INTRODUCTION TO DISTORTION PARAMETERS IN LINEAR NETWORKS

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OBJECTIVES

"EXPLAIN"

HOW A NETWORK MODIFIES A SIGNAL

WHAT PARAMETERS GENERATE DISTORTION

HOW TO MINIMIZE THEIR EFFECT

FIGURE OF MERIT OF A SYSTEM

FIDELITY WITH WHICH A SIGNAL OR INFORMATION IS TRANSMITTED OR REPRODUCED.

DEVIATIONS FROM PERFECT TRANSMISSION:

- HARMONIC DISTORTION
- CROSSTALK, INTERFERENCE
- ERROR RATE, S/N-DEGRADATION

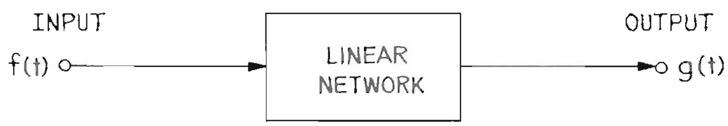
DISTORTION IS DUE TO ONE OR MORE OF THE BASIC NETWORK CHARACTERISTICS

1	2	3
LEVEL DEPENDING	TIME DEPENDING	FREQUENCY DEPENDING
NON LINEAR TIME VARIANT		LINEAR TIME INVARIANT

EFFECTS:

NON LINEAR TIME VARIANT	LINEAR TIME INVARIANT
NEW SIGNAL COMPONENTS ARE GENERATED	SIGNAL COMPONENTS ARE MODIFIED ?

LINEAR NETWORKS



IF	LINEAR	THEN
$\left.\begin{array}{l} f_1(t) \rightarrow g_1(t) \\ f_2(t) \rightarrow g_2(t) \end{array}\right\}$	$a_1 f_1(t) + a_2 f_2(t)$	$\rightarrow a_1 g_1(t) + a_2 g_2(t)$

SUPERPOSITION APPLIES

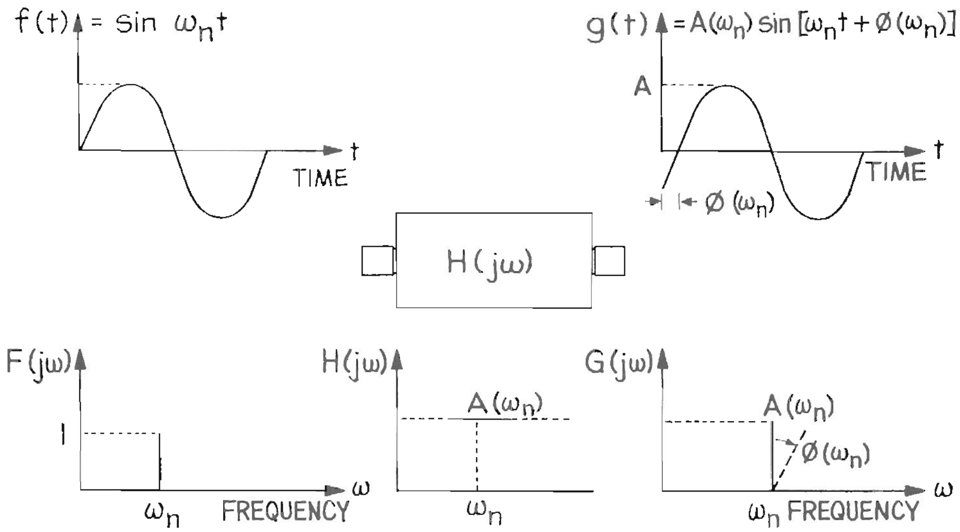
FOR A SINUSOIDAL INPUT FUNCTION $f(t) = e^{-j\omega t}$ THE STEADY STATE OUTPUT FUNCTION $g(t)$ IS GIVEN BY THE TRANSFER FUNCTION $H(j\omega)$ MULTIPLIED BY THE INPUT FUNCTION $e^{-j\omega t}$:

$$g(t) = H(j\omega) \cdot e^{-j\omega t} = \underbrace{A(\omega) \cdot e}_{\substack{\text{AMPLITUDE} \\ \text{CHARACTERISTIC}}}{}^{\underbrace{-j\emptyset(\omega)}_{\substack{\text{PHASE} \\ \text{CHARACTERISTIC}}}} \cdot e^{-j\omega t} = A(\omega)\, e^{-j[\omega t + \emptyset(\omega)]}$$

$$G(j\omega) = H(j\omega) \cdot F(j\omega)$$

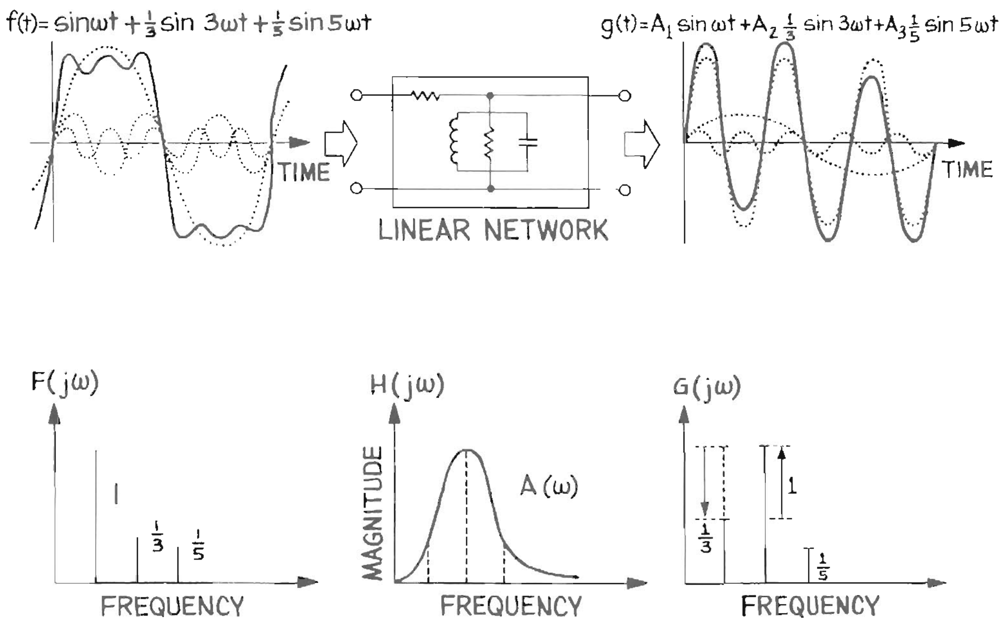
$$g(t) = h(t) * f(t)$$

LINEAR NETWORKS

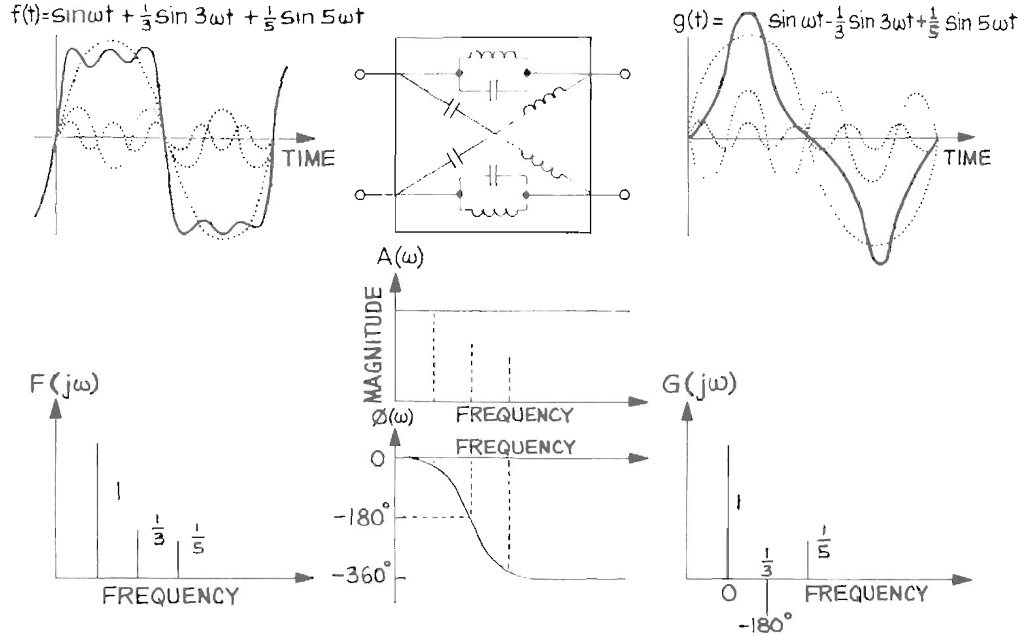


DOES A LINEAR NETWORK IMPLY NO DISTORTION?
YES FOR CW SIGNALS, HOWEVER...

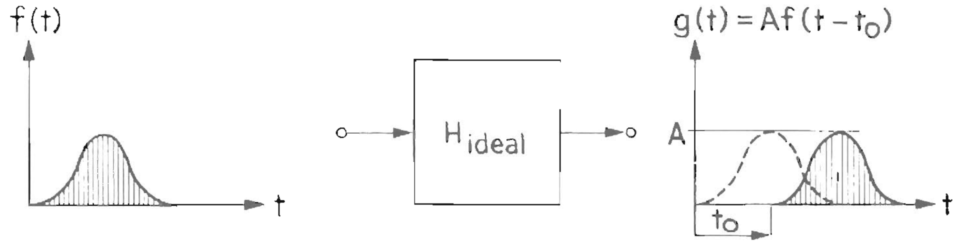
MAGNITUDE VARIATIONS WITH FREQUENCY



PHASE VARIATIONS WITH FREQUENCY



DISTORTIONLESS TRANSMISSION



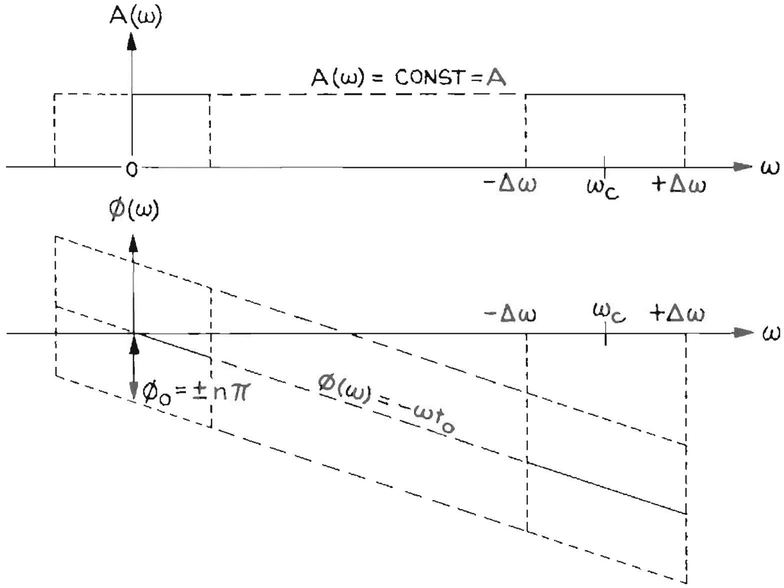
$$H(j\omega)_{ideal} = \frac{G_i(j\omega)}{F(j\omega)} = \frac{\mathfrak{F}g_i(t)}{\mathfrak{F}f(t)} = \frac{\frac{1}{2\pi}\int_{-\infty}^{\infty} g_i(t)e^{-j\omega t}dt}{\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt}$$

