple calculations that can be implemented in a spreadsheet can be expressed as follows[4]:

Assume:
(a) P_{in}-P_{out} curve is linear with small signals, and
(b) The curve compresses quickly to approach P_{sat} asymptotically.

$$P_{out} = \frac{G_T \cdot P_{in}}{1 + \frac{\beta(G_T \cdot P_{in})}{P_{sat}}} \quad (13)$$

where:

$$\beta = \exp\left[\frac{-\alpha \cdot P_{sat}}{G_T \cdot P_{in}}\right] \quad (14)$$

$$\alpha = \left(\frac{k_1 \cdot P_{-1dB}}{P_{sat}}\right)\ln\left(\frac{k_2 \cdot P_{sat}}{P_{-1dB}}\right) \quad (15)$$

$$k_1 = -1.258925412 \quad (16)$$

$$k_2 = 0.2056717653 \quad (17)$$

When using these formulas for calculating P_{out} there are some caveats to take into account. First, the P_{-1dB} and P_{sat} levels need to be realistic. Generally, to achieve good convergence if these equations are used for numerically calculating the gain compression of a chain of components, the P_{-1dB} and P_{sat} numbers should be within 3 dB of each other. The input power cannot be greatly above (or below) the input P_{-1dB} power level, as many numerical solvers will overflow under those conditions. For those interested, more discussion about the constants, k_1 and k_2, can be found in *Appendix A*.

With an inherently broadband amplifier, power output as a function of power input does not vary discontinuously as a function of frequency. Typically, a wideband microwave power amplifier that could deliver in excess of several Watts required a solution where numerous narrowband amplifiers were either multiplexed or switched; often introducing undesired issues, such as power curve discontinuities, at frequency cross-over points.

Standing Wave Ratio (SWR) and Return Loss

The standing wave ratio, often referred to interchangeably as *VSWR*, is the result of wave interference. Peaks and troughs in a given field pattern remain in a static position as long as the sources of interference do not change with respect to each other. The interfering field patterns give rise to voltages and currents that also exhibit this static interference pattern. A classic demonstration in an electro-magnetic laboratory would use a slotted coaxial transmission line with a source at one end, and a reflecting element at the other end. The reflected wave interferes with the original source wave, creating a pattern of voltage peaks and troughs that can easily be measured with a detector probe. The ratio of these peaks and troughs is the definition of SWR[5]:

$$SWR \equiv \frac{E_{\max}}{E_{\min}} = \frac{E_{inc} + E_{refl}}{E_{inc} - E_{refl}} \quad (18)$$

The standing wave ratio is almost exclusively associated with *voltage* hence the common usage VSWR; however, it could just as well be interpreted in terms of current, or other electro-magnetic field terms. In the *Gain/Gain Flatness* section, reflection coefficient was defined in terms of incident and reflected waves. Likewise, SWR is very closely related to reflection coefficient:

$$\Gamma_L = \rho = \frac{\beta_L}{\alpha_L} = \frac{(V_L - Z_0 \cdot I_L)/2 \cdot \sqrt{Z_0}}{(V_L + Z_0 \cdot I_L)/2 \cdot \sqrt{Z_0}} \quad (19)$$

Canceling the common factors, and dividing the numerator and denominator by I_L:

$$\rho = \frac{V_L/I_L - Z_0}{V_L/I_L + Z_0} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (20)$$

Since E_{max} occurs when the load impedance, Z_L, approach infinity, and E_{min} occurs when the load impedance approached zero:

$$\left| \rho|_{E_{\max}} \right| = \left| \lim_{Z_L \to \infty} \left[\frac{Z_L - Z_0}{Z_L + Z_0} \right] \right| = +1 \quad (21) \quad \text{and,}$$

$$\left| \rho|_{E_{\min}} \right| = \left| \lim_{Z_L \to 0} \left[\frac{Z_L - Z_0}{Z_L + Z_0} \right] \right| = -1 \quad (22)$$

SWR can then be expressed in terms of reflection coefficient as:

$$SWR = \frac{1 + |\rho|}{1 - |\rho|} \quad (23)$$

Another common way of quantifying the quality of the match is to express the reflection coefficient (a ratio) in decibels. This is commonly referred to as *return loss* (RL):

$$RL = -20 \cdot \log|\rho| = -20 \cdot \log\left[\frac{SWR - 1}{SWR + 1}\right] \quad (24)$$

A few extra formulas that may be helpful when considering SWR in a system of components:

$$SWR|_{\max} = SWR_1 \cdot SWR_2 \quad (25)$$

$$SWR|_{\min} = SWR_1 / SWR_2 \quad (26)$$

where $SWR_1 > SWR_2$, and:

$$\varepsilon_a = 20 \cdot \log\left(1 \pm \left|\Gamma_1 \cdot \Gamma_2\right|\right) \quad (27) \quad \textit{amplitude error in dB}$$

$$\varepsilon_\phi = \left(180/\pi\right) \cdot \Gamma_1 \cdot \Gamma_2 \quad (28) \quad \textit{phase error in degrees}$$

MMIC distributed amplifiers by design possess broadband terminations, and non-dispersive baluns and power splitters/combiners do not create inherent SWR discontinuities. Low input/output SWR is especially difficult to achieve in classic microwave power amplifier designs that cover a broad bandwidth. In comparison, low-loss broadband transitions using spatial combining techniques in an amplifier insure excellent power delivery to the load, and provide a well behaved system whenever out-of-band reflections from a high SWR load require a broadband source termination from the driving amplifier.

Stability

Naturally, it is expected that any amplifier is stable; however, often it is difficult to determine if this is the case. The active devices used in most microwave power amplifiers have substantial gain from less than 1 MHz up to 30 GHz (and higher in certain cases). Gain is one part of the two necessary conditions for oscillation; the other is a means of feedback. The feedback may take the form of a conducted path through a bias network, or a radiated path in the form of a waveguide cavity in which the active elements are embedded.

For example, while it may appear that there are no overt oscillations emanating from an amplifier, it can be the case that the oscillation frequency is low enough that the DC blocking caps attenuate the signal sufficiently to make it very difficult to measure directly; however, sidebands appear to be modulated on a carrier of a desired signal. Or in the case of high frequency oscillations, there may appear an unexplained spurious signal in the output spectrum of the amplifier that is the mixing product of the desired signal and an oscillation tone that is out-of-band of the measuring receiver. Insuring stability in an amplifier, especially an amplifier with substantial gain over a wide frequency range requires very careful attention to design details to insure that all conducted and radiated feedback paths are sufficiently attenuated. Using the concepts developed earlier in the sections on Gain and SWR, a formal set of conditions for unconditional stability can be expressed in a set of formulas as:

$$K = \frac{1 - \left|S_{11}\right|^2 - \left|S_{22}\right|^2 + \left|\Delta\right|^2}{2 \cdot \left|S_{12} \cdot S_{21}\right|} > 1 \quad (29)$$

$$\left|\Delta\right| = \left|S_{11} \cdot S_{22} - S_{12} \cdot S_{21}\right| < 1 \quad (30)$$

The two above equations, referred to as the Rollett Criteria for Unconditional Stability[6], are two necessary and sufficient conditions for characterizing stability in a two port network[7]. This set of conditions apply to any source or load impedance found inside the Smith Chart; i.e., source or load reflection coefficients whose magnitudes are less than one. These conditions are not necessarily sufficient for a source or load that has reflection gain. A more detailed analysis, beyond the scope of this paper, is required to consider the condition where a source or load possesses reflection gain. A spatially combined MMIC amplifier array, with its intrinsically low SWR and high reverse isolation, is guaranteed to be unconditionally stable operating into any source or load whose reflection coefficient magnitude is less than one.

Noise Figure

Noise Figure may not be a primary design goal for a typical microwave power amplifier, but it must be understood to specify for system use. The definition of noise factor is[8]:

$$F \equiv \frac{S_{in} / N_{in}}{S_{out} / N_{out}} = 1 + \frac{N_a}{G \cdot N_{in}} > 1 \qquad (31)$$

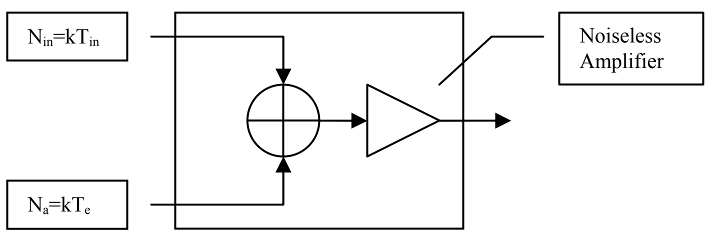

Figure 3: Noisy Amplifier Model

S_{in} = Signal at input
N_{in} = Noise at input
S_{out} = Signal at output
N_{out} = Noise at output
N_a = Noise added
G = Available Gain

Noise Figure (NF) is defined for a noise input power corresponding to kT_oB from a passive device (no gain) held at 290K (about 16.8°C or 62.3°F), where k is Boltzmann's constant (1.380658 x 10^{-23} J·K^{-1}) and B is noise bandwidth in Hertz. Expressed in decibels:

$$NF = 10 \cdot \log(F) \qquad (32)$$

To illustrate a point, the noise factor for multiple cascaded stages of components can be expressed as[9]:

$$F_{casc} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \ldots + \frac{F_n}{G_1 \cdot G_2 \cdots G_{n-1}} \qquad (33)$$

A power amplifier, using spatial combining techniques, will have a low loss input. As a result, the noise factor of the first element (the first term, F_1 in equation 33) within such a power amplifier has a much lower loss than a typical planar power combiner (such as a Wilkinson combiner or a Lange coupler). Thus the important first term in the cascaded noise factor is minimized. Furthermore, an amplifier with intrinsically broadband transitions has relatively flat gain as a function of frequency; therefore the noise figure does not change abruptly as a function of frequency.

Harmonics, Intercept Point, and Inter-modulation Distortion

Harmonics

Harmonics, intercept point, and inter-modulation distortion are expressions of linearity, or the lack thereof, for an amplifier. To one degree or another, all microwave circuits generate signal distortion, as a result of non-linear behavior. It is highly desirable to minimize this non-linear behavior in a microwave power amplifier; however, since it is inevitable that it exists, it must be characterized. The means of generating distortion as a result of non-linear behavior are beyond the scope of this paper, but it is instructive to consider the non-linear memory-less transfer characteristic expressed as a power series[10]:

$$g(v) = g_0 + g_1 \cdot v + g_2 \cdot v^2 + g_3 \cdot v^3 + \ldots \quad (34)$$

Assuming an undistorted sinusoidal source and an ideal linear load, the voltage output may be expressed as a Fourier series, with harmonic components that are multiples of the input carrier frequency, ω_c:

$$v_{out}(t) = g_0 + g_1 \cdot A \cdot \cos(\omega_c t) + g_2 \cdot A^2 \cdot \cos^2(\omega_c t) + g_3 \cdot A^3 \cdot \cos^3(\omega_c t) + \ldots \quad (35)$$

$$= g_0 + \frac{g_2 A^2}{2} + \left(g_1 A + \frac{3 g_3 A^3}{4} \right) \cdot \cos(\omega_c t) + \frac{g_2 A^2}{2} \cos(2\omega_c t) + \frac{g_3 A^3}{4} \cos(3\omega_c t) + \ldots \quad (36)$$

From inspection it can be seen that the non-linear transfer characteristic contains the fundamental frequency component (ω_c), along with a DC term and harmonic components of the carrier frequency ($n \cdot \omega_c$). The effects of the non-linear transfer function can be summarized in Table 1.

Term	Amplitude	Qualitative Effect
DC	$g_0+g_2 \cdot A^2/2$	Offset added due to RF rectification
Fundamental	$20\log(g_1A+3g_gA^3/4)$	Amplitude compression
2nd Harmonic	$40\log(g_2A^2/2)$	2:1 slope on P_{in}/P_{out} curve
3rd Harmonic	$60\log(g_3A^3/4)$	3:1 slope on P_{in}/P_{out} curve

Table 1: Non-linear Terms

Intercept Point

As described in Table 1, the slope of the amplitude of the harmonics at the output increases more rapidly with respect to the fundamental tone as input power is increased. This gives rise to a figure of merit called *intercept point*. Referring to Figure 4, for fundamental, second, and third order products, their respective amplitudes can be plotted depicting their respective slopes on a log-log scale. If the higher order products could increase linearly without compression, then there exists a point where they would intersect a line with the 1:1 slope of the fundamental tone. Of course, as discussed earlier in the section on power output, all of the signals compress well before this could occur. Specifically, these intersections are referred to as *second order intercept (SOI or IP_2) and third order intercept (TOI or IP_3)*. These intercept points have units of dBm or Watts less commonly, and are interpolated from measured data under conditions where the unit under test is operating as linearly as possible.

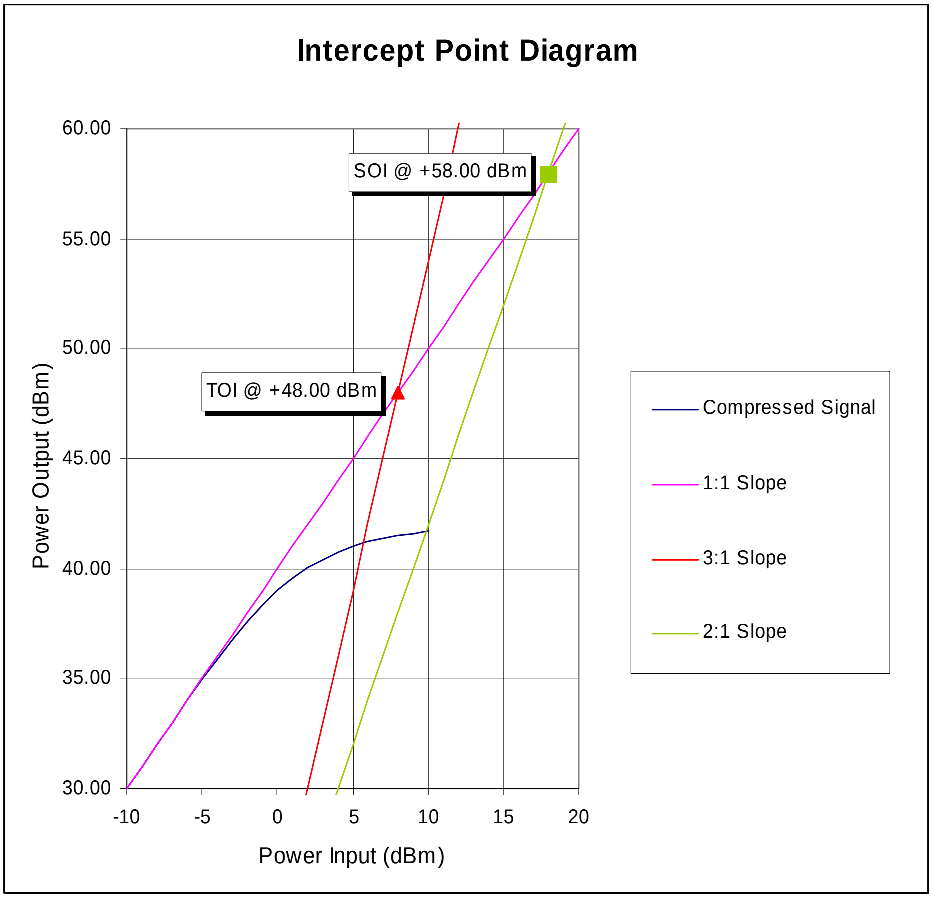

Figure 4: Intercept Point Diagram

Inter-modulation Distortion

When multiple signals occur at the input of an amplifier, an additional distortion product becomes evident at the output of the amplifier in the form of mixing products of the input signals. This form of signal distortion is called inter-modulation distortion (IMD). While it is closely related to harmonic and intercept point issues, from a system standpoint it is more difficult to deal with IMD than harmonic distortion. Harmonics can be filtered from the output spectrum, but inter-modulation products occur close to the desired signals, and typically cannot be filtered without introducing very significant losses or group delay issues, if at all. Often, inter-modulation distortion is characterized in terms of two-tone IMD, having units of decibels referenced to the carrier levels (dBc). If a two-tone input signal is applied to a non-linear memory-less system, with the two-tones having equal amplitude:

$$v_{in}(t) = A\cos(\omega_1 t) + A\cos(\omega_2 t) \quad (37)$$

Re-writing equation (37) using a trigonometry identity:

$$v_{in}(t) = 2 \cdot A\cos(\omega_c t) \cdot \cos(\omega_m t) \quad (38)$$

where:

$$\omega_c = \frac{\omega_1 + \omega_2}{2} \quad \text{and} \quad \omega_m = \left|\frac{\omega_1 - \omega_2}{2}\right| \qquad \text{(39a and 39b)}$$

Assuming ω_1 and ω_2 are close in frequency, then ω_c can be thought of as a carrier tone in the median between the two tones, and ω_m as a frequency modulation envelop. If the input power level of the two tones is low (< 10 dB below P_{-1dB}), and the AM to PM conversion is negligible, it can be shown that odd order terms are the dominant distortion products. The result for the lowest order odd term is, using the same cubic trigonometry identity as used in equation (36):

$$v_{out}(t) = \left[\left(g_1 A + \frac{3g_3A^3}{4}\right)\cos(\omega_m t) + \frac{3g_3A^3}{4}\cos(3\omega_m t)\right]\cos(\omega_c t) \qquad (40)$$

The distortion tones evidenced in equation (40) are two times the modulation envelope frequency above and below the two tones incident at the input of the amplifier. A more thorough treatment of this topic may be found in textbooks treating power amplifier design[11].

Most microwave power amplifiers are composed of a number of amplifier stages, sometimes cascaded directly, sometimes indirectly with passive or other active microwave components between stages. It is possible, but tedious, to analyze the entire cascaded chain of components as a “black-box”, whereby deriving a transfer function for the entire entity. Aside from the mathematical challenge of doing so, the assumptions made at each stage have to be challenged at successive stages, making the task more daunting. Another approach is to use individual stage characteristics (akin to a behavioral model) to predict the distortion performance for the entire cascade. To do so requires a few definitions of nomenclature; e.g., let g_n equal the gain for the n^{th} stage, g_{n+1} equal the gain for the following stage (n+1), ipk_n equal the k^{th} order intercept point for the n^{th} stage, and so forth. Variables in lower case letters indicate that the quantity is in numerical terms, whether absolute (e.g. Watts) or a ratio (unit-less), as opposed to a decibel value. Variables in upper case letters are in decibels for ratios and in dBm for power. From the above described definition of intercept point, the power level of the inter-modulation product (pk_n) of the n^{th} stage is given as:

$$pk_n = \frac{p_n^k}{(ipk\ )^{k-1}} \qquad (41)$$

or in decibels,

Error! Bookmark not defined. $IMD_{dBc} = 2(P_{out,dBm} - IP_{3,dBm}) \qquad (42)$

To reiterate, the nomenclature p_n^k is the power level of the n^{th} stage raised to the k^{th} power. At the output of the cascade, the inter-modulation products are increased by the gain of each successive stage. By defining a new parameter, *Intercept Point Susceptibility*, the housekeeping of the IMD products can be simplified[12]:

$$ips_n = ip_n(g_{n+1} \cdot g_{n+2} \cdot \ldots \cdot g_m) \qquad (43)$$

where g_m is the gain of the final stage in the cascade. The intercept point for the total cascade is given as (using the subscript "T" to indicate "total"):

$$(ipk_T)^{-1} = (ips_1 + ips_2 + ... + ips_m)^{-1} \quad (44)$$

Much like resistors added in parallel, the total intercept point is always less than the lowest intercept point in the cascade. Furthermore, since gain (or loss) after each successive stage affects intercept point, loss after an amplification stage adversely affects the intercept point, which in turn increases the inter-modulation distortion.

Using a very low-loss broadband combining network minimizes degradation to the intercept point for the active amplifier elements. The broadband non-dispersive nature of the combiner also minimizes AM to PM conversion. A spatially combined amplifier is well suited for amplifying signals with complex modulation or deep amplitude modulation.

Performance Verification Measurements

Aside from the fundamental performance parameters discussed above, there are numerous other parameters that may require verification. Verification of performance, through testing, of the fundamental parameters is not necessarily a trivial task and is usually necessary before more esoteric measurements are made.

Small-Signal Gain and Flatness

Of all of the parameters for an amplifier to measure, perhaps the gain of an amplifier is the most fundamental parameter to measure. Given the critical nature of the measurement, the techniques for making gain measurements are numerous. It is beyond the scope of this paper to attempt to cover all of the techniques; however, there are issues to consider before selecting a general class of gain measurement technique. Some a priori knowledge of the anticipated performance of the device under test (DUT) is helpful in selecting a technique.

One of the workhorse techniques for measuring gain and gain flatness for a microwave amplifier is to use a scalar network analyzer (SNA). This technique, while probably not the last word on accuracy, does have several advantages to consider.
The basic premise of the measurement is to adjust the output of a precision microwave signal generator, such that its output power is as flat as possible at the reference plane of the measurement. Then a diode sensor, for largest measurement range, can be used to detect the power after the device under test. Knowing the input power level and measuring the output power level allows the scalar network analyzer to compute the gain across the DUT.

During the calibration for the gain/flatness test, if required, an electrically short transmission line, usually in the form of a coaxial adapter may be inserted between the precision source and the power sensing element. Such an adapter may be required if the DUT cannot be mated directly to either the source or the sensor element. It is important to minimize the number of adapters necessary to insure that the SWR between the source and detecting element is as low as possible. For this class of measurement, SWR is typically the largest source of measurement uncertainty.

Applying the same reasoning, it is necessary to select a signal generator and detecting element that have the lowest possible SWR for minimizing measurement uncertainty. The signal generator must possess a high quality automatic leveling control (ALC) loop to insure that its source match is as close as possible to 50Ω over the widest possible dynamic range. Giga-tronics manufactures the 2400B and 2500A series precision microwave signal generators, each suitable as a source for scalar network analysis measurements. Both series of Giga-tronics signal generators work with the Giga-tronics 8003 Precision Scalar Network Analyzer, as well as the popular HP/Agilent 8757 series of scalar network analyzers. Giga-tronics also manufactures wide dynamic range diode-based power sensors that are fully compatible with the Giga-tronics 8003 SNA.

One of the advantages of a scalar network analysis measurement, over other techniques to be discussed later, is the ability to drive the DUT at higher power signal levels commensurate with the output power available from the microwave signal generator, particularly if a simultaneous return loss measurement is not necessary. Furthermore, it will be possible to make accurate gain/flatness measurements at drive levels up to +40 dBm and beyond, by using a wideband high power microwave amplifier, such as the Giga-tronics GT-1000A, and external ALC. Combined with Giga-tronics high power-high dynamic range diode sensors, this creates a unique scalar analysis measurement system for testing DUTs that require accurate gain or insertion loss measurements at high power levels. For more details regarding extended dynamic range scalar network analysis, please refer to the *Endnotes and References* section[13,14,15].

Another technique for measuring small signal gain uses a vector network analyzer (VNA). This technique can yield very accurate results, as both the amplitude and the phase of the scattering parameters for the DUT are measured. This technique does require a significantly more expansive calibration compared to that required for a gain measurement using a scalar network analyzer. With the latest generation of vector network analyzers, measurements can be made that are not strictly linear, as implied by measuring scattering parameters. In essence, this involves using the source and receivers within the VNA to make precision amplitude measurements, while keeping track of the time delay to maintain phase information. A complete discussion of vector network analysis is beyond the scope of this paper, but a citation within the *Endnotes and References* section can provide a more in-depth discussion, as well as citations to further references[16].

Power and Compression Measurement Techniques

As discussed in the previous *Power Output* section of this paper, the transfer function for power output has both linear and non-linear characteristics. Typically, the region where a microwave power amplifier is characterized is at the point where the power behavior transitions from linear to non-linear, as well as its fully saturated power output. Since power output is also a very key parameter for a microwave amplifier, there are several techniques for making these measurements as well. Nevertheless, the most dominant technique is to make the measurements with a power sensor. These techniques are well documented in the literature from the manufacturers of power meters and/or SNAs; however, one technique deserves a little more discussion: the Ratio Power Measurement.

The ratio power measurement is a swept power measurement. If set-up using a scalar network analyzer, a display of the gain and power output can measured, allowing the user to see a very graphical view of the amplifier behavior as it goes into compression. A signal generator capable of driving the DUT amplifier into compression is required. Prior to the DUT, a directional coupler is used to provide a sense of the swept input power as it is increased into the DUT. A second sensor is placed at the output of the DUT. Using a built-in mathematical capability of the SNA (or alternatively, a two channel power meter), the ratio of the detected power at the output sensor, over the detected power at the coupled input sensor provides the gain measurement.

Both paths have to have a through calibration as discussed in the prior section *Gain and Gain Flatness*. If using a SNA, a marker is placed in the gain region where the amplifier is linear, then using the marker-delta feature, the point where the gain drops one decibel is marked. The SNA is then switched back to measuring absolute power. The marker on the output power trace is at the P_{-1dB} point. Other compression points can be measured in a similar fashion. For more details on this measurement technique, please refer to Giga-tronics application note AN 00-02[17].

SWR Measurements

Like the small-signal gain measurements, SWR can be measured either with a SNA or VNA. With the VNA, the advantage, aside from accuracy, is that the measurement includes phase information. This information can be used to judge what is happening to the input and output matches as a function of frequency; i.e., how does the input/output complex impedance vary from an ideal 50Ω match. In most instances, this information has to be used judiciously; it is highly likely that the source of the deviation from an ideal match is not immediately located at the electrical measurement plane. Therefore, some knowledge of the design is required to judge how to move the electrical reference plane to the area of interest. Many VNAs come equipped with a feature, time domain reflectometry (TDR), which can be used to help judge where an undesired source of reflection is located.

A SNA can be used to measure SWR; however, it is neither as precise, nor does it provide any phase information. One advantage to using a SNA is that it may be possible to obtain a little better signal to noise ratio when measuring the output match of a broadband power amplifier. The SNA uses a device commonly referred to as a “directional bridge” to facilitate making SWR measurements.

The directional bridge is a three port device that is connected to a signal source, such as a bench-top synthesizer, the DUT, and an input port on the SNA. Between the source input and the test port of the directional bridge that is connected to the DUT, there is typically a substantial amount of loss. To a point, this can be overcome by increasing the source output power. Be sure to check the specification for the directional bridge to make sure its input power is not exceeded. The port that is connected to the SNA behaves much like a highly directional detector, in that it detects signals reflected from the DUT, but is isolated from the signal source.

Prior to making a measurement of output SWR, it is a good idea to measure the broadband noise power present at the amplifier output, while the input in terminated with a broadband 50Ω load. Next, measure the signal level strength present at either the VNA or SNA test ports. Depending on how much accuracy is required, the signal level needs to be substantially greater than the noise power present in the detected bandwidth. The accuracy of the measurement can be improved with the VNA by narrowing the IF (intermediate frequency) bandwidth, if the VNA is so equipped with this feature. Information about the IF bandwidth, and the capacity to modify it, can usually be found in the operations manual for the VNA.

Most SNAs do not have selectable IF bandwidths per se; however, almost all have an AC detection mode that offers a similar benefit. In AC detection mode, the signal source output is chopped (pulse modulation mode) at a relatively low rate (e.g. 27.778 kHz for the HP8757 series SNAs). The directional bridge detector amplitude demodulates (envelop detection) the reflected signal, and the SNA band-pass filters the AM detected signal, improving the signal-to-noise ratio by reducing the noise bandwidth. Care must be exercised when using AC detection mode with a SNA; amplifiers that have automatic leveling or gain control can become unstable with a low-frequency chopped RF signal.

Stability Measurements

As discussed earlier, stability in an amplifier can be difficult to measure. Rarely is instability an issue in the designed for frequency range, as it is straightforward to mitigate this design issue. As mentioned earlier, the more pernicious issues are out-of-band oscillation and bias oscillation. While the measurement techniques for detecting these issues are relatively simple measurements to make, they can be tedious to complete. Ultimately, direct measurements will need to be made with a spectrum analyzer; however, it can be sometimes useful to include an indirect measurement technique.

One technique to help detect oscillation, or meta-stability (excess noise gain peaking) is to measure the noise power of the amplifier. It requires a priori knowledge of the expected noise power to judge if there is more noise than there should be, or an outright oscillation. Often, a power sensor detector is quite broadband, even if it is not rated for the frequency range where the oscillation occurs. Furthermore, if it is not possible to directly measure the oscillation with a spectrum analyzer, an indirect measurement may justify the effort to mix-down the suspected out-of-band oscillation, or obtain a wider frequency range spectrum analyzer.

When looking for bias oscillations, making the direct measurement with a spectrum analyzer requires a tedious search at various offset frequencies from the carrier tone injected at the input of the amplifier. Sometimes it may be more expedient to amplitude demodulate an undesired bias oscillation with a diode detector and an oscilloscope. The drawback of this technique is that is not sensitive enough to detect low level bias oscillation. Nevertheless, it is usually very simple to quickly adjust the oscilloscope time base to see in the time domain a bias oscillation. An advantage to this technique is that if such an issue has been found, often it may be necessary to make direct measurements with an oscilloscope to troubleshoot the origin of the issue.

When making direct measurements for stability issues with a spectrum analyzer, patience is required. Frequently, the first attempt for an oscillation search is to sweep the entire spectrum that the analyzer is capable of measuring. It is important to make note of the analyzer noise floor for a given resolution bandwidth prior to the measurement. Subsequent sweeps require successively reducing the resolution bandwidth, and the video bandwidth (auto-couple the RBW and VBW).

It may also be necessary to reduce the input attenuation of the analyzer to improve its receiver noise figure. Extreme care must be taken before completely removing all attenuation from the front-end of the spectrum analyzer to make sure that there are no out-of-band oscillations; it is critical to measure the broadband noise power first. If any narrowband gain peaking is observed, center it within the span of the analyzer, and successively reduce the span until the gain peak is nearly filling the swept span. Continue reducing the resolution bandwidth to insure that what may appear to be “noise” is not in fact a group of closely spaced oscillation tones. Automation of these measurements is highly desirable, but may not be completely foolproof.

One last measurement technique may be necessary to insure stability. Depending on how the amplifier is specified, it may be necessary to make a *pulling-figure* test. For example, if the amplifier is guaranteed to be stable into a 3:1 SWR, then it must be stable when looking into a return loss of 6 dB at all phases. To test this, refer to Figure 5. The sliding short circuit must be electrically long enough to represent a 180° phase shift at the lowest frequency of interest. The spectrum analyzer must be set so as to allow a reasonably fast sweep across the frequency of interest, but maintain enough dynamic range to see low level oscillation issues of interest.

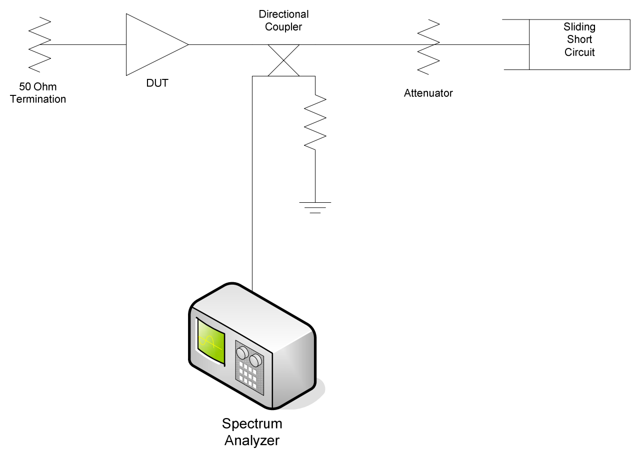

Figure 5: Pulling Diagram

Noise Figure Measurement

Several techniques exist for measuring noise figure. In most instances, the techniques are focused on making the most accurate measurement possible for the lowest noise figure devices. While many of these techniques may work well for measuring the noise figure of a microwave power amplifier, they may be more complicated or expensive than is necessary for the application at hand. Using a calibrated excess noise source, and a spectrum analyzer, it is possible to measure the noise figure of a typical microwave power amplifier using the *Y-Factor* method[18].

Using the test set-up in Figure 6, the excess noise source is turned on and off, while monitoring the noise floor on a spectrum analyzer. For a microwave solid state power amplifier, a calibrated excess noise source of approximately 15 dB is appropriate. Some experimenting may be necessary; however, the spectrum analyzer can be set as follows:

SPAN = 1 MHz
RBW = 2 MHz
VBW = 10 Hz

The noise figure can then be calculated as:

$$NF = ENR(f) - 10 \cdot \log\left[\frac{P_{out}\big|_{on}}{P_{out}\big|_{off}} - 1\right] \qquad (45)$$

where P_{out} is in Watts, rather than dBm. Written another way:

$$NF = ENR(f) - 10 \cdot \log\left(10^{\Delta dB/10} - 1\right) \qquad (46)$$

where ΔdB is the change in displayed power in decibels from when the ENR source was on and off.

Prior to making this measurement, it may be necessary, depending on the gain and power of the amplifier DUT to adjust the input attenuation of the spectrum analyzer. As a rule of thumb, unless otherwise specified, a typical spectrum analyzer may have a NF of about 30 dB, without input attenuation. Thus, to negate the effect of the spectrum analyzer noise figure, the preceding gain stage should have at least 40 dB of gain. Again, when removing all input attenuation on a spectrum analyzer, it is critical to verify that the output power from the DUT does not exceed the specified input overload level for the spectrum analyzer (and certainly not to exceed the damage level).

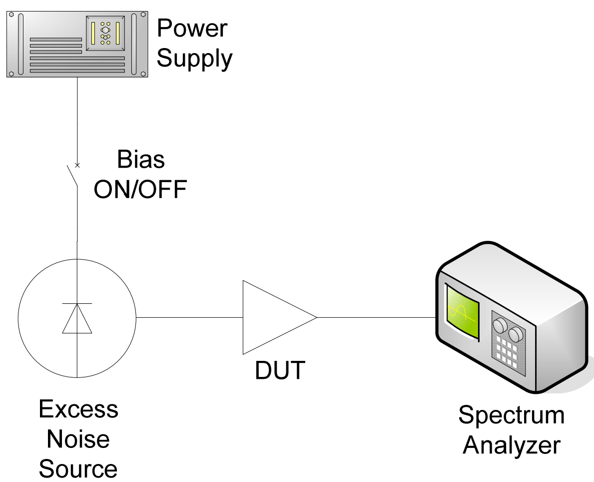

Figure 6: Noise Figure Measurement

Distortion Products Measurements

There are numerous techniques for characterizing non-linearity in microwave power amplifiers. Often the technique is determined by the application for the amplifier, as well as the resources on-hand. A common technique for characterizing IMD is to measure mixing products from a two tone input. Referring to Figure 7, two signal generators are used to drive a DUT, and a spectrum analyzer is used to measure the resulting distortion products. A coupler and power meter are used to accurately measure the power output of the DUT; however, they may be omitted if the spectrum analyzer is carefully calibrated for amplitude accuracy.

To improve the accuracy of the measurement, a few issues should be considered. To partially mitigate AM to PM conversion issues, the two signal generators should be phase locked to a common 10 MHz reference from the spectrum analyzer. Note that partial mitigation may only be possible depending on the architecture of the signal generators.

Care must be taken to minimize phase differences in the connections from the two signal sources to the power combiner to minimize AM to PM conversion as well. The isolators help mitigate any issues with generating IMD at the signal generators themselves. Once the test system is put together, the IMD of the test system itself should be characterized to insure that the measurement system has sufficiently low IMD compared with the intended range of measurements for characterizing the DUT.

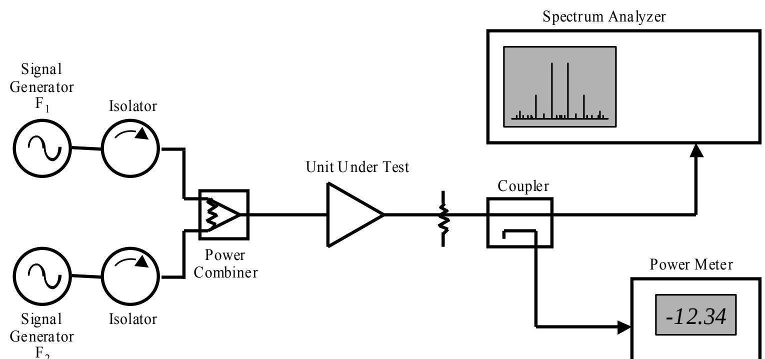

Figure 7: IMD Test Set-up

Summary

The GT-1000A Microwave Power Amplifier, using CAP Wireless patented Spatium™ technology, provides an elegant solution with a remarkable level of performance compared to many other microwave power amplifier solutions. Unlike a TWTA, the solid state GT-1000A does not require a high voltage power supply, resulting in an inherently more reliable design, as well as requiring significantly less warm-up time and with negligible aging related issues.

With many classic microwave power amplifiers, power combining is accomplished using planar circuit techniques. While these techniques can be made quite broadband, they are nearly universally very lossy when compared with a spatial combiner. Without exotic super-cooling techniques, the losses are unacceptably high for instances where more than four amplifiers are combined. This limits the usefulness of classic planar circuit techniques to a few Watts for broadband designs.

Furthermore, as discussed in earlier sections, high losses or lack of bandwidth also have many undesirable effects upon the linearity and noise figure for amplifiers with classic power combining architectures. The spatially combined GT-1000A Microwave Power Amplifier does not suffer from these disadvantages.

For other solid state microwave power amplifiers to achieve similar performance often requires switching between narrow-band amplifiers, with deleterious effects to the overall linearity and gain/power flatness. Switches themselves embody performance compromises. Switching between narrowband amplifiers requires external stimulus with the software control complication that entails, as well as other performance compromises. The GT-1000A does not require any external stimulus, as there are no internal switches, nor are there any other negative issues associated with internal switching.

The GT-1000A Microwave Power Amplifier embodies a new design technique, scalable as MMIC technology advances, permitting a combination of performance parameters unchallenged by conventional technology.

Appendix A: P_{out} Calculation

For reference, equation (13) is repeated:

$$P_{out} = \frac{G_T \cdot P_{in}}{1 + \frac{\beta(G_T \cdot P_{in})}{P_{sat}}} \qquad (A1)$$

$$1 + \frac{\beta(G_T \cdot P_{in})}{P_{sat}} = 1 + \frac{G_T P_{in}}{P_{sat}} e^{-\alpha \frac{P_{sat}}{G_T P_{in}}} \qquad (A2)$$

$$= 1 + \frac{G_T P_{in}}{P_{sat}} \cdot e^{-\left(k_1 \frac{P_{-1dB}}{P_{sat}} \cdot \ln\left(k_2 \frac{P_{sat}}{P_{-1dB}}\right) \cdot \frac{P_{sat}}{G_T P_{in}}\right)} \qquad (A3)$$

$$= 1 + \frac{G_T P_{in}}{P_{sat}} \cdot e^{-\left(k_1 \frac{P_{-1dB}}{P_{sat}} \cdot \ln\left(k_2 \frac{P_{sat}}{P_{-1dB}}\right)\right)} \qquad (A4)$$

$$= 1 + \frac{G_T P_{in}}{P_{sat}} \cdot \left(e^{\ln\left(k_2 \frac{P_{sat}}{P_{-1dB}}\right)}\right)^{-k_1 \frac{P_{-1dB}}{G_T P_{in}}} \qquad (A5)$$

$$= 1 + \frac{G_T P_{in}}{P_{sat}} \cdot \left(k_2 \frac{P_{sat}}{P_{-1dB}}\right)^{-k_1 \frac{P_{-1dB}}{G_T P_{in}}} \qquad (A6)$$

Recall, 1dB = $10^{1/10}$ = 1.259825412… Thus when:

$$G_T P_{in} \cong P_{-1dB} \cdot 1.258925412 \qquad (A7)$$

$$1 + \frac{\beta(G_T \cdot P_{in})}{P_{sat}} = 1 + \frac{P_{-1dB} \cdot 1.2589...}{P_{sat}} \left(k_2 \frac{P_{sat}}{P_{-1dB}}\right)^{-k_1 \frac{P_{-1dB}}{P_{-1dB}(1.2589...)}} \qquad (A8)$$

$$= 1 + \frac{P_{-1dB}(1.2589...)}{P_{sat}} \cdot k_2 \cdot \frac{P_{sat}}{P_{-1dB}} \qquad (A9)$$

$$= 1 + (1.2589...)(0.20567...) \qquad (A10)$$

$$= 1.2589\ldots$$

This situation arises when the following condition is met:

$1 + (-k_1)(k_2) = -k_1 = 10^{1/10} = 1$ dB

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