# Time Domain Reflectometry 

The measurements we make are limited by the tools we have at our disposal. Often measurements must be made by roundabout methods because no instruments exist with which to make them directly. Such difficulties are typical of pioneering work in any field. It is then the business of the instrument maker to devise tools to simplify frequent measuring needs.

Sometimes the cumbersome methods first required become so habitual, so ingrained, that the development of improved techniques is overlooked. This seems to be the case with respect to transmission and reflection measurement in the UHF and microwave range. Methods long in use here are now obsolete but are slow to give way to more modern direct methods.

The time-honored method of measuring reflections on a transmission line is to measure, as a function of frequency, the standing wave ratio (SWR) produced by the reflections. The resulting curves of magnitude and phase of the SWR can, in simple cases, be unscrambled to give the location and nature of the reflections, but the interpretation is difficult at best. With each alteration or attempted improvement, a com-


Fig. 1. Time domain reflectometer setup using -hp-Model 1415A plug-in unit with -hp- Model 140A Oscilloscope. Plugin unit generates a fast step which is fed to external circuit. Energy reflected by discontinuities and impedance changes in external circuit returns to plug-in and is displayed on oscilloscope, permitting magnitude, sign and physical location of discontinuity to be read from scope face.


Fig. 2. A Typical Time Domain Reflectometer.
pletely new set of data must be taken and re-interpreted. The process is very time-consuming.

The swept-frequency reflectometer speeds the measurement considerably but does nothing to simplify the task of interpretation. Since no phase information is obtained, the results from even a single simple discontinuity are ambiguous.

The direct method of measuring reflections is to send out a pulse and listen for the echoes. This is what we do in radar, this is what the dolphin does, and the blind man who taps his cane. If the pulse is short enough each reflection produces a characteristic echo distinct from all others. The interpretation is extremely simple and the effect of changes can be seen instantly.

The pulse echo method has been used for many years for the location of faults in wide-band transmission systems such as coaxial cables. Here the time scale is such that microsecond pulses and megacycle bandwidths suffice. But in the laboratory setup, where reflections may be separated only an inch or less, nanosecond pulses and gigacycle bandwidths are needed. Thus pulse echo reflectometry as a laboratory tool has had to await the development of fast pulse generators and oscilloscopes. With these the era of time domain reflectometry has arrived and it is time for engineers to become familiar with this new measurement technique.

As shown in Figure 2, the time domain reflectometer consists merely of a fast rise-time pulse generator to

## SEE ALSO:

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drive the system under test and a fast rise-time oscilloscope to display the reflections. No complicated directional couplers are needed to separate the incident and reflected waves; they are already separated by time, so a simple probe suffices to pick up the signals on the line. In order not to complicate the picture with re-reflections the probe should have a high impedance, and the generator should be a matched source. Reflections from the system under test are then viewed only once before being absorbed by the generator.
The time difference, $t$, between two successive echoes is $2 \mathrm{~s} / \mathrm{v}$, where $s$ is their separation along the line and $v$ is the velocity of propagation. We will show later that if $t>\tau / 2$, where $\tau$ is the system rise time, the echoes can be resolved. The minimum separation for accurate measurement is therefore $\mathrm{s}=\mathrm{v} \tau / 4$. Since $\mathrm{v}=2$ to $3 \times 10^{10} \mathrm{~cm} / \mathrm{sec}$ and, at the present state of the art, $\tau<10^{-10}$ seconds, we find $s$ to be about 5 mm . Discontinuities separated by less than this distance produce seriously overlapping echoes and this complicates the interpretation. However, such close separation complicates the frequency domain measurement equally. To resolve such echoes, SWR measurements would have to be made over a frequency range of 3 to 10 Gc ., i.e. over a band comparable to the spectrum of the pulse


Fig. 4. After the reflection from a resistive termination the trace has the height $1+\rho$ $=2 \mathrm{R} /\left(\mathrm{R}+\mathrm{Z}_{0}\right)$. This is the scale law of the simple ohmmeter shown in Fig. 3. Such a scale on the scope face permits impedance to be read directly.

Fig. 3 (at left), A simple ohmmeter using a two volt battery produces the same relation between terminal volt. age and resistance $R$ as a transmission line excited by a unit step.

Fig. 5 (at right). The reflections produced by simple resistive and reactive terminations.

used in the time domain case.
Irregularities separated by more than the minimum distance produce distinct echoes. Each discontinuity then writes its own signature on the trace: a characteristic pulse shape that immediately tells what kind of discontinuity is present. Further, the position of the echo gives the location of the discontinuity. In exposed circuits one can touch the line to produce an added echo. Then, by running the point of contact along the line till this added echo coincides with the system echoes, one can literally put his finger on the troubles. In a coaxial cable one can produce a reflection by squeezing the cable.
Reflections occurring at intermediate points, unless quite large, may be ignored. Reflections from connectors, a great bugaboo of SWR measurements, are of no concern in time domain reflectometry. Nor do reflections, no matter how large, from later discontinuities affect the interpretation of a given echo. As a result, in cleaning up a system, one usually eliminates the first echo first, then the second, and so on to the end. Because the measurement is dynamic one can find the required cure for each echo very quickly experimentally. Often an entire system can be corrected in less time than would be required to make the first set of SWR measurements.

A modern sampling scope can accurately display signals of $100 \mu$ volts or less in the presence of signals of 1 volt or more. It can therefore detect reflections that are down 80 db or more. This corresponds to $\mathrm{R} \leq 10^{-4}$ and to SWR $\leq 1.0002$, and is beyond the capability of the most elaborate frequency domain reflectometer or slotted line.

## ELEMENTARY ECHOES

The test signal may be either an impulse or a step function. If the system is
linear the impulse response will be the derivative of the step response, so both waves contain essentially the same information. However, the step response is generally simpler in appearance, easier to interpret, and displays more clearly such things as gradual impedance variation with distance. Unless otherwise noted we will assume a step function test signal.

If the "system under test" in Figure 2 is simply a terminating impedance, $\mathbf{Z}(\mathrm{p})$, the reflection coefficient will be

$$
\rho(\mathrm{p})=\frac{\mathbf{Z}-\mathbf{Z}_{0}}{\mathbf{Z}+\mathbf{Z}_{0}} .
$$

The reflection of a step function will be a wave, $f(t)$, whose spectrum $\mathrm{F}(\mathrm{p})=\rho(\mathrm{p}) / \mathrm{p}$. At a time $2 \ell / \mathrm{v}$ after the incident unit step passed the probe the reflected wave returns. The total response $h(t)$ for $t>2 \ell / v$ is the sum of the


FRONT PAGE PHOTO
Fig. 1 on the front page shows -hpengineers Harley L. Halverson and George $H$. Blinn, Jr., using the -hp-reflectometer to determine the dimensions needed to achieve a desired impedance level in stripline. A stripline test section is constructed with the width of its center conductor varying linearly from one end to the other. By connecting this section to the reflectometer and then sliding a probe such as a pencil along the line, the point whose impedance is 50 ohms is determined from the reflectometer. The dimensions of the section at this point can then be measured. No measurements or assumptions about dielectric constant are required. The method was conceived by Wayne Grove of -hp- Associates.


Fig. 6. Typical reflections produced by simple discontinuities.
incident unit step and the reflection, i.e.,

$$
\begin{equation*}
h(t)=1+f(t), t>2 \ell / v \tag{I}
\end{equation*}
$$

Let us now analyze a few simple cases. We shall consider the coaxial cable to be lossless so that $Z_{o}$ is constant and real.

If Z is a pure resistance then $\rho$ is a constant independent of frequency. Thus $f(t)$ is also a step function of magnitude $\left(\mathrm{R}-\mathrm{Z}_{\mathrm{o}}\right) /\left(\mathrm{R}+\mathrm{Z}_{\mathrm{o}}\right)$, and

$$
\begin{equation*}
\mathrm{h}=1+\rho=\frac{2 \mathrm{R}}{\mathbf{R}+\mathbf{Z}_{\mathrm{o}}} \tag{2}
\end{equation*}
$$

for $t>0$. We may invert this relation to get $\mathrm{R} / \mathrm{Z}_{\mathrm{o}}=\mathrm{h} /(2-\mathrm{h})$.

Figure 3 shows a simple ohmmeter consisting of a 2 -volt battery, an internal impedance $Z_{o}$, and a meter to read the terminal voltage, $e$, when an unknown, R, is connected. The expression for the terminal voltage (and therefore the ohmmeter scale law) is identical with (2). Thus we may use an ordinary ohmmeter scale normalized to $\mathrm{Z}_{\text {o }}$ at mid-scale, as shown in Figure 4, to read the terminating resistance. Alternatively we may compute the resistance from (3).

If the termination is a pure capacitance, $Z=\frac{1}{\mathrm{pC}}, p=\frac{1-\mathrm{pCZ}_{\mathrm{o}}}{1+\mathrm{pCZ}}$ and from $\mathrm{F}(\mathrm{p})=\rho(\mathrm{p}) / \mathrm{p}$ we find

$$
f(t)=1-2 \exp \left(-t / C Z_{0}\right)
$$

Note that $|\rho|=1$ at all frequencies but that its phase changes from 0 to $\pi$ as P increases from zero to infinity. For high frequencies the capacitor looks like a short circuit, while for low frequencies it looks like an open circuit. Accordingly $f(t)$ changes exponentially from -1 at $\mathrm{t}=0$ to +1 at $\mathrm{t}=\infty$.

For an inductive termination,

$$
\mathrm{Z}=\mathrm{pL}, \rho=\frac{\mathrm{pL}-\mathrm{Z}_{\mathrm{o}}}{\mathrm{pL}+\mathrm{Z}_{0}}
$$

and $f(t)=2 \exp \left(-Z_{0} t / L\right)-1$. The reflection is the negative of that for a capacitance, changing from +1 at $t=0$ to -1 at $t=\infty$. This is to be expected since an inductance looks like an open circuit for high frequencies and a short circuit for low frequencies. Again $|\rho|=1$ at all frequencies.

The above cases for $\mathrm{R}, \mathrm{C}$, and L terminations are shown in Figure 5. The equations written by the curves are for $h(t)=1+f(t)$.

A simple discontinuity in a line adds an impedance, $\mathrm{Z}_{\mathrm{d}}$, if in series, or an admittance, $\mathrm{Y}_{\mathrm{d}}$, if in shunt. Thus the line carrying the incident wave faces an impedance $\mathrm{Z}=\mathrm{Z}_{\mathrm{d}}+\mathrm{Z}_{\text {o }}$ or an admittance $\mathrm{Y}=\mathrm{Y}_{\mathrm{d}}+\mathrm{Y}_{\mathrm{o}}$. The reflection coefficient is therefore

$$
\frac{Z_{d}}{Z_{d}+2 Z_{o}} \text { or }-\frac{Y_{d}}{Y_{d}+2 Y_{o}} .
$$

Figure 6 shows the various cases of pure discontinuities, and the corresponding waveforms produced. In all cases one may draw the waveform immediately by determining whether the reflection coefficient at infinite frequency and at zero frequency is $+1,0$, or -1 . The result gives the value of $f(t)$ at zero and infinite time respectively. The transition is an exponential whose time constant is determined by the value of the reactive element and the impedance it faces.

The first two cases of Figure 6 can occur because of poor contacts, bad insulation, or circuit loading. When $\mathrm{R} \ll \mathrm{Z}_{0}$ or $\mathrm{G} \ll \mathrm{Y}_{\mathrm{o}}$, the reflection has the magnitude $\mathrm{R} / 2 \mathrm{Z}_{0}$ or $\mathrm{G} / 2 \mathrm{Y}_{0}$. If we take $\rho=10^{-4}$ as a reasonable lower limit and $Z_{0}=50$ ohms we find we can detect a series resistance of one hun-
dredth of an ohm, or a shunt resistor of one megohm!
The middle two cases of Figure 6 frequently occur as a result of cable defects, connector tolerances, or dimensional errors in design. Often the added reactance or susceptance is so small that the time constant of the exponential is much less than the pulse rise time. This important case will be discussed in the next section.

The last two cases in Figure 6 generally occur only from accidental open or short circuits, in which case the time constant will be short, or from design elements in a high pass structure, in which case the time constant may be quite long.

## EFFECTS OF FINITE RISE TIME

The waveforms derived in the previous section are ideal in that they assume a zero rise-time step. The actual observed waveform is the same as would be obtained by passing the ideal waveform through a filter whose step response is that of the generator-oscilloscope combination. The impulse


Fig. 7. As the reversed system impulse response (a) scans the ideal curve (b) the integral of the product (shaded area) produces the actual response shown in the lower curve.

response of this equivalent filter is therefore the derivative of the observed step response.

As is well known, the output of a linear filter is the convolution of the input with the filter impulse response ${ }^{1,2}$ (that is, the integral of the product of the input wave and the impulse response reversed in time, as the two functions scan past each other). Each step in the input integrates the impulse response to produce the step response; each input impulse produces an output impulse response. Figure 7 shows a typical impulse response (a) reversed in time and scanning an ideal step and pair of reflections (b). The step integrates the impulse response to produce the actual observed step response (c). The two reflections scan the impulse response to produce a bump (d) and a dip (e) each of which is a replica of the impulse response and has the same area as the reflection producing it.

The reflections in Figure 7 are typical of a small series inductance and shunt capacitance and have the areas $\mathrm{L} / 2 \mathrm{Z}_{o}$ and $\mathbf{C Z} / 2$ respectively. Rather than trying to estimate the area of (d) or (e) in order to find L or C , it is easier to estimate the peak amplitude, a , of the observed reflection and the maximum slope, $m$, of the step response. Since the latter is also the peak value of the response to a unit area impulse, it follows that $a$ is $m$ times the area, $\mathrm{L} / 2 \mathrm{Z}_{\text {o }}$ or $C Z_{0} / 2$, of the actual impulse. Therefore,

$$
\begin{align*}
& \mathrm{L}=2 \mathrm{Z}_{\mathrm{o}} \frac{\mathrm{a}}{\mathrm{~m}}  \tag{4}\\
& \mathrm{C}=\frac{2 \mathrm{a}}{\mathrm{Z}_{\mathrm{o}} \mathrm{~m}} \tag{5}
\end{align*}
$$

[^0] printed in expanded form.)

Fig. 8 (at left). Pairs of closely spaced reflections as they would be seen with a zero rise time system, (a) and (c), and (b) and (d) as seen on a system having the impulse response shown by the dotted curves. Height measurements indicated by the arrows are still accurate, but further overlap would cause error.

Fig. 9 (at right). The successive reflections and transmissions produced by a pair of discontinuities in impedance. The first, from $\mathrm{Z}_{0}$ to $\mathrm{Z}_{1}$ produces a reflection coefficient
$\rho_{1}=\left(Z_{1}-Z_{0}\right) /\left(Z_{1}+Z_{0}\right)$ while the second from $\mathrm{Z}_{1}$ to $\mathrm{Z}_{2}$ produces $\rho_{2}=\left(Z_{2}-Z_{1}\right) /\left(Z_{2}+Z_{1}\right)$. The sum of all the reflections is

$$
\rho=\frac{\rho_{1} \rho_{2}}{1+\rho_{1} \rho_{2}}=\frac{Z_{2}-Z_{0}}{Z_{2}+Z_{0}} .
$$

The sum of all the transmissions is $1+\rho$.

If $\mathrm{Z}_{0}=50 \Omega, \mathrm{~m}=10^{10} / \mathrm{sec}$, and the smallest observable value of $a$ is $10^{-4}$, we find the minimum detectable $\mathrm{L}=10^{-12}$ henry, and $\mathrm{C}=4 \times 10^{-16}$ farad!
Reflections closer together than the width of the impulse response (the rise time of the step response) will overlap. Figure 8 shows two reflections (a) of opposite sign and (c) of the same sign. The corresponding responses are shown in (b) and (d). Note that the peak of the impulse response from one reflection occurs just before or after the response from the other. Thus height measurements taken at the arrows will give correct measures of L or C . Closer separation will not permit accurate measurement. We can take the practical limit of resolution to be one half the width of the impulse response, i.e., a little more than half the nominal rise time. Obviously any period of overshoot or ring. ing will further lengthen the minimum separation required for accurate measurement.

## MULTIPLE REFLECTIONS

Whenever more than one discontinuity is present multiple reflections occur, which, if ignored, can cause errors of interpretation. To see at what point these become serious let us consider the case of three transmission lines of different impedances, $Z_{o}, Z_{1}$ and $Z_{2}$, connected in tandem. Two discontinuities having reflection coefficients $p_{1}=\left(\mathrm{Z}_{2}-\mathrm{Z}_{o}\right)$ ) $\left(Z_{1}+Z_{0}\right)$ and $\rho_{2}=\left(Z_{2}-Z_{1}\right) /\left(Z_{2}+Z_{1}\right)$ will be present and the series of reflections these cause is shown as the set of arrows emerging to the left in Figure 9.


Ignoring multiple reflections we would expect only two reflections of amplitudes $\rho_{1}$ and $p_{2}$. The first actually occurs but the second is diminished by the factor ( $1-\rho_{1}^{2}$ ). There follows a geometric series of reflections each weaker than the last by the factor $-\rho_{1} \rho_{2}$. From this we can conclude that, since each disturbing multiple reflection involves at least two more reflections than the primary echoes, we can neglect multiple reflections to the extent that $\rho_{\mathrm{i}} \rho_{\mathrm{j}} \ll 1$ for all $i, j$.

The successive reflections (and transmissions) in Figure 9 are of course delayed by multiples of $\tau$, the propagation time of the middle line. Before adding the coefficients in the frequency domain, each should be multiplied by e-nTp where n is the number of crossings of the middle line that has occurred. In the time domain each coefficient produces a distinct impulse or added step. Figure 10 shows the manner in which the successive reflections accumulate for several impedance combinations. The convergence to the final value is in an alternating geometric series if $\rho_{1}$ and $\rho_{2}$ have the same sign, and in a monotone series if they are of opposite sign.
From curves (a) and (f) we notice that, even with a matched generator, reflections can more than double or cancel the voltage or current on a line. In the cases shown the overshoot is $25 \%$ and is the greatest amount obtainable using a different impedance uniform line as a termination.

## IMPEDANCE PROFILES

If we read the height of the traces of Figure 10 on the "ohmmeter" scale of

(f)

| Curve | $\mathbf{Z}_{1} / \mathbf{Z}_{0}$ | $\mathbf{Z}_{1} / \mathbf{Z}$ | 1 | 2 |
| :---: | :---: | :---: | :---: | ---: |
| a | 3 | $\infty$ | $1 / 2$ | 1 |
| b | 2 | 4 | $1 / 3$ | $1 / 3$ |
| c | 2 | 1 | $1 / 3$ | $-1 / 3$ |
| d | $1 / 2$ | 1 | $-1 / 3$ | $1 / 3$ |
| e | $1 / 2$ | $1 / 4$ | $-1 / 3$ | $-1 / 3$ |
| f | $1 / 3$ | 0 | $-1 / 2$ | -1 |

Fig. 10. Incident wave and reflection pattern produced by multiple reflections shown in Figure 9.

Figure 4, then before the first reflection all traces coincide at the value $\mathbf{Z}_{0}$. After the first reflection all traces have the value $Z_{1}$, and all assume the final value $Z_{2}$. If we take distance along the x -axis to represent distance along the line, the scale factor being the ratio of sweep speed to $\mathbf{v} / 2$, then cases (b) through (e) may be interpreted as a reasonably accurate profile of impedance along the line. Under certain conditions this interpretation is quite accurate and useful.

In order for the impedance profile to be accurate, multiple reflections must be small. In the foregoing example the greatest error occurs immediately after the second reflection.

Using the ohmmeter scale to read traces (c) and (e) immediately after the second reflection yields the result $\mathrm{Z}_{2}=$ $4.4 \mathrm{Z}_{0}$ and $\mathrm{Z}_{\mathrm{o}} / 4.4$ instead of $4 \mathrm{Z}_{\mathrm{o}}$ and $\mathrm{Z}_{o} / 4$ : an error of only $10 \%$. For smaller total impedance change the error is less.

For the impedance profile to be valid, all enduring reflections must be produced by impedance changes in a physical structure. Discontinuities such as series resistance and capacitance, or shunt conductance and inductance produce permanent reflections that do not represent changes in the characteristic impedance of the line. Small shunt capacitive or series inductive discontinui-

$\frac{\mathrm{kvt}}{2}=k x \rightarrow$
Fig. 11. The actual reflection from an infinite exponentially tapered line (solid), and the line impedance profile plotted on the vertical scale $\mathrm{y}=\left(\mathrm{Z}-\mathrm{Z}_{0}\right)$ / ( $\mathrm{Z}+\mathrm{Z}_{0}$ ) (dashed curve). The error incurred from reading impedance off the solid curve is under $7 \%$ for $\mathrm{Z} / \mathrm{Z}_{0}<3$.
ties on the other hand produce short spike reflections before and after which the impedance profile may still be valid.

Often a time domain reflectometer is used to inspect cable quality or the reflections from some other supposedly smooth line. Here the reflections,
though possibly numerous, are normally all small and the impedance profile concept may be used with impunity. For small variations about $Z_{0}$ we find from (3) that

$$
\begin{equation*}
\Delta \mathbf{Z}=2 \mathrm{a} \mathbf{Z}_{\mathrm{o}} \tag{6}
\end{equation*}
$$

where $a$ is the amplitude of the departure from the incident unit step height.


Fig. 12. Reflection from an infinite lossy line. With no conductance the reflection eventually corresponds to an open circuit; with no resistance, a short circuit. The distortionless line with $R / L=G / C$ produces no reflection.


Fig. 13. Three feet of RG-9/U open circuited. Sweep speed 2.5 ns/division. The round trip delay is 3.8 divisions or 9.5 nanoseconds. The propagation velocity is thus 193 meters/microsecond or $64 \%$ of the velocity of light. The dielectric constant is therefore $(1 / .64)^{2}$ or 2.4.


Fig. 14. Double exposure showing the incident step (below) and the reflection from an open circuit at the end of 15 feet of $R G-9 / U$. The slower rise and rounding of the top caused by cable loss are clearly evident. $2.5 \mathrm{~ns} / \mathrm{cm}$ sweep speed.


Fig. 15. Fifteen inches of 100 ohm cable spliced into a 50 -ohm cable to give the case depicted in Fig. 10c. The slight multiple reflection can be seen following the main bump. Aside from this, the trace is an accurate impedance profile of the line.


Fig. 16. Reflection from a lumped inductance of 330 nanohenries in series with a 50 -ohm line. Sweep speed is $2.5 \mathrm{~ns} / \mathrm{cm}$. The time constant is $L / 2 Z_{o}$ or 3.3 nanoseconds. The finite rise time of the step prevents the voltage from quite doubling.

## TAPERED SECTIONS

When large impedance changes are necessary, and narrow band operation is permissible, quarter-wave matching sections are often used. (Figure 10 (b) and (e) show the profile that would be obtained for a matching section between impedances having a 4 to 1 ratio.) When wide-band operation is desired a section of line having a tapered impedance is commonly used. Such sections can be designed to produce very little reflection above some minimum frequency and thus be good high-pass impedance transformers.


Fig. 17. Reflection from 100 picofarads shunting a 50 -ohm line. Sweep speed 2.5 $n s / \mathrm{cm}$. Time constant is $C Z_{0} / 2$ or 2.5 nanoseconds. The finite rise time prevents the voltage from dropping quite to zero.


Fig. 18. A long section of lossy 60 -ohm line. The jump from 50 ohms to 60 ohms is evident at the start, followed by an irregular rise due to loss and cable defects. On the vertical scale shown, the incident step would be 35 cm high. Much more vertical magnification could be used without distortion.

If the impedance of an infinite tapered section varies exponentially with distance, $x$, the impedance at any point will be
$\mathrm{Z}=\mathrm{Z}_{0} \mathrm{e}^{\mathrm{kx}}\left\{\sqrt{1+\left(\frac{\mathrm{kv}}{2 \mathrm{p}}\right)^{2}} \pm\left(\frac{\mathrm{kv}}{2 \mathrm{p}}\right)\right\}(7)$ where the upper sign applies for waves travelling in the positive direction of x , the lower for waves travelling in the negative direction. The bracket has unity magnitude and assumes complex conjugate values for the two cases. Thus when joined at $\mathrm{x}=0$ to a uniform line of impedance $\mathbf{Z}_{\mathrm{o}}$, there is a mismatch in angle of impedance and

$$
\begin{equation*}
\rho=\frac{\frac{\mathrm{kv}}{2}}{\sqrt{\mathrm{p}^{2}+\left(\frac{\mathrm{kv}}{2}\right)^{2}}+\mathrm{p}} \tag{8}
\end{equation*}
$$

The inverse transform of this expression is the reflection produced by a unit impulse and is


Fig. 19. A 50 -ohm (unbalanced) to 200-ohm (balanced) balun followed by a section of twin lead tapering from 200 to 300 ohms. The balun produces only the minor reflections before the slope, but changes the reference impedance level to 200 ohms. The slope is produced by the tapered section. Small inductive and capacitive discontinuities produce the two pips in the 300 -ohm portion of the trace. The former was produced by stitting the twin lead and spreading the conductors, the latter by the top of a metal partition over which the lead was draped.

$$
\begin{equation*}
\mathrm{f}^{\prime}(\mathrm{t})=\frac{1}{\mathrm{t}} \mathrm{~J}_{1}\left(\frac{\mathrm{kv}}{2} \mathrm{t}\right) \tag{9}
\end{equation*}
$$

A step function therefore produces the integral of (9) or:

$$
\begin{equation*}
f(t)=\int_{0}^{k v t / 2} \frac{J_{1}(z)}{z} d z \tag{10}
\end{equation*}
$$

Figure 11 shows the actual reflection (solid curve) as well as the shape it would have to have (dashed curve) to be a true impedance profile. The error made by interpreting the reflection as impedance on the ohmmeter scale is shown as the dotted curve. For $\mathrm{kx}<1.4$, i.e., for impedance ratios less than $\mathrm{e}^{1.4}$ $\approx 4$, the error is less than $15 \%$.

The step function response, $\mathrm{Si}(\mathrm{x})$, of an ideal low pass filter naturally contains no frequencies above the cutoff. If the reflection from a tapered section had the shape of the sine-integral (or the step response of any completely low pass filter) this would imply no reflection (and hence complete transmission) above a critical frequency. Using a time domain reflectometer and a razor blade one could presumably empirically shape the impedance $\mathbf{Z}(x)$ of a strip line, say, to produce the desired reflection.

## LOSSY LINES

Distributed series resistance and shunt conductance, unless properly balanced, will cause the characteristic impedance to be complex and cause reflection at the junction with a distortionless line. For the lossy line we have

$$
\begin{aligned}
Z & =\sqrt{\frac{R+P L}{G+p C}}=\sqrt{\frac{L}{C}} \sqrt{\frac{P+R / L}{P+G / C}} \\
& =Z_{0} \quad \begin{array}{l}
P+\eta \\
P+\sigma
\end{array}
\end{aligned}
$$

When connected to a line of impedance $Z_{0}$, the reflection coefficient


Fig. 20. (a) A V-antenna with two conductors per side tapered to give a constant 300 ohm impedance for the first two feet after which the impedance rises. (b) The same antenna with end conductors and mesh added to make side surfaces rather than rods. The sides form a conical 300 -ohm transmission line.

$$
p=\frac{\sqrt{\mathrm{P}+\eta}-\sqrt{\mathrm{P}+\sigma}}{\sqrt{\mathrm{P}+\eta}+\sqrt{\mathrm{P}+\sigma}}
$$

results. Taking the Fourier transform we find for the reflection of a unit impulse
$\mathrm{f}^{\prime}(\mathrm{t})=\frac{1}{\mathrm{t}} \mathrm{e}^{-\frac{\eta+\sigma}{2} \mathrm{t}} \mathrm{I}_{1}\left(\frac{\eta-\sigma}{2} \mathrm{t}\right)$
Again, the step response is the integral of (11). For $\alpha=0$ we find
$\mathrm{f}(\mathrm{t})=1-\mathrm{e}^{\left.-\frac{\eta \mathrm{t}}{2}\left\{\mathrm{I}_{0}\left(\frac{\eta \mathrm{t}}{2}\right)+\mathrm{I}_{1}\left(\frac{\eta \mathrm{t}}{2}\right)\right\},{ }^{2}\right)}$
while for $\eta=0$
$f(t)=e^{-\frac{\sigma \mathrm{t}}{2}}\left\{1_{1}\left(\frac{\sigma \mathrm{t}}{2}\right)+1_{1}\left(\frac{\sigma \mathrm{t}}{2}\right)\right\}-1$ (13)
These two functions are shown in Fig. ure 12.

As time goes on, the line with only series resistance ( $\sigma=0$ ) looks more and more like an open circuit while the line with only shunt conductance ( $\eta=0$ ) looks more and more like a short circuit.

Normally one encounters lengths and losses such that only the initial part of these curves is seen. The initial slope is found from (II)

$$
\begin{equation*}
I^{\prime}(0)=\frac{\eta-\sigma}{4}=\frac{1}{4}\left(\frac{\mathrm{R}}{\mathrm{~L}}-\frac{\mathrm{G}}{\mathrm{C}}\right) \tag{14}
\end{equation*}
$$

After a time $t$, this slope will produce a departure $a=\mathrm{tf}^{\prime}(0)$ from the height of the incident unit step. If $\mathrm{G}=0$ we can interpret this departure in terms of accumulated series resistance. Thus from (6)

$$
\begin{equation*}
\Delta Z=2 \text { a } Z_{0}=\frac{1}{2} \frac{\mathrm{R}}{\sqrt{\mathrm{LC}}} \mathrm{t}=\mathrm{R} \frac{\mathrm{vt}}{2} \tag{15}
\end{equation*}
$$

$R$ is the resistance per unit length and $\mathrm{vt} / 2$ is the distance the incident wave had travelled for the time displayed. A similar result can be derived for $G$ if $\mathrm{R}=0$, but no such simple picture exists if neither $R$ nor $G$ is negligible.

How well an actual physical system can display the theoretical responses discussed above may be seen from the photographs reproduced as Figures 13 to 21.

## SOME EXPERIMENTAL TECHNIQUES

Using a time domain reflectometer one soon develops techniques for improving the speed and accuracy of measurements. Reflections are quickly located by touching or squeezing along the line as mentioned earlier. If no discontinuity is evident at or near the spot located, the observed echo may be a multiple reflection. This can often be checked by altering a few line lengths and noting if the echo in question moves with respect to the others.

When looking at reflections far down a system or after many earlier reflections, one can always check the effective incident step size and rise time by shorting or opening the line at the point in question to produce unity reflection. The resulting step should be used as the reference rather than the incident step first seen by the scope.

Highly accurate evaluation of shunt irregularities may be made by duplicating the observed reflection at some nearby point using known components.


Fig. 21. (a) The reflection from the antenna of Fig. 20(a). The tapering impedance causes a $20 \%$ overshoot (as in Figs. 10 a and 11 ) and there is a rapid initial rise caused by too little fringing capacity at the open end.
(b) The reflection from the modified antenna of Fig. 20 (b). No reflection occurs at the throat; in fact none occurs until the wave reaches the open end. The increased end capacity due to the end rods and the

Series irregularities may be evaluated by cancelling the reflections they produce using known components in shunt at the same point. Thus a small shunt capacitance will cancel the reflection of a small series inductance; a small shunt conductance will cancel a small series resistance, etc.

Such techniques enable one largely to dispense with quantitative measurements of the trace and deal directly in terms of the test circuit elements and locations. The oscilloscope becomes, in a sense, a qualitative indicator telling where to look for trouble and what kind it is. The remedy is then determined by a quick succession of trials, guided by the display.

In narrow-band resonant structures TDR is of little use because the reflections are apt to be overlapping oscillatory transients of long duration. Here the time domain picture is complex and the frequency domain picture is usually simpler. But in the wide-band structures which comprise the majority of cases today, TDR is incomparably faster and simpler than conventional SWR or reflectometer methods.

## ACKNOWLEDGMENT

The author wishes to acknowledge the contributions of Harley L. Halverson of the -hp-laboratories who was quick to appreciate the potentialities of time domain reflectometry and who has done much with an earlier paper ${ }^{3}$ to popularize the technique.
-B. M. Oliver
"Harley Halverson, "Time Domain Reflectometry," Cleveland Electronics Conference, April 16, 1963.

(b)
constant characteristic impedance produce a smooth reflection with very little overshoot. The reflection may be approximated by an exponential rise of 16 -nanosecond time constant. Thus the return loss is 3 db at 10 Mc and falls at $6 \mathrm{db} /$ octave above this frequency to become 14 db at 50 Mc , 20 db at 100 Mc , and 26 db at 200 Mc . Sueep speed is $5 \mathrm{~ns} / \mathrm{cm}$; incident step height is 5 cm .

## TIME DOMAIN REFLECTOMETRY WITH A PLUG-IN FOR THE 140A OSCILLOSCOPE

TTime domain reflectometry measurements are easily made with the $-h p-$ Model 140A general-purpose Oscilloscope and a plug-in designed specifically for such measurements. To make a time domain reflectometry measurement with this plug-in, -hp- Model 1415A, it is only necessary to connect the system to be tested to a front panel coaxial connector and to set the controls for the desired time base and amplitude range.

As shown in the block diagram, the voltage step from the internal generator in the plug-in passes through the sampling channel to the external system under test. Reflections from any impedance mismatches in the external system travel back through the sampling gate and are displayed on the cathode-ray tube in the oscilloscope main frame. The reflections subsequently are absorbed by the matched source impedance of the step generator.

Sweep magnifier controls enable any portion of the train of reflections to be


Voltage step generator in -hp-1415A Plugin connects directly to feed-through sampling channel. Plug-in also has rate generator and necessary synchronizing functions.


Self-contained -hp-1415A Time Domain Reflectometer Plug-in for -hp-140A Oscilloscope makes detailed measurements on transmission systems up to 1000 feet in length (incident step top is flat within $1 \%$ for 2000 nanosecs).
expanded for detailed examination. The controls are calibrated to indicate the distance to the section of line under examination, simplifying identification of the physical location of observed discontinuities. The wide dynamic range of the sampling channel allows expansion in the vertical dimension so that reflections that are even as small as $0.1 \%$ of the incident step may be examined in detail.

Overall system rise time is less than 150 picoseconds, enabling resolution of discontinuities that are spaced about 1 cm apart in coaxial systems.

The plug-in also provides outputs that permit replicas of the displayed waveform to be recorded in permanent form on an external X-Y recorder. For this application, the horizontal sweep is generated manually with a front panel control. Manual control of the sweep also enables the amplitude of any part of the observed waveform to be measured precisely with a digital voltmeter connected to the Y -axis recorder output.

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TENTATIVE SPECIFICATIONS
                        -hp-
        MODEL 1415A
TIME DOMAIN REFLECTOMETER
                        PLUG-IN
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SYSTEM (in reflectometer configuration): RISE TIME: Less than 150 ps. OVERSHOOT. $<5 \%$ (down to $1 / 2 \%$ in 1 ns ).
REP RATE: 200 kc . REP RATE: 200 kc
SIGNAL CHANNEL:
RISE TIME: Approximately 90 ps .
SENSITIVITY.
SENSITIVITY: $1 \mathrm{mv} / \mathrm{cm}$ to $0.1 \mathrm{v} / \mathrm{cm}$. NOISE: Less than 0.2 mv peak-to-peak
DYNAMIC RANGE: $\pm 1.0 \mathrm{v}$.

## TDR WITH -hpSAMPLING SCOPES

The -hp-Model 188A Plug-in for the 185A/B Sampling Oscilloscopes realizes the highest resolution yet attained in a TDR system. When the incident step is supplied by the -hp-Model 213B tun-nel-diode Pulse Generator, overall system rise times approaching 120 picoseconds are achieved with the 188A.

Well-defined time domain reflectometry measurements also are made with the 187A/B plug-ins and the 185A/B scopes. The high impedance probes of these plug-ins are well-fitted for bridg. ing 50 -ohm lines (see Fig. 2, page 1). System rise time with the $185 \mathrm{~B} / 187 \mathrm{~B}$ is less than $1 / 2$ nsec when the 213B Pulse Generator supplies the incident step, or 2 nsec if the 185B sync pulse serves by itself as the step.


Fast-rise feed-through sampling channels of -hp-188A dual-channel plug-in for-hp185A / B Sampling Oscilloscopes bridge 50ohm lines without attenuation.


TDR measurements are easily made with $-h p-188$ A plug-in since plug-in allows incident step and returning reflections to pass through in opposite directions.


[^0]:    'B. M. Oliver, "Square Wave and Pulse Testing of Linear Systems," Hewlett-Packard Journal, Vol. 7, No. 3, November, 1955
    z "Table of Important Transforms," Hewlett-Packard Journal, Vol. 5, No. 3-4, Nov-Dec., 1953. (Also re-