$$= \frac{\frac{A}{2\pi}\int_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t}dt}{\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt} = \frac{A\cdot F(j\omega)\cdot e^{-j\omega t_o}}{F(j\omega)}$$

shift theorem

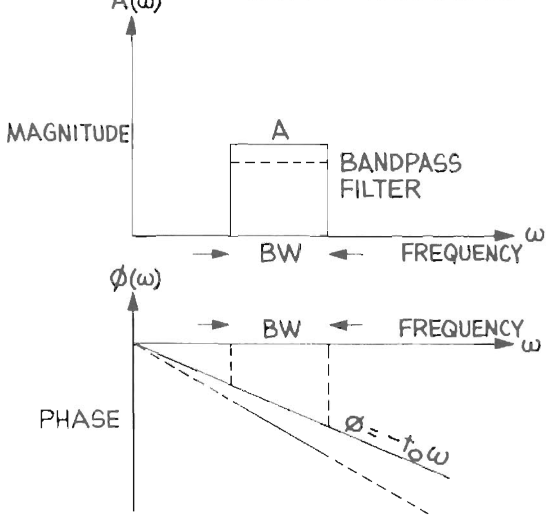
$$H(j\omega)_{ideal} = A(\omega)e^{-j\phi(\omega)} = \underbrace{A\cdot e^{-j\omega t_o}}_{\text{ideal characteristic}}$$

IDEAL TRANSFER CHARACTERISTIC



CRITERIA FOR DISTORTIONLESS TRANSMISSION OVER LIMITED BANDWIDTH

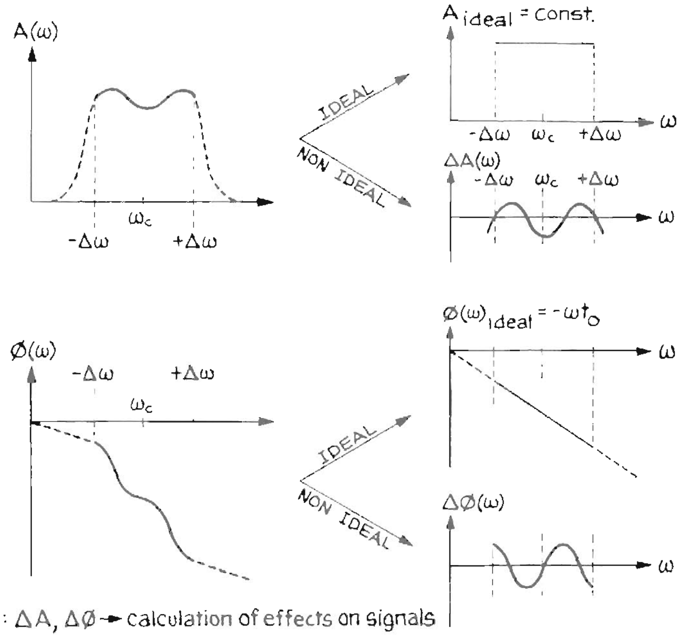
NETWORK TRANSFER FUNCTIONS



1. CONSTANT AMPLITUDE OVER BANDWIDTH OF INTEREST $A(\omega) = A =$ CONSTANT

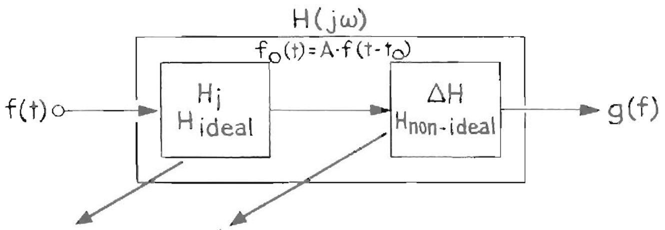
2. LINEAR PHASE OVER BANDWIDTH OF INTEREST $\phi(\omega) = -\omega t_o =$ LINEAR WITH ω $t_o =$ PROPAGATION DELAY OF AN IDEAL NETWORK

NON IDEAL TRANSFER CHARACTERISTIC



Cause : ΔA, ΔØ → calculation of effects on signals

IDEAL AND NON-IDEAL PART OF A NETWORK



$$H(j\omega) = \overbrace{H_i(j\omega)} \cdot \overbrace{H_{ni}(j\omega)}$$

$$= \pm A \cdot e^{-j\omega t_o} \cdot (1+\Delta A(\omega))\underbrace{e^{-j\Delta\phi(\omega)}}$$

$$[1+\Delta A(\omega)] \cdot \left[1 - j\Delta\phi(\omega) + \tfrac{1}{2}\Delta\phi^2(\omega) + \cdots\right]$$

$$= \underbrace{\pm A \cdot e^{-j\omega t_o}} \cdot \underbrace{\left[\mathbf{1} + \mathbf{\Delta A(\omega)} - j\mathbf{\Delta\phi(\omega)} + \ldots \text{higher order terms}\right]}$$

$$H(i\omega) = H_i(j\omega) \quad \cdot \quad H_{ni}(j\omega) \text{ or } \Delta H(j\omega)$$

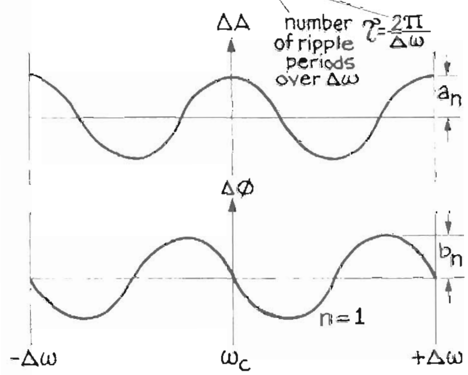
APPROXIMATIONS FOR $\Delta A(\omega)$ & $\Delta\phi(\omega)$:

① FOURIER EXPANSION:

$$\Delta A(\omega) = \sum_{n=0}^{\infty} a_n \cdot \cos n\tau\omega$$

$$\Delta\phi(\omega) = \sum_{n=0}^{\infty} b_n \cdot \sin n\tau\omega$$

number of ripple periods over $\Delta\omega$ $\qquad \tau = \frac{2\pi}{\Delta\omega}$



$$\boxed{g(t) = \mathcal{F}^{-1} F_o(j\omega)\cdot[1+\Delta A - j\Delta\phi + \ldots]}$$

CALCULATE EFFECT OF ΔA, $\Delta\phi$ ON SIGNAL $f(t) \rightarrow g(t)$

EXAMPLE:

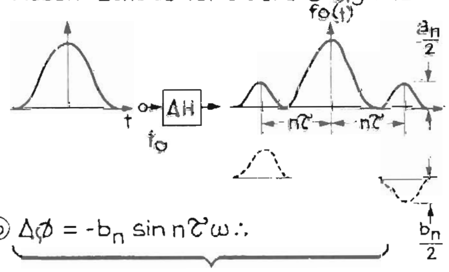
ⓐ $\Delta A = a_n \cos n\tau\omega$ for $-\Delta\omega < \omega < +\Delta\omega$

$$g(t) = \mathcal{F}^{-1} F_o(j\omega)\left[1 + \frac{a_n}{2} e^{jn\tau\omega} + \frac{a_n}{2} e^{-jn\tau\omega}\right]$$

$\therefore$ Shift Theorem: $\underset{\omega}{x n\tau|} \leftrightarrow \underset{t}{\pm n\tau|}$

$$g(t) = f_o(t) + \frac{a_n}{2} f_o(t-n\tau) + \frac{a_n}{2} f_o(t+n\tau)$$

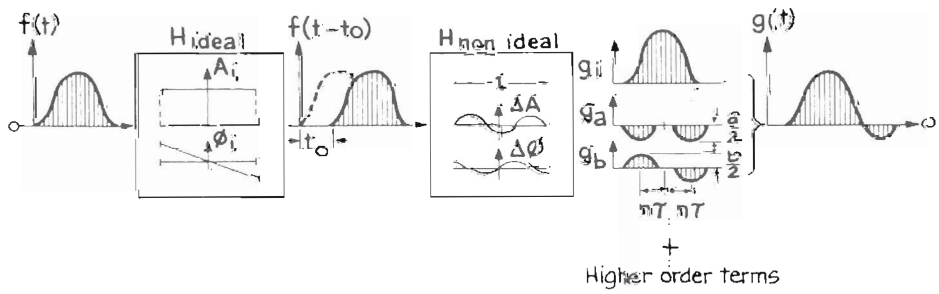
Result: Echoes for discrete signals



ⓑ $\Delta\phi = -b_n \sin n\tau\omega \therefore$

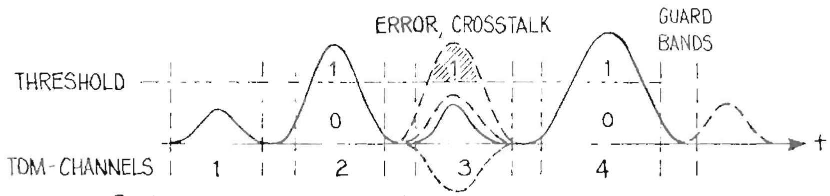
Discrete Signals

SIGNAL DISPERSION DUE TO A NON IDEAL AMPLITUDE AND PHASE CHARACTERISTIC



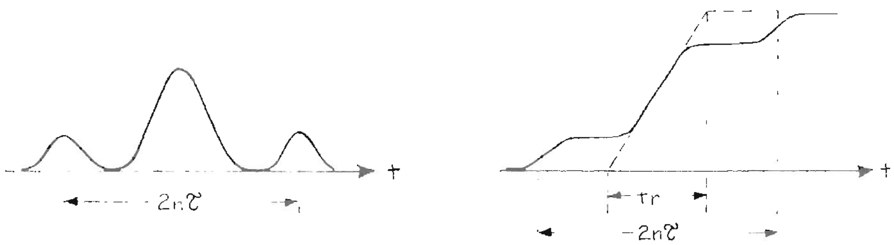
EFFECTS OF ECHOS ON DISCREET SIGNALS

CROSSTALK



@: 1 dB ripple generates echoes only ~20 dB down

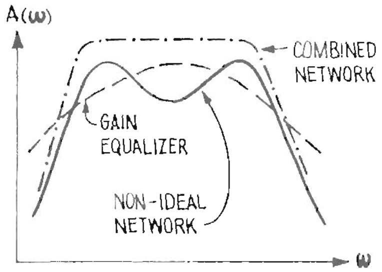
RISE TIME DEGRADATION

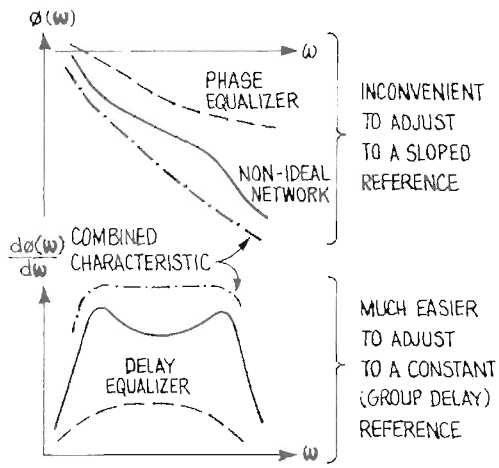


HOW CAN DISTORTION BE MINIMIZED?

1. MEASURE ΔA, $\Delta\phi$, (Γ^*)
2. EQUALIZE/MINIMIZE ΔH

EXAMPLE: GAIN (AMPLITUDE) AND PHASE EQUALIZING





* MISMATCH AND MULTI PATH TRANSMISSION

TWO ASPECTS OF GROUP DELAY

(1) SIGNAL PROPAGATION

$$t_g = -\frac{d\phi}{d\omega}$$

THE GROUP DELAY $-\frac{d\phi}{d\omega}$ DETERMINES THE PROPAGATION DELAY OF SIGNAL ENERGY OR INFORMATION (ENVELOPE DELAY).

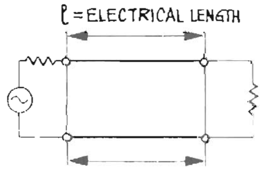
THE PHASE DELAY $-\frac{\phi}{\omega}$ DETERMINES THE STEADY STATE PHASE RELATIONSHIP BETWEEN INPUT AND OUTPUT (CARRIER DELAY) FOR AN IDEAL NETWORK $t_g = t_p$.

(2) SIGNAL DISTORTION

$$\frac{d\phi(\omega)}{d\omega} = \underbrace{t_0}_{\substack{\text{constant:} \\ \text{ideal}}} + \underbrace{\frac{d[\Delta\phi(\omega)]}{d\omega}}_{\substack{\text{deviation:} \rightarrow \Delta\phi \\ \text{non ideal}}}$$

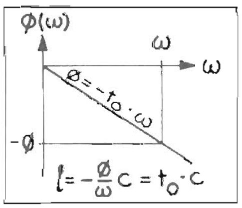
THE DERIVATIVE OF THE PHASE CHARACTERISTIC CAN BE USED TO SPECIFY AND MEASURE THE DEVIATION FROM THE IDEAL (LINEAR) PHASE CHARACTERISTIC.
IT IS A PHASE LINEARITY MEASUREMENT. ONLY THE DEVIATION FROM THE CONSTANT (GROUP DELAY) IS IMPORTANT IN THIS CASE.

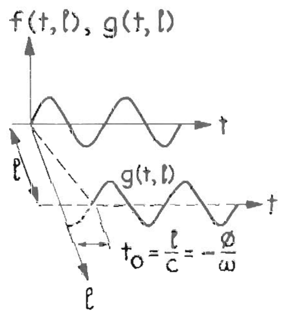
PROPAGATION DELAY AND ELECTRICAL LENGTH FOR AN IDEAL PHASE CHARACTERISTIC



$$t_0 = \frac{\ell}{c} = \frac{\ell_p}{c}\sqrt{\mu_r \cdot \varepsilon_r}$$

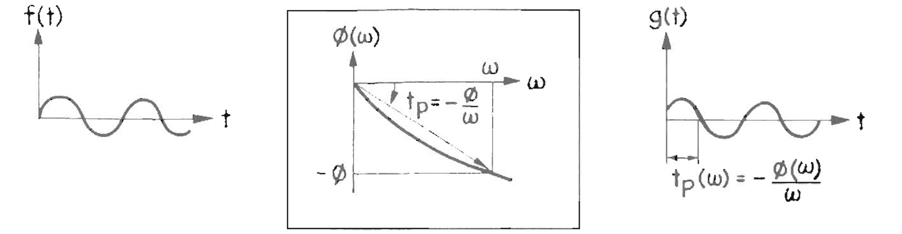
$\sim 3 \cdot 10^8 \left[\frac{m}{s}\right]$ in vacuum





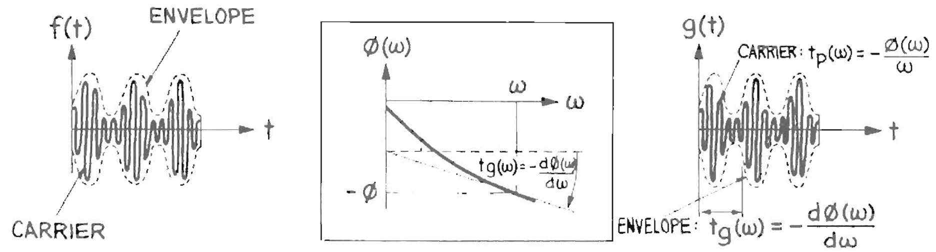
$$\underbrace{t_0}_{\substack{\text{PROPAGATION} \\ \text{DELAY}}} = \frac{\ell}{c} = \underbrace{-\frac{\phi}{\omega}}_{\substack{\text{PHASE} \\ \text{DELAY}}} = \underbrace{-\frac{d\phi}{d\omega}}_{\substack{\text{GROUP} \\ \text{DELAY}}}$$

PHASE DELAY AND GROUP DELAY



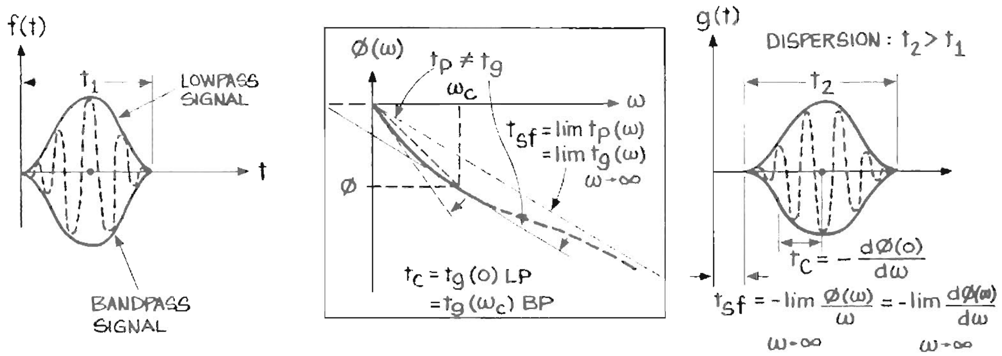
ENVELOPE CARRIER

$$g(t) = e(t - t_g)\ \cos\left[\omega_c (t - t_p)\right]$$



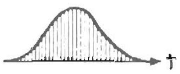
PROPAGATION DELAY FOR A NON IDEAL PHASE CHARACTERISTIC

$$\frac{ø(\omega)}{\omega} \neq \frac{dø(\omega)}{d\omega}$$



DISTORTION EFFECTS ON MODULATED SIGNALS:

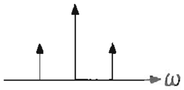
1. TIME DOMAIN MODULATION:
   (PULSE MODULATION)
   INFORMATION IN SIGNAL
   AMPLITUDE AND TIME:

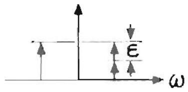


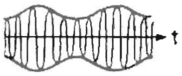
$$f(t) = \begin{cases} 1: & -\Delta t < t < \Delta t \\ 0: & t > |\Delta t| \end{cases} \rightarrow$$

DISCRETE SIGNALS:
FOURIER APPROACH FOR ΔH
ECHO DISTORTION

2. FREQUENCY DOMAIN MODULATION:
   (CARRIER MODULATION)
   INFORMATION IN CARRIER AMPLITUDE/ANGLE
   (ENVELOPE)







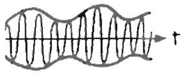
ENVELOPE CARRIER

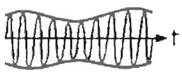
$$f(t) = [1 + \alpha(t)] \cdot e^{-j\omega_c t} \quad \text{AM}$$

$$f(t) = e^{-j[\omega_c t + \varphi(t)]} \quad \text{PM/FM}$$

CARRIER ENVELOPE

CONTINUOUS SIGNALS:
POWER SERIES APPROACH
ENVELOPE DISTORTION

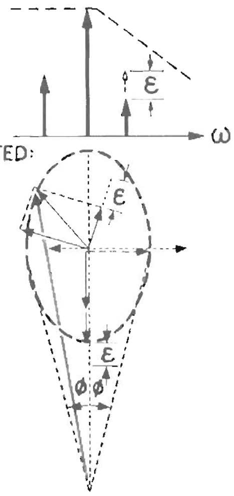


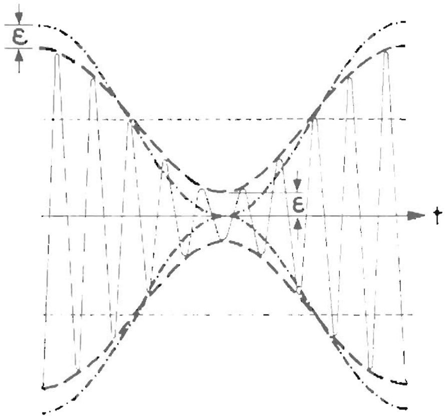


SIGNAL DISTORTION DUE TO LINEAR BUT FREQUENCY DEPENDING TRANSFER CHARACTERISTIC

AM DISTORTION

USB ATTENUATED:
e.g. L.P.F.





EFFECT: CHANGE OF INDEX MODULATION (m)

ODD HARMONIC DISTORTION OF ENVELOPE FOR SMALL INDEX m.

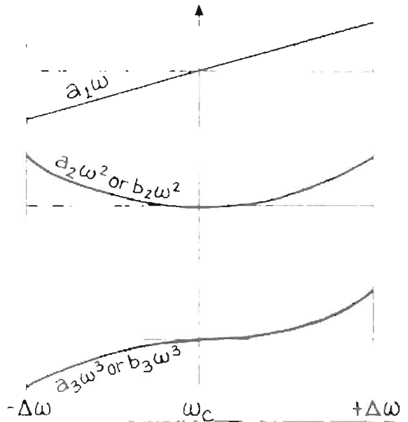
AM TO PM CONVERSION

APPROXIMATIONS FOR ΔA(ω) & ΔØ(ω)

(2) POWER SERIES EXPANSION

$$\Delta A(\omega) = a_1\omega + a_2\omega^2 + a_3\omega^3 + \dots$$

$$\Delta Ø(\omega) = b_o + b_2\omega^2 + b_3\omega^3 + \dots$$



$$g(t) = \mathfrak{F}^{-1}F_o(j\omega)\cdot[1 + \Delta A - j\Delta Ø + \dots]$$

CALCULATE EFFECT OF ΔA, ΔØ ON g(t)

EXAMPLE:

$$\Delta A = a_1\omega + a_2\omega^2$$

$$g(t) = \mathfrak{F}^{-1}F_o(j\omega)\cdot[1 + a_1\omega + a_2\omega^2]$$

∴ DIFFERENTIATION THEOREM

$$a_1\cdot\omega \rightarrow a\cdot\dot{f}(t)$$

$$a_2\omega^2 \rightarrow a_2\ddot{f}(t)$$

$$g(t) = f_o(t) + a_1\dot{f}_o(t) + a_2\ddot{f}_o(t)$$

RESULT: TABLE FOR BASIC SIGNALS:

$$\underbrace{\begin{aligned} AM &\rightarrow f(t) = R_e\left[[1+\alpha(t)]e^{-j\omega_c t}\right] \\ \varphi M &\rightarrow f(t) = R_e\left[e^{-j[\omega_c t + \varphi(t)]}\right] \end{aligned}}_{\text{MODULATED SIGNALS}}$$

DISTORTION ON AM-SIGNALS

$$f(t) = R_e[1+d(t)]e^{-j\omega_c t} \circ\!\!\rightarrow \boxed{\Delta H} \rightarrow\!\!\circ\; g(t) = R_e\sqrt{[1+\alpha(t)+P(t)]^2 + Q(t)^2}\cdot e^{-j\left[\omega_c t + \mathrm{arctg}\left[\frac{Q(t)}{1+\alpha(t)+P(t)}\right]\right]}$$

$$\Delta A(\omega) = a_1\omega + a_2\omega^2 + a_3\omega^3$$

$$\Delta A(\omega) = b_2\omega^2 + b_3\omega^3$$

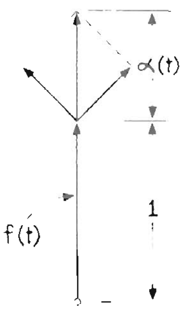
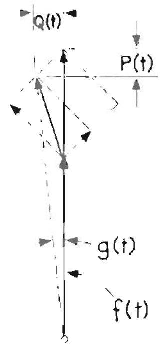


TABLE 1

α(t) PURE AM φ = CONST.	P_α	Q_α
a_1		$a_1\dot{\alpha}$
a_2	$-a_2\ddot{\alpha}$	
a_3		$a_3\dddot{\alpha}$
b_2		$-b_2\ddot{\alpha}$
b_3	$b_3\dddot{\alpha}$	
a_1b_2	$a_1b_2\dddot{\alpha}$	



EXAMPLE:

$\alpha(t) = m\cos\omega_m t$

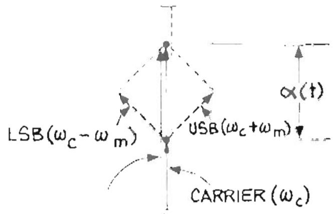
$\dot{\alpha} = -m\cdot\omega_m \sin\omega_m t$ Quadrature Phase

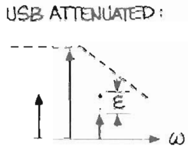
$\ddot{\alpha} = -m\,\omega_m^2\cos\omega_m t$ In Phase

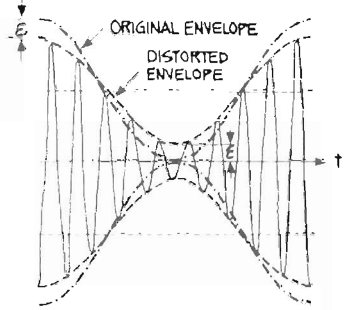
$\dddot{\alpha} = m\omega^3_m \sin\omega_m t$ Quadrature Phase

DISTORTION ON AM-SIGNALS:

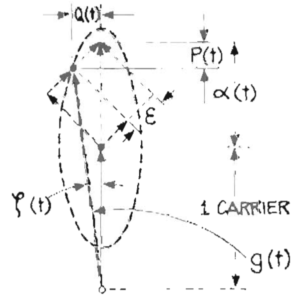
① EXAMPLE: USB ATTENUATED BY ε







$$f(t) = [1+\alpha(t)]\, e^{-j\omega_c t}$$



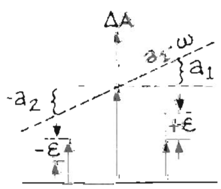
EFFECT: CHANGE OF INDEX MODULATION (m)
HARMONIC DISTORTION OF ENVELOPE
@: ε ≅ −6dB → d_2 ≅ 3%
AM TO PM CONVERSION

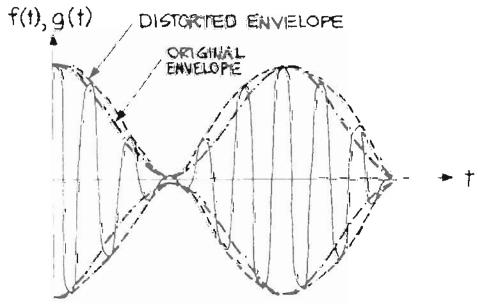
$$g(t) = \underbrace{\left[[1+\alpha(t)+P(t)]^2+Q(t)^2\right]^{\frac{1}{2}}}_{\text{MODIFIED ENVELOPE}} e^{-j\left[\omega_c t - \overbrace{\operatorname{arctg}\frac{Q(t)}{1+\alpha(t)+P(t)}}^{\varphi(t)}\right]}$$

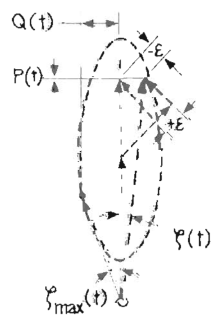
MODIFIED CARRIER PHASE (AM/PM)

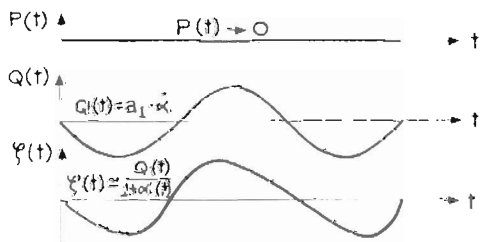
DISTORTION ON AM-SIGNALS:

② EXAMPLE: $\Delta A = a_1 \cdot \omega$



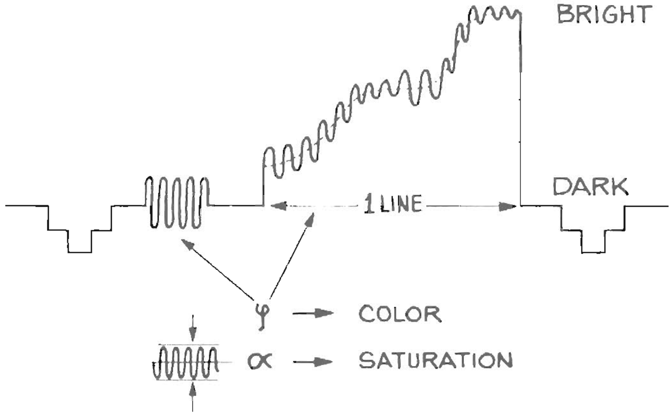






EXAMPLE FOR AM TO PM CONVERSION

COLOR TV SIGNAL:



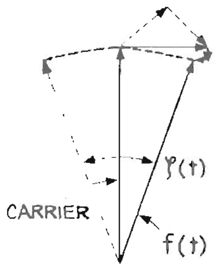
PROBLEM: COLOR CHANGE AS A FUNCTION OF BRIGHTNESS

DISTORTION ON ANGULAR MODULATION

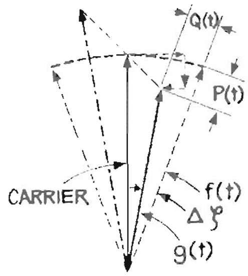
$$f(t) = R_e\left[e^{-j[\omega_c t + \varphi(t)]}\right] \circ\!\!\rightarrow \boxed{\Delta H} \rightarrow\!\!\circ\; g(t) = R_e\sqrt{[1+P(t)]^2 + Q(t)^2}\cdot e^{-j\left[\omega_c t + \varphi(t) + \operatorname{arctg}\left[\frac{Q}{1+P}\right]\right]}$$

$$\Delta A(\omega) = a_1\omega + a_2\omega^2 + a_3\omega^3$$

$$\Delta\varnothing(\omega) = b_2\omega^2 + b_3\omega^3$$



$\varphi(t)$ PURE φM α = CONST. = 1	P_φ	Q_φ
a_1	$a_1\varphi$	
a_2	$a_2\dot{\varphi}^2$	$-a_2\ddot{\varphi}$
a_3	$a_3(\dot{\varphi}^3 - \dddot{\varphi})$	$-3a_3\dot{\varphi}\ddot{\varphi}$
b_2	$b_2\ddot{\varphi}$	$b_2\dot{\varphi}^2$
b_3	$3b_3\dot{\varphi}\ddot{\varphi}$	$b_3(\dot{\varphi}^3 - \dddot{\varphi})$
$a_1 b_2$	$3a_1b_2\dot{\varphi}\ddot{\varphi}$	$a_1b_2(\dot{\varphi}^3 - \dddot{\varphi})$



EXAMPLE:

$\varphi(t) = m\cos\omega_m t$

$\dot{\varphi} = -m\,\omega_m \sin\omega_m t \qquad \dot{\varphi}^2 = \frac{m^2\omega_m^2}{2}\left[-\cos 2\omega_m t + 1\right] \qquad \dot{\varphi}^3 = \frac{m^3\omega_m^3}{4}\left[\sin 3\omega_m t - 3\sin\omega_m t\right]$

$\ddot{\varphi} = -m\,\omega_m^2 \cos\omega_m t$

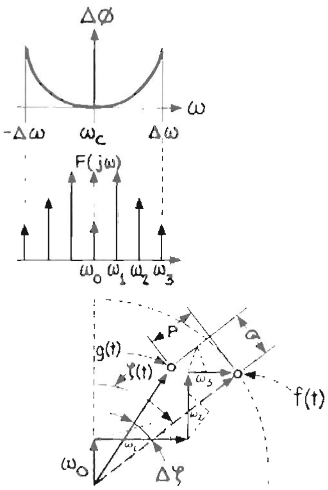
$\dddot{\varphi} = m\,\omega_m^3 \sin\omega_m t$

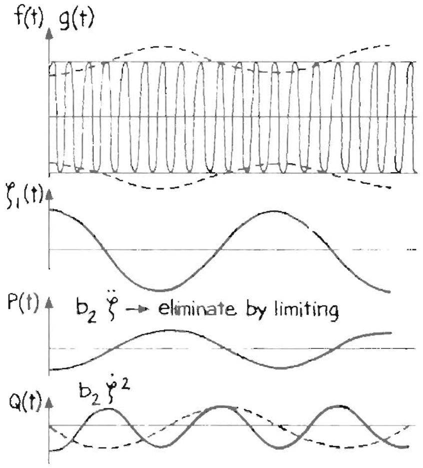
DISTORTION ON ANGULAR MODULATION

① EXAMPLE: PM – SIGNAL

$\Delta\varnothing = b_2\omega^2$

$\Delta A = 0$



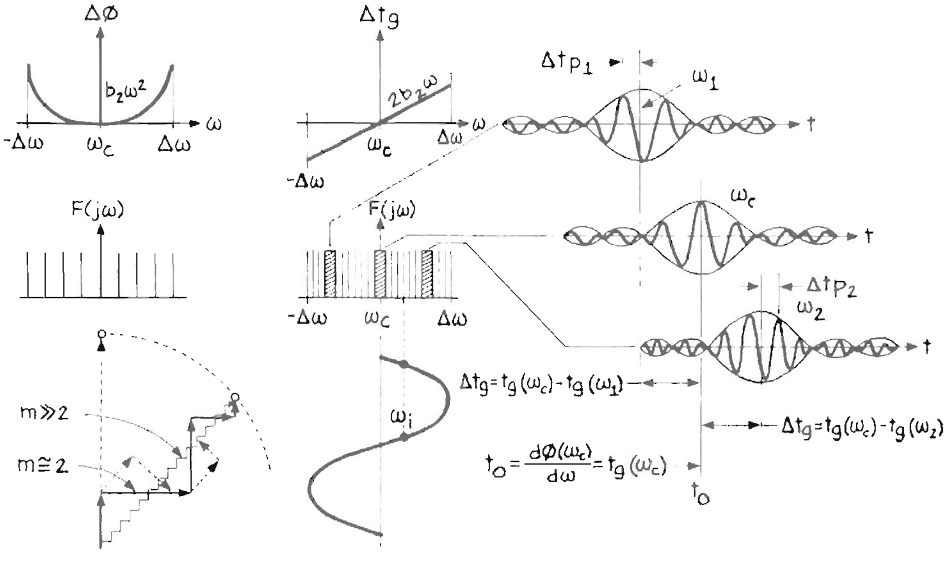


HARMONIC DISTORTION:

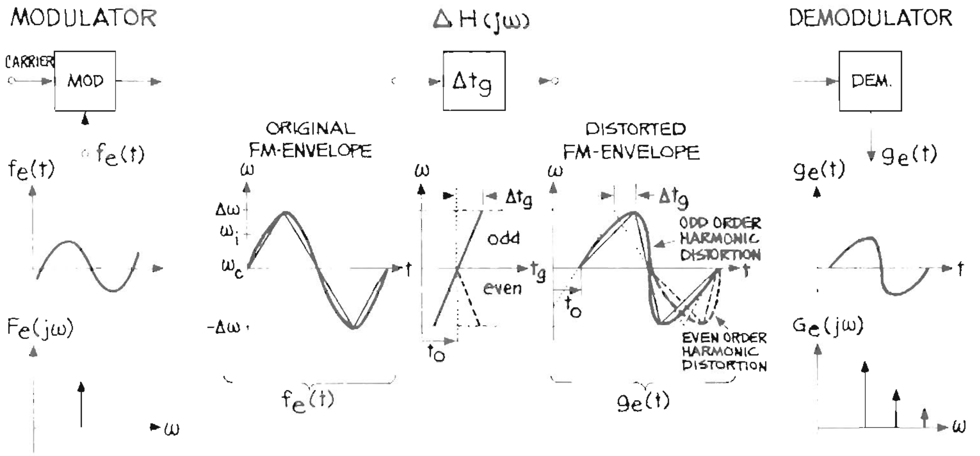
ⓐ: $\omega_m = 2\pi \cdot 10\,\text{kHz} \rightarrow b_2 = 0.5\,\text{deg} \rightarrow d_2 \cong 1\%$

DISTORTION ON ANGULAR MODULATION

② EXAMPLE: FM – SIGNAL, $\Delta\varnothing = b_2\omega^2$, $\Delta A = 0$



DISTORTION ON ANGULAR MODULATION



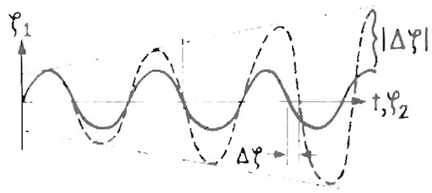
EXAMPLE: HARMONIC DISTORTION

$$\left.\begin{array}{l} @: \Delta t_g \approx 300\text{ ns} \\ \omega_m \cong 2\pi \cdot 10\text{ kHz} \end{array}\right\} d_2 \cong 1\% \longrightarrow \text{SECOND HARMONIC DISTORTION}$$

WHAT HAPPENS WHEN MORE THAN ONE SIGNAL IS PRESENT:

$$f(t) = e^{-j\left[\omega_c t + \varphi_1(t) + \varphi_2(t)\right]} \longrightarrow g(t) = \sqrt{(1+P)^2 + Q^2} \cdot e^{-\ldots \operatorname{arctg}\frac{Q}{1+P}}$$

TABLE	P_φ	Q_φ
a_3	$a_3[\dot{\varphi}^3 - \ddot{\varphi}]$	$-3a_3\dot{\varphi}\ddot{\varphi}$



$$\dot{\varphi}\ddot{\varphi} = (\dot{\varphi}_1 + \dot{\varphi}_2)(\ddot{\varphi}_1 + \ddot{\varphi}_2) = \dot{\varphi}_1\ddot{\varphi}_2 + \dot{\varphi}_2\ddot{\varphi}_1 + \ldots$$

$$\text{INTERMODULATION} \left.\begin{array}{l} \rightarrow Q_1(\varphi_2)\text{ etc} \\ \rightarrow P_1(\varphi_2)\text{ etc} \end{array}\right\} \Delta\varphi_1(\varphi_2)$$

$$\dot{\varphi}^3 = [\dot{\varphi}_1 + \dot{\varphi}_2]^3 = \ldots$$

$\Delta\varphi\,(\varphi_2) \approx Q_1(\varphi_2)$ = DIFFERENTIAL PHASE

$|\Delta\varphi|(\varphi_2) \approx P_1(\varphi_2)$ = DIFFERENTIAL GAIN

SUMMARY AND CONCLUSIONS

DISTORTION EFFECTS ON MODULATED SIGNALS:

- CHANGE OF INDEX MODULATION
  - → CHANGE IN BRIGHTNESS (TV) OR LOUDNESS VS. FREQUENCY
- NON-LINEAR ENVELOPE DISTORTION
  - → HARMONIC DISTORTION
- MODULATION CONVERSION (AM TO PM)
  - → COLOR CHANGE VS. INTENSITY, CROSSTALK, ETC.
- INTERMODULATION: DIFFERENTIAL GAIN AND PHASE
  - → MORE THAN ONE SIGNAL COMPONENT PRESENT

RESPONSIBLE PARAMETERS:

- $\Delta A(\omega)$, $\Delta\phi(\omega)$ OR $\Delta tg(\omega)$

MEASUREMENT PRINCIPLES FOR: ΔA, Δφ & Δtg

Ⓐ SEPARATION BY SUBSTITUTION

$$\underbrace{A(\omega)}_{\text{REAL}} = \underbrace{A_O}_{\text{IDEAL}} + \underbrace{\Delta A(\omega)}_{\text{NON IDEAL}} \Big\}\text{ SEPARATE FROM } A_O$$

$$\Delta A(\omega) = A(\omega) - \underbrace{A_O}_{\text{SUBSTITUTED CONSTANT}}$$

$$\Delta\phi(\omega) = \phi(\omega) - \overbrace{\omega t_O}^{\text{SUBSTITUTED LINEAR TERM}}$$

$$\Delta tg(\omega) = tg(\omega) - \overbrace{t_O}^{\text{SUBSTITUTED CONSTANT}}$$

Ⓑ SEPARATION BY DIFFERENTIATION

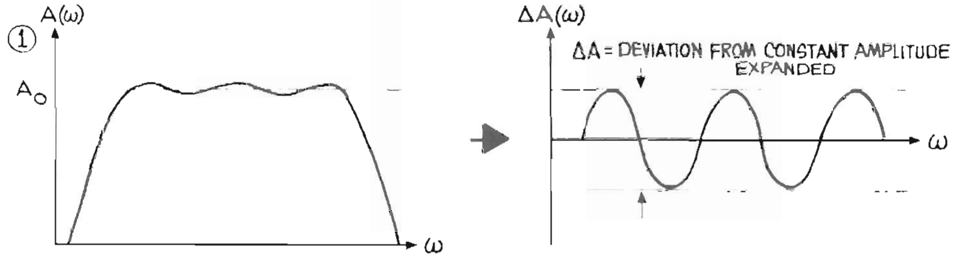
$$\frac{dA}{d\omega} = 0 + \underbrace{\frac{d[\Delta A(\omega)]}{d\omega}}_{\text{AMPLITUDE LINEARITY}}$$

$$\frac{d\phi}{d\omega} = \underbrace{\frac{d[\Delta\phi(\omega)]}{d\omega}}_{\text{PHASE LINEARITY}} + t_O$$

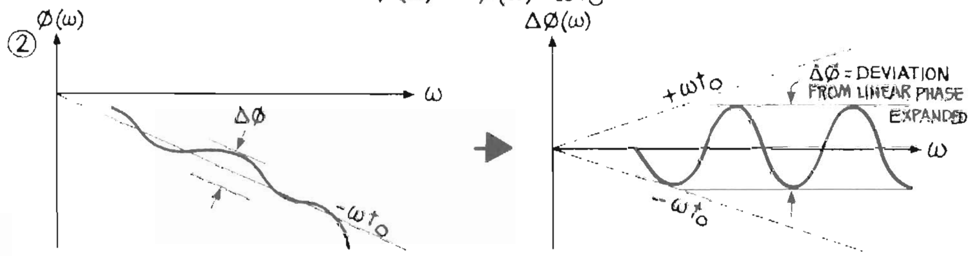
$$\frac{-d[\Delta\phi(\omega)]}{d\omega} = t_g(\omega) - t_O = \Delta t_g$$

Ⓐ ΔA, ΔØ MEASUREMENT BY SUBSTITUTION

$$\Delta A(\omega) = A(\omega) - A_o$$

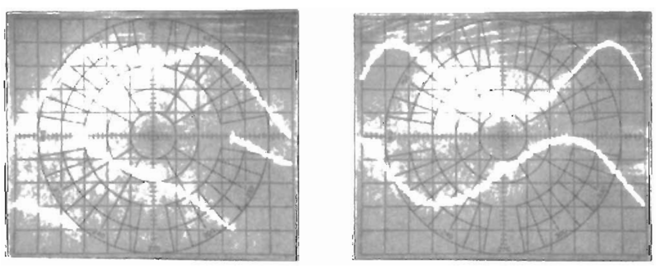


$$\Delta \phi(\omega) = \phi(\omega) - \omega t_o$$



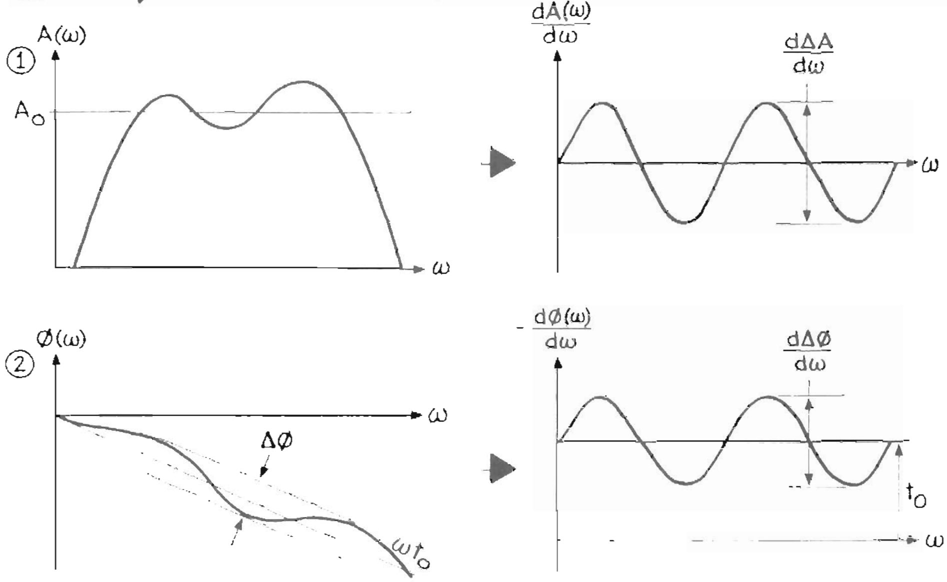
MEASUREMENT EXAMPLE FOR DEVIATION FROM LINEAR PHASE

MAGNITUDE AND PHASE CHARACTERISTIC OF BAND PASS FILTER



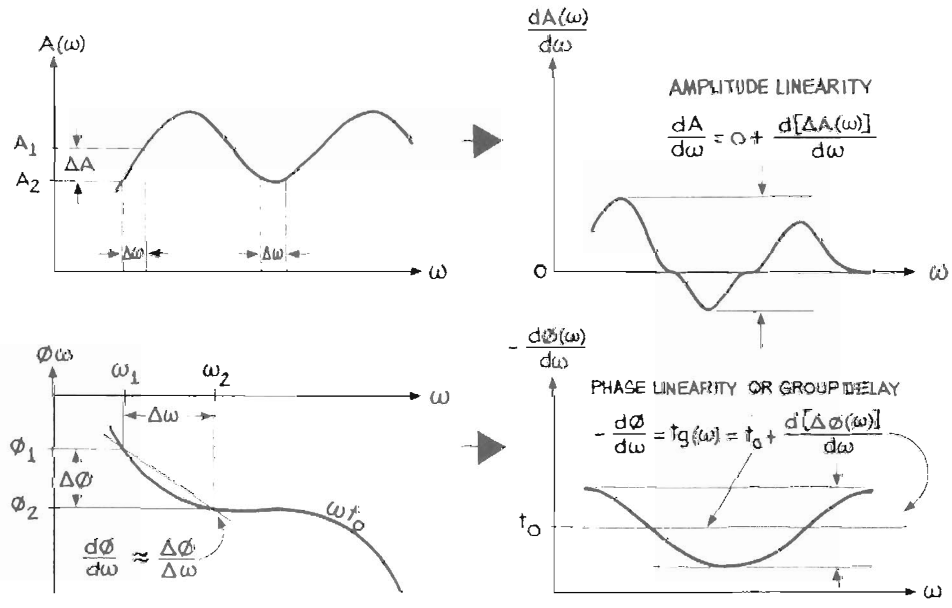
A = 5 dB/div (top)	ΔA = 1 dB/div (top)
Ø = 90°/div (bottom)	ΔØ = 10°/div (bottom)
ℓ = 0	ℓ = 4.2 m
CW = 1.1 GHz	CW = 1.1 GHz
±ΔF = 50 MHz	±ΔF = 50 MHz

(B) ΔA, ΔØ MEASUREMENT BY DIFFERENTIATION



DIFFERENTIATION PROCESS: Analog, Numerical, Modulation

ΔA, ΔØ MEASUREMENT BY DIFFERENTIATION



GROUP DELAY AS DISTORTION PARAMETER

CRITERIA FOR DISTORTION LESS TRANSMISSION:

$$H_{ideal} = \begin{cases} A(\omega) = A = \text{constant} \\ \emptyset(\omega) = \omega t_o = \text{linear with } \omega \end{cases}$$

NECESSARY CONDITION:
(SUFFICIENT IF $\frac{\Delta\omega}{\omega} \ll 1 \triangleq$ NARROW BAND)

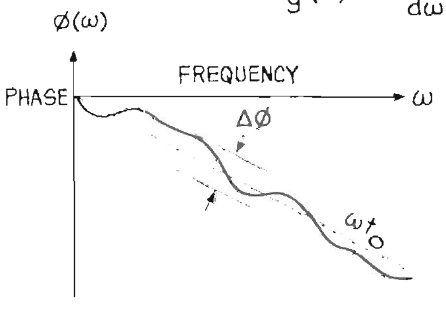
$$\frac{d\emptyset(\omega)}{d\omega} = t_o = \text{const}$$

$$\Delta H_{non\ ideal} = \begin{cases} A(\omega) - A_o = \Delta A(\omega) \\ \emptyset(\omega) - \omega t_o = \Delta\emptyset(\omega) \end{cases}$$

$$t_g(\omega) - t_o = \Delta t_g(\omega) = \frac{-d[\Delta\emptyset(\omega)]}{d\omega}$$

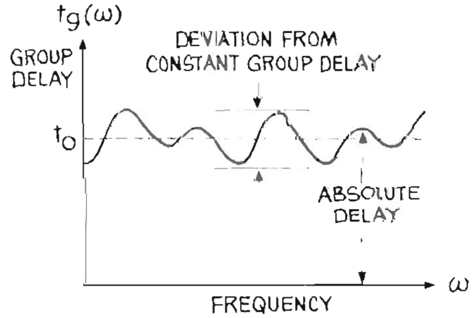
GROUP DELAY MEASUREMENT PRINCIPLES:

$$t_g(\omega) = -\frac{d\emptyset(\omega)}{d\omega} \text{ BY DEFINITION}$$



Ⓐ DIFFERENTIATION

Ⓑ ENVELOPE DELAY

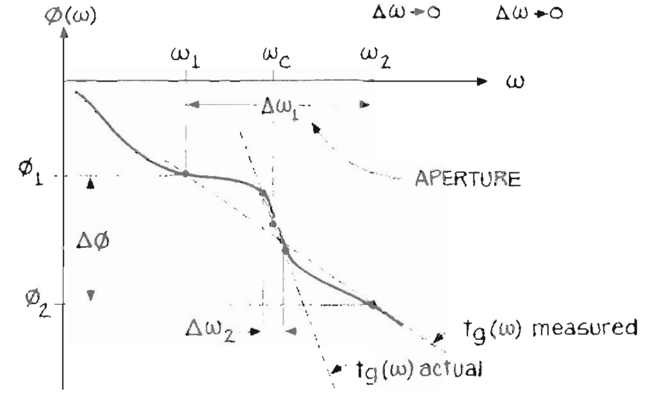


DEVIATION FROM CONSTANT GROUP DELAY INDICATES DISTORTION

TOTAL DELAY INDICATES TRANSIT TIME

DIFFERENTIATION TECHNIQUES FOR GROUP DELAY MEASUREMENTS

Ⓐ NUMERICAL DIFFERENTIATION $t_g(\omega) = -\frac{d\varnothing(\omega)}{d\omega} = -\lim_{\Delta\omega \to 0} \frac{\Delta\varnothing}{\Delta\omega} = -\lim_{\Delta\omega \to 0} \frac{\varnothing_2 - \varnothing_1}{\omega_2 - \omega_1}$



$$\frac{d\varnothing(\omega_c)}{d\omega} \neq \frac{\Delta\varnothing(\omega_c)}{\Delta\omega} = \text{FUNCTION OF } \Delta\omega, \varnothing(\omega)$$

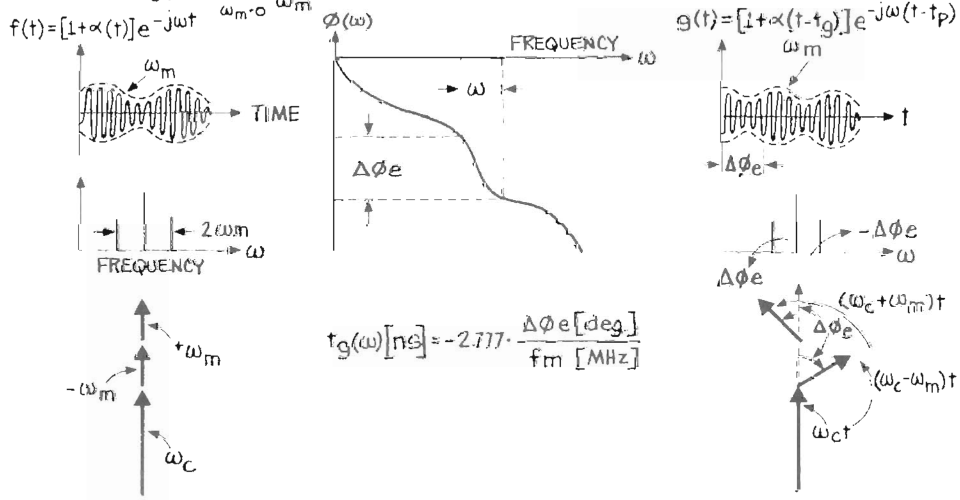
TRADE OFFS:

- For a finite resolution in $\varnothing$, $\Delta\omega$ cannot be arbitraraly small, otherwise $\Delta\varnothing$ goes to zero.
- Increasing $\Delta\omega$ leads to an average value $\Delta\varnothing$ over the interval $\Delta\omega$ (aperture) rather than the true value at ω_c.

GROUP DELAY MEASUREMENT PRINCIPLES

Ⓑ ENVELOPE DELAY WITH AM OR FM MODULATION

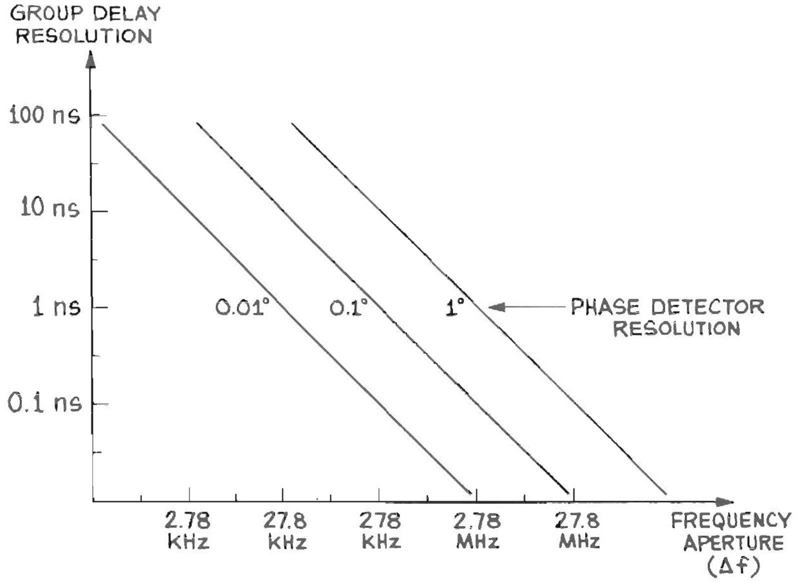
$t_g(\omega) = -\lim_{\omega_m \to 0} \frac{\Delta\varnothing e}{\omega_m}$



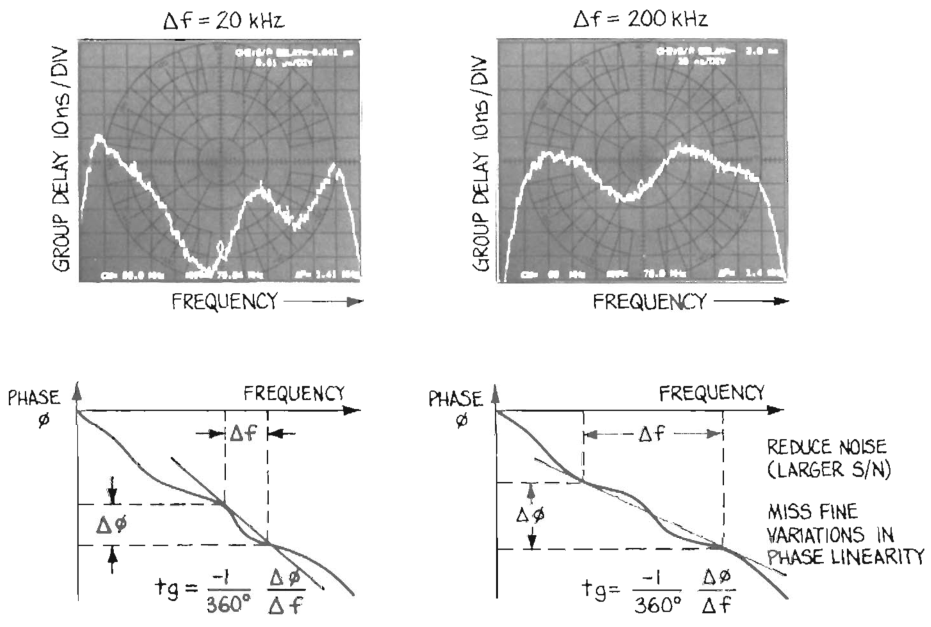
$$t_g(\omega)[ns] = -2.777 \cdot \frac{\Delta\varnothing e[deg]}{f_m\,[MHz]}$$

GROUP DELAY RESOLUTION

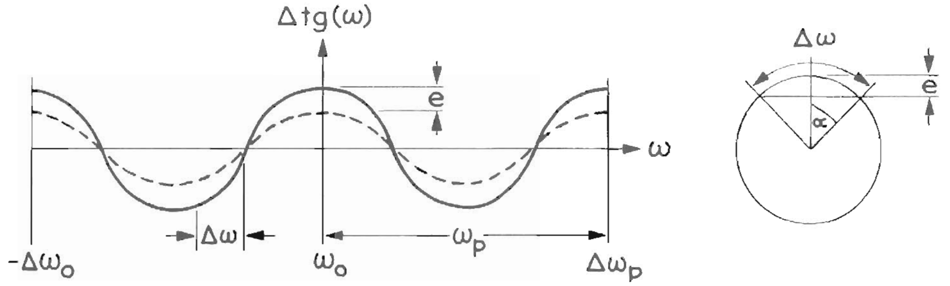
AS A FUNCTION OF APERTURE AND PHASE RESOLUTION



EFFECTS OF INCREASING APERTURE

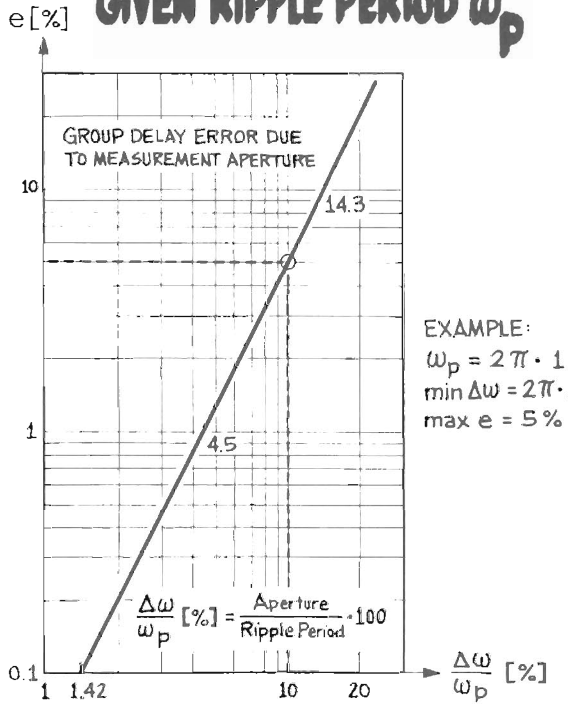


LIMITATION DUE TO MEASUREMENT APERTURES



$$e = 1 - \cos \alpha = 1 - \cos \frac{\Delta\omega}{2\omega_p}$$

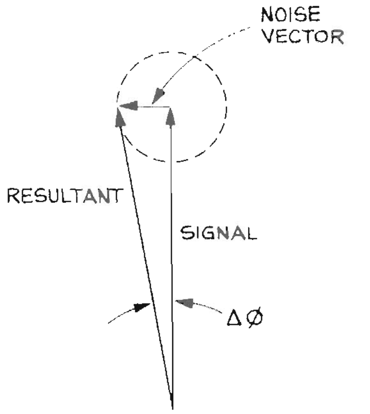
PEAK TO PEAK ERROR IN % AS A FUNCTION OF THE MEASUREMENT APERTURE FOR A GIVEN RIPPLE PERIOD ω_p



EXAMPLE:
$\omega_p = 2\pi \cdot 1$ MHz
min $\Delta\omega = 2\pi \cdot 100$ kHz
max $e = 5$ %

NOISE LIMITATIONS

GROUP DELAY MEASUREMENTS



SIGNAL/NOISE (FOR A GIVEN BANDWIDTH)	ΔØ	Δt_g *
80 dB	±0.006°	±0.06 ns
60 dB	±0.06°	±0.6 ns
40 dB	±0.6°	±6.0 ns
20 dB	±6.0°	±60.0 ns

* FOR Δf = 278 KHz

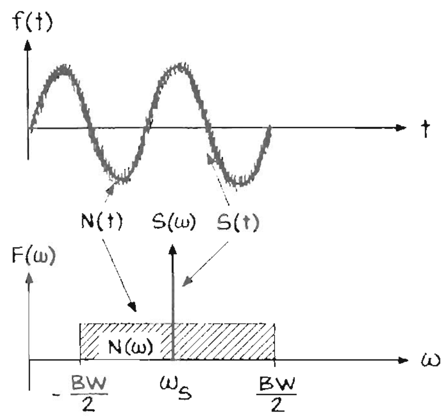
WHERE $t_g = \frac{-1}{360°} \cdot \frac{\Delta Ø}{\Delta f}$

NOTE: AS $\Delta f \to 0$, THE SIGNAL/NOISE → 0 dB

CAUSES OF NOISE

- SOURCE RESIDUAL FM
- D.U.T. NOISE FIGURE
- PHASE DETECTOR NOISE

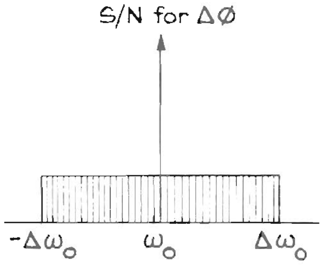
SIGNAL TO NOISE DEGRADATION DUE TO DIFFERENTIATION

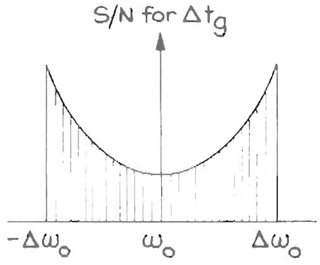


$$f(t) = S(t) + N(t) \quad @:\ f(t) = |S| \sin \omega_s t + \sum_{n=-\frac{BW}{2}}^{\frac{BW}{2}} |N_n| \sin \omega_n t$$

$$\frac{df}{dt} = \omega_s |S| \cos \omega_s t + \sum_{n=\frac{BW}{2}}^{\frac{BW}{2}} \omega_n |N_n| \cos \omega_n t$$

LIMITATIONS DUE TO THE DIFFERENTIATION PROCESS





GROUP DELAY AND DEVIATION FROM LINEAR PHASE FOR THE SAME SIGNAL TO NOISE RATIO

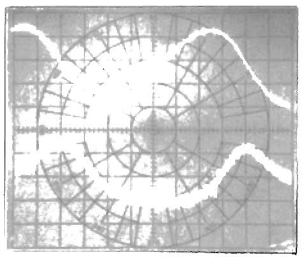
DEVIATION FROM LINEAR PHASE:

$\Delta\phi$: 10 deg/div

GROUP DELAY

Δt_g: 50ns/div

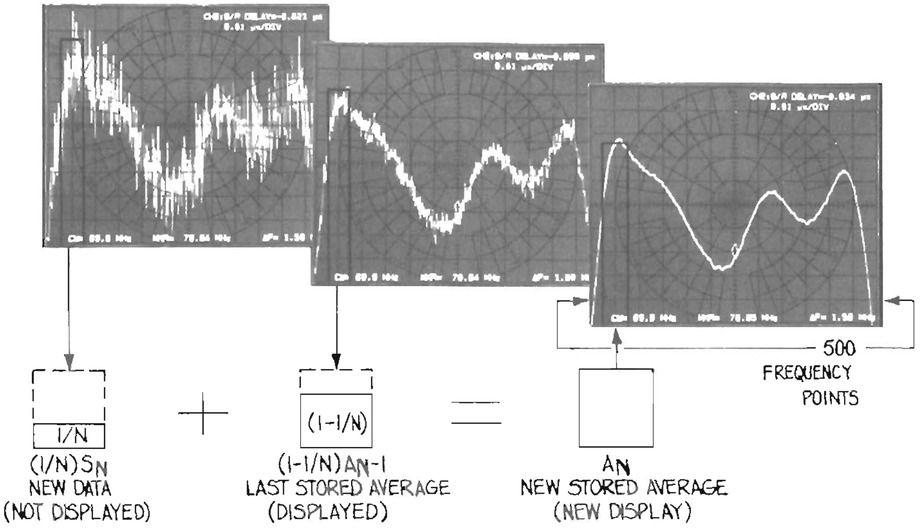
Aperture: 200 KHz



CW: 70 MHz

$\pm\Delta F$: 5 MHz

NOISE REDUCTION USING SIGNAL AVERAGING



SUMMARY: MEASUREMENT LIMITATIONS IN GROUP DELAY MEASUREMENTS

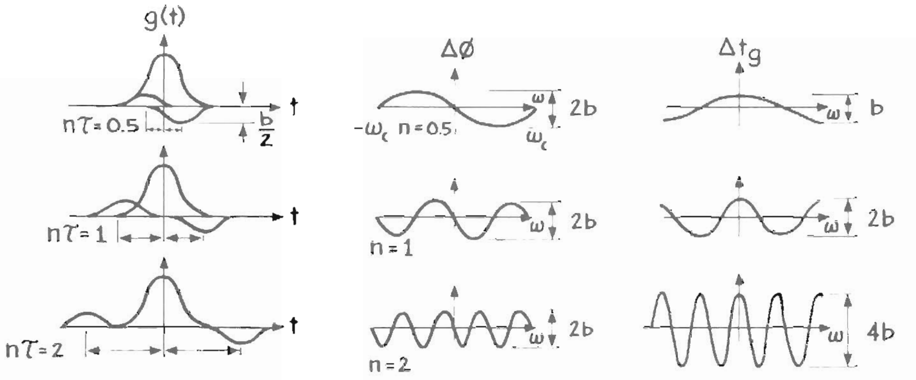
APERTURE:	MEASUREMENT APERTURE DUE TO $\Delta\omega$
NOISE:	S/N RATIO IN SOURCE, RECEIVER, DEVICE, ETC.
HARDWARE LIMITATIONS:	DETECTOR RESOLUTION, SOURCE RESOLUTION, ETC.
PRINCIPLE RELATED LIMITATIONS:	DIFFERENTIATION, NON LINEAR DISTORTION

HOW DOES Δt_g COMPARE TO $\Delta\phi$

$\Delta\phi$: WHEN EVER POSSIBLE SINCE IT HAS MUCH LESS AMBIGUETY.

Δt_g: S/N CAN BE IMPROVED BY AVERAGING.

RELATIONSHIP BETWEEN DISPERSION AND GROUP DELAY DEVIATIONS



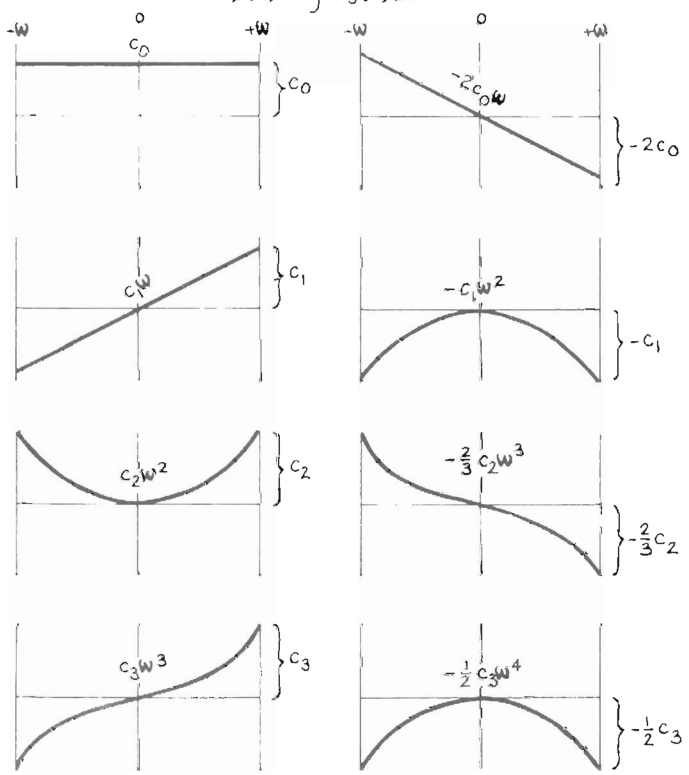
$\tau = \frac{2\pi}{\omega_c}$ $\Delta\phi(\omega) = -b\cdot\sin(n\tau\omega)$ $\Delta t_g(\omega) = n\tau b\cdot\cos(n\tau\omega)$

CONVERSION TABLE I

DEVIATION FROM CONSTANT GROUP DELAY — to → DEVIATION FROM LINEAR PHASE

$\Delta\, tg(W) \longrightarrow \Delta\phi(W)$

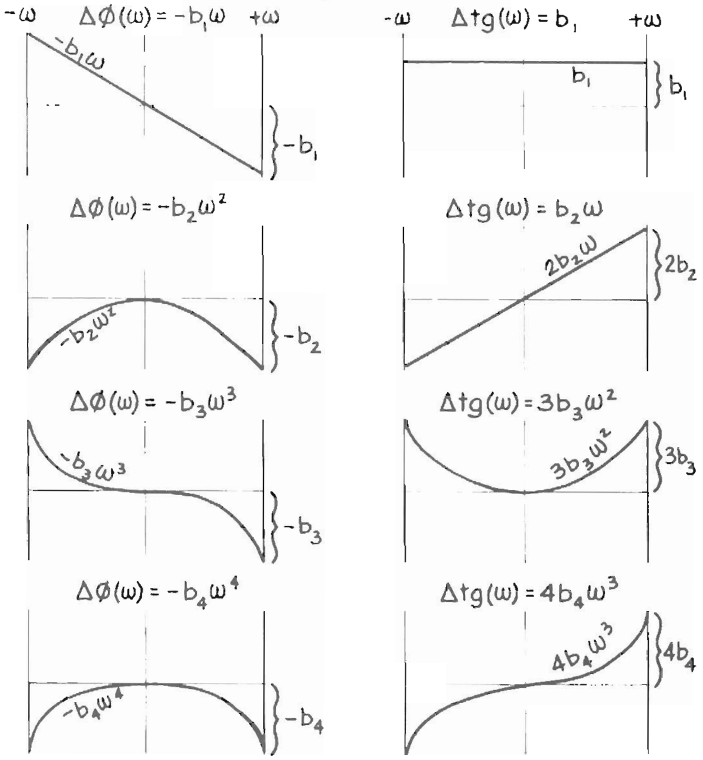
$$\Delta\phi(W) = -\int \Delta tg(W)\,dW$$



CONVERSION TABLE II

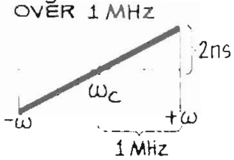
DEVIATION FROM LINEAR PHASE $\xrightarrow{\text{TO}}$ DEVIATION FROM CONSTANT GROUP DELAY

$$\Delta t_g(\omega) = \frac{-d}{d\omega}\left[\Delta\phi(\omega)\right]$$



EXAMPLE CONVERSIONS
$\Delta t_g(\omega) \rightarrow \Delta\phi(\omega)$

a) LINEAR DELAY GIVEN SPEC:
$\Delta t_g < 1\,\text{ns}$
OVER 1 MHz



WHAT DEVIATION FROM LINEAR PHASE CAN WE SPECIFY?

SOLUTION:

STEP 1: WHAT IS THE NORMALIZED GROUP DELAY FUNCTION?

$$\Delta t_g(\omega) = C_1\omega$$

STEP 2: WHAT IS THE CORRESPONDING PHASE CHARACTERISTIC?

$$\Delta\phi(\omega) = \frac{-1}{2}C_1\,\omega^2$$

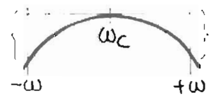
STEP 3: CALCULATE C_1 FROM SPEC. AND SUBSTITUTE IN $\Delta\phi(\omega)$

$$C_1 = \frac{\Delta t_g}{\omega} = \frac{[S]}{\left[\frac{rad}{S}\right]}$$

STEP 4: SUBSTITUDE NUMERICAL VALUES AND SOLVE FOR [rad] or [deg]

$$\Delta\phi(\omega) = \frac{-1}{2}\cdot\frac{\Delta t_g}{\omega}\cdot\omega^2 = \frac{-\Delta t_g\cdot\omega}{2}$$

NEW SPEC:

