# A Novel Impedance-Measuring System Using Standard - $h p$ - Instruments 

WHEN an impedance is fed from a con-stant-current source, the voltage across the impedance is a function of the magnitude of the impedance.

This basic relation will permit a combination of two standard -hp-instruments to be used to measure extremely wide ranges of L and C as well as moderate ranges of R and Z. C's from 800 microfarads to a few micromicrofarads, L's from 200 henrys to 100 mi crohenrys, and R's and Z's from 30,000 ohms to a few ohms can be measured. A feature of the system is that, in general, the measured values can be read directly.

The two instruments required for the measurements are an oscillator and a sensitive voltmeter. Several different combinations of -hp-oscillators and voltmeters can be used. Probably the combination most commonly available is one of the $-h p-200$ series audio oscillators used with the -hpModel 400C VTVM or Model 330 Distortion Analyzer. For these measurements the Model 330 Distortion Analyzer is used as a sensitive voltmeter having a maximum sensitivity of 300 microvolts full scale. Other possible combinations of instruments are discussed later. Some of these combinations


Fig. 1. Basic set-up for measuring impedances with constant-current method.
will give even wider ranges of measurements than those listed above.

The basic set-up for the measurements is indicated in Fig. 1. The -hp- Model 200 audio oscillator is converted to a constantcurrent source by insertion of a 1 -megohm resistance in series with the generator output terminals. The voltage across an unknown impedance connected into the circuit will then be proportional to the magnitude of the unknown impedance.

As part of the set-up procedure, the oscillator output voltage is adjusted by the panel controls to be 10 volts. Since the circuit impedance is 1 megohm, a generator voltage of 10 volts will cause a constant current of 10.0 microamperes to flow through the unknown as long as it is small compared to 1 megohm. The significance of achieving a constant current of 10.0 microamperes is that this value of current will permit $\mathrm{R}, \mathrm{Z}$ and L to be read directly on the -hp-voltmeter.

How this direct-reading feature occurs can be described by considering the expression for the voltage across the unknown. If the constant current is 10 microamperes, the voltage across the unknown will be

$$
|\mathbf{E}|=\mathrm{IZ}=\left(10 \times 10^{-6}\right)(\mathbf{Z})=\mathbf{Z} \times 10^{-5}
$$

If the unknown is, say, 3,000 ohms, the voltage across the unknown will be 0.03 volt. When the voltmeter range switch is positioned to the 0.03 volt range, the voltmeter pointer will be deflected to read " 3 ." If the unknown is changed to, say, 1,500 ohms, the voltage across the unknown will be 0.015 volt. The voltmeter pointer will


Fig. 2. Basic set-up for measuring capacity with constant-voltage method.
then be deflected to read "1.5." In other words, the significant figures of the voltage value are the same as the significant figures of the impedance value. The voltmeter thus becomes a direct-reading impedance meter. Using the -hp- Model 400 C VTVM with one of the 200 series oscillators, full-scale resistance or impedance values of $100,300,1,000$, $3,000,10,000$, and 30,000 ohms are obtained. Table I at the end of the article gives the full-scale values of the various positions of the voltmeter range switch.

## 1 measurements

So far no mention has been made of frequency, since the resistance or impedance measurements can be made at any frequency within the range of the equipment.

However, the adjustable frequency of the oscillator can be used in such a way as to permit direct readings both of L and C . Consider first the measurement of L . When an inductance is connected into the circuit of Fig. 1, the voltage across the inductance will be

$$
|E|=(I)(2 \pi f)(L)
$$

Now the factor ( $2 \pi \mathrm{f}$ ) in the above expression can be adjusted to be a power of 10 by suitable adjustment of frequency. For example, if $f$ is adjusted to be 159.2 cps , (or in round numbers 160 cps ), the factor ( $2 \pi \mathrm{f}$ ) will be equal to $10^{3}$; if $f$ is adjusted to be $1600 \mathrm{cps},(2 \pi \mathrm{f})$ will be $10^{4}$, etc.

When ( $2 \pi \mathrm{f}$ ) is made equal to $10^{\mathrm{m}}$, the voltage across the inductance will be

$$
\begin{aligned}
|\mathbf{E}| & =\left(10 \times 10^{-6}\right)(\mathrm{L})\left(10^{\mathrm{m}}\right) \\
& =(\mathrm{L})\left(10^{\mathrm{n}}\right)
\end{aligned}
$$

This is the same type of relation obtained earlier in the measurement of resistance and impedance where $|\mathrm{E}|$ was equal to $(\mathrm{Z})\left(10^{-5}\right)$. In those measurements it was unnecessary to
adjust frequency to obtain a directreading impedance-measuring system. But in this case, if frequency is adjusted to be 160,1600 , or 16,000 cps, the significant figures in the voltage and in the value of $L$ are the same. Therefore, direct measurements can be made of L.

To take an example, assume that an inductance of 0.3 henry is to be measured. The generator could be set to 160,1600 , or $16,000 \mathrm{cps}$, but it is convenient to set it to 160 cps . The voltage across the inductance will then be

$$
|E|=\left(10 \times 10^{-6}\right)(6.28 \times 160)(0.3)
$$ or $|\mathrm{E}|=0.003$ volt

When the voltmeter range switch is set to the 0.003 - volt full scale position, the meter pointer will directly indicate the value of the inductance ( 0.3 h ).

The range of values of $L$ that can be measured using various frequencies is shown in Table I. It should be noted that the two lowest frequencies listed in the inductance portion of the table are not direct-reading.

## c measurements

The constant-current system can not be used to make direct-reading measurements of C , because the voltage across $C$ and the value of $C$ are related inversely rather than directly. However, the constant-current system can be used to measure C by use of a simple nomograph. This arrangement is described later.

To obtain a system for direct-reading measurements of $C$, it is necessary to use the inverse or dual of the constant-current system; that is, to use a constant-voltage system in which the current will be a function of C. A suitable arrangement is illustrated in Fig. 2. The oscillator is a low-impedance constant-voltage type such as the -bp-Models 200A, 200B, 200C, etc. The large series resistance of the constant-current system is replaced with a small precision resistance across which is connected the voltmeter. The current flowing in such a circuit is

$$
(I-E / X-(E)(2-f)(C)
$$

The current is thus proportional to the value of C .

To make the system direct-reading, it is necessary that the significant figures in the value of I be the same as the significant figures in the value of $C$. This condition can be achieved either by making the factors (E) and ( $2 \pi \mathrm{f}$ ) both equal to powers of 10 or by making the prod$u c t$ of (E) and ( $2 \pi \mathrm{f}$ ) a power of 10 . To accommodate the frequency range of most -hp- oscillators, it is convenient to make the product of (E) and ( $2 \pi \mathrm{f}$ ) a power of 10 . This condition is best met by choosing an (E) of 8 volts and an $f$ of 20,200 , etc.

To describe how the method operates, assume that a C of 1 microfarad is to be measured at a frequency of 20 cps . The current flowing in the circuit of Fig. 2 will be

$$
|\mathbf{I}|=(E)(2 \pi f)(C)=(8)\left(125.7 \times 10^{-6}\right)
$$

$$
=1 \text { milliampere }
$$

One milliampere flowing through the 100 -ohm precision resistor impresses a voltage of 100 millivolts across the voltmeter. The voltmeter thus reads " 1 ," the value of C . If the value of $C$ were half as large, i.e., 0.5 microfarad, only half as large a current would flow and the voltmeter reading would be only half as large.

Using the constant-voltage circuit, C's from 3 microfarads to approximately 3 mmf can be measured. The full-scale values of the voltmeter positions in terms of C are given in Table II.

## CONSTANT-CURRENT <br> C MEASUREMENTS

Where the constant-voltage method described above is suitable for


Fig. 3. Inherent error caused by voltmeter loading and by reduced current. Plotted for case where unknown is a pure resistance. Voltmeter impedance is assumed to be entircly resistive.
measuring very small values of $\mathbf{C}$, the constant current method is suitable for measuring very large values of C . The range of values that can be measured with the set-up of Fig. 1 is shown in Table II.

Constant-current C measurements are made in the same general manner as constant-current measurements of R, Z, and L. However, the resulting voltmeter readings must be converted by use of the nomograph (see back page). For convenience, C measurements are made at frequencies of $20 \mathrm{cps}, 200 \mathrm{cps}$, etc. To facilitate use of the nomograph, the con-stant-current portion of Table II is arranged to indicate the maximum and minimum value of each switch setting at each frequency.

## calibration

Fig. 4 shows the operating set-up for making measurements with either the constant-current or con-stant-voltage methods. The constantcurrent set-up can be calibrated by plugging the -hp- Model 470D 100 ohm precision shunt into the voltmeter terminals and adjusting the oscillator voltage until the voltmeter indicates exactly 1 millivolt. The desired current of 10 microamperes will then be obtained.

For constant-voltage measurements, the 470 D can be used as the 100 -ohm precision resistor across which the voltmeter is connected. To calibrate the set-up, adjust the oscillator voltage to be 8 volts when the shunt is removed and the capacity shorted.

## ERRORS

In general, the error inherent in the constant-current system of measurement increases with the magnitude of the impedance being measured. Greatest error occurs in the measurement of resistance. Where the impedance being measured is substantially reactive, i.e., in measurements of $\mathrm{C}, \mathrm{L}$, and Z , the error is considerably less than that obtained in measurements of R.

There are two principle factors
that contribute to the inherent error in constant-current type measurements. These are (a) the relation of the impedance of the unknown to the 1 -megohm impedance of the current source; and (b) the loading effect on the unknown by the voltmeter impedance.

An indication of error is given in Fig. 3. The errors shown have been calculated for the case where the unknown is a pure resistance, the worst case, and for several voltmeter impedances. In calculating these values, the voltmeter was assumed to have a pure resistive input. Actually, VTVM's have associated with them a capacitive component which makes their input impedance a function of frequency. Therefore, greater accuracy will, in general, be obtained in constant-current type measurements at the lower frequencies.

Measurements of C with the di-rect-reading constant-voltage system can be made with excellent accuracy. In general, the inherent error will not exceed a few tenths of a per cent. This small error will be swamped by practical errors such as errors in frequency setting, in voltmeter readings, etc.

## general

Full scale-values of $\mathrm{R}, \mathrm{Z}$, and L are given in Table I for the various positions of the voltmeter switch.

The table is designed for use with a constant current of 10 microamperes. If a series resistance of 1 megohm is used, nearly any of the -hpaudio oscillators can be used to provide the 10 volts necessary to obtain 10 microamperes.

A number of -hp- oscillators will provide considerably more than 10 volts. The Models 200A and 200B provide approximately 25 volts, while the Model 201B will provide more than 40 volts. Where one of these instruments is available, the additional voltage can be used in either of two ways to increase the range of measurements. First, the series resistance can be made several times larger than 1 megohm so that


Fig. 4. Generalized set-up for coustantcurrent or constant-voltage measurements. -hp-470D 100-obm shunt is conveniently arranged to be plugged directly into volt. meter terminals.
the range of the measurements can be extended upward. Or, second, the constant current can be increased from 10 to 31.6 microamperes so that the minimum value of measurable impedances can be extended downward. With a constant current of 31.6 microamperes, it is convenient to make readings on the "wrong" scale of the meter face. The "wrong" scale is used in this case because the voltage across the unknown will be $3.16(10 \mathrm{db})$ times as great as with a current of 10 microamperes and because the meter scales of - $h p$ - voltmeters are separated by 10 db .

The widest range of measurements will be obtained by using the $-b p$. Models 205A or 205AG Audio Signal Generators. The Model 205 provides a maximum output voltage of 150 volts. With such a large generator voltage the source impedance can be made very large and at the same time a large constant current can be obtained so that small impedances can be measured.

Two of the $-h p$ - voltmeters are especially suited for measuring the voltage across the unknown. The -hp. Model 400 C has a full-scale sensitivity of 1 millivolt and can be used with a current of 10 microamperes to make measurements as low as 100 ohms full scale. The -hpModel 330 distortion analyzers, which are capable of being operated as high-sensitivity voltmeters, provide a full-scale sensitivity of 300 microvolts. The 330 can be used to make measurements as low as 30 ohms full scale. However, the input impedance of the 330 is 0.2 meg .
ohm in parallel with approximately 40 mmf so that the 330 is less suitable than the Model 400 C for measurements of large impedances.

A little experimenting with the particular combination of instruments at hand will indicate the most convenient method of making measurements.

## ACKNOWLEDGMENT

The foregoing arrangement for measuring impedances with the con-stant-current system was suggested by Mr. Mario Novajra of Poirino, Italy.

A résumé of the above system printed on durable paper will be supplied free upon request while supply lasts.

TABLE I

| VOLT <br> METER <br> RANGE | R \& Z (Full Scale Value) | INDUCTANCE (FULL SCALE VALUE) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} 24 \\ \text { CPS } \end{gathered}$ | $\begin{gathered} 48 \\ \text { CPS } \end{gathered}$ | $\begin{aligned} & 160 \\ & \text { CPS } \end{aligned}$ | $\begin{aligned} & 1600 \\ & \text { CPS } \end{aligned}$ | $\begin{aligned} & 16,000 \\ & \text { CPS } \end{aligned}$ |
| 0.3 MV | 30 | - | 100 MH | 30 MH | 3 MH | $300 \mu \mathrm{H}$ |
| 1.0 MV | 100 | $\square$ | 333 MH | 100 MH | 10 MH | 1 MH |
| 3.0 MV | 300 | - | 1 H | 300 MH | 30 MH | 3 MH |
| 10 MV | 1,000 | $\longrightarrow$ | 3.33 H | 1 H | 100 MH | 10 MH |
| 30 MV | 3,000 | - | 10 H | 3 H | 300 MH | 30 MH |
| 0.1 V | 10,000 | - | 33.3 H | 10 H | 1 H | 100 MH |
| 0.3 V | 30,000 | 200 H | 100 H | 30 H | 3 H | 300 MH |

## TABLE II

| C MEASUREMENTS |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| VOLT <br> METER <br> RANGE | CONSTANT VOLTAGE METHOD (7.96 VOLTS) |  |  |  | CONSTANT CURRENT METHOD (10 MICROAMPERES) |  |  |  |
|  | $\begin{gathered} 20 \\ \text { CPS } \end{gathered}$ | $\begin{aligned} & 200 \\ & \text { CPS } \end{aligned}$ | $\begin{gathered} 2000 \\ \text { CPS } \end{gathered}$ | $\begin{gathered} 20,000 \\ \text { CPS } \end{gathered}$ | $\begin{gathered} 20 \\ \text { CPS } \end{gathered}$ | $\begin{aligned} & 200 \\ & \text { CPS } \end{aligned}$ | $\begin{gathered} 2000 \\ \text { CPS } \end{gathered}$ | $\begin{gathered} 20,000 \\ \text { CPS } \end{gathered}$ |
| 0.3 MV | $0.003 \mu \mathrm{~F}$ | $300 \mu \mu \mathrm{~F}$ | $30 \mu \mu \mathrm{~F}$ | $3 \mu \mu \mathrm{~F}$ | $800 \mu \mathrm{~F}-250 \mu \mathrm{~F}$ | $80 \mu \mathrm{~F} \cdot 25 \mu \mathrm{~F}$ | $8 \mu \mathrm{~F}-2.5 \mu \mathrm{~F}$ | $0.8 \mu \mathrm{~F}-0.25 \mu \mathrm{~F}$ |
| 1.0 MV | $0.01 \mu \mathrm{~F}$ | $0.001 \mu \mathrm{~F}$ | $100 \mu \mu \mathrm{~F}$ | $10 \mu \mu \mathrm{~F}$ | $250 \mu \mathrm{~F}-80 \mu \mathrm{~F}$ | $25 \mu \mathrm{~F}-8 \mu \mathrm{~F}$ | $2.5 \mu \mathrm{~F}-0.8 \mu \mathrm{~F}$ | $0.25 \mu \mathrm{~F}-0.08 \mu \mathrm{~F}$ |
| 3.0 MV | $0.03 \mu \mathrm{~F}$ | $0.003 \mu \mathrm{~F}$ | $300 \mu \mu \mathrm{~F}$ | $30 \mu \mu \mathrm{~F}$ | $80 \mu \mathrm{~F}-25 \mu \mathrm{~F}$ | $8 \mu \mathrm{~F}-2.5 \mu \mathrm{~F}$ | $0.8 \mu \mathrm{~F}-0.25 \mu \mathrm{~F}$ | $0.08 \mu \mathrm{~F}-0.025 \mu \mathrm{~F}$ |
| 10 MV | $0.1 \mu \mathrm{~F}$ | $0.01 \mu \mathrm{~F}$ | $0.001 \mu \mathrm{~F}$ | $100 \mu \mu \mathrm{~F}$ | $25 \mu \mathrm{~F}-8 \mu \mathrm{~F}$ | $2.5 \mu \mathrm{~F}-0.8 \mu \mathrm{~F}$ | $0.25 \mu \mathrm{~F}-0.08 \mu \mathrm{~F}$ | $0.025 \mu \mathrm{~F}-0.008 \mu \mathrm{~F}$ |
| 30 MV | $0.3 \mu \mathrm{~F}$ | $0.03 \mu \mathrm{~F}$ | $0.003 \mu \mathrm{~F}$ | $300 \mu \mu \mathrm{~F}$ | $8 \mu \mathrm{~F}-2.5 \mu \mathrm{~F}$ | $0.8 \mu \mathrm{~F}-0.25 \mu \mathrm{~F}$ | $0.08 \mu \mathrm{~F} \cdot 0.025 \mu \mathrm{~F}$ | $0.008 \mu \mathrm{~F}-0.0025 \mu \mathrm{~F}$ |
| 0.1 V | $1 \mu \mathrm{~F}$ | $0.1 \mu \mathrm{~F}$ | $0.01 \mu \mathrm{~F}$ | $0.001 \mu \mathrm{~F}$ | $2.5 \mu \mathrm{~F}-0.8 \mu \mathrm{~F}$ | $0.25 \mu \mathrm{~F} \cdot 0.08 \mu \mathrm{~F}$ | $0.025 \mu \mathrm{~F} \cdot 0.008 \mu \mathrm{~F}$ | $0.0025 \mu \mathrm{~F} \cdot 800 \mu \mu \mathrm{~F}$ |
| 0.3 V | $3 \mu \mathrm{~F}$ | $0.3 \mu \mathrm{~F}$ | $0.03 \mu \mathrm{~F}$ | $0.003 \mu \mathrm{~F}$ | $0.8 \mu \mathrm{~F}-0.25 \mu \mathrm{~F}$ | $0.08 \mu \mathrm{~F}-0.025 \mu \mathrm{~F}$ | $0.008 \mu \mathrm{~F}-0.0025 \mu \mathrm{~F}$ | $800 \mu \mu \mathrm{~F} \cdot 250 \mu \mu \mathrm{~F}$ |

## C MEASUREMENTS

NOMOGRAPH FOR "1" RANGES OF VOLTMETER


## NOMOGRAPH FOR " 3 " RANGES OF VOLTMETER



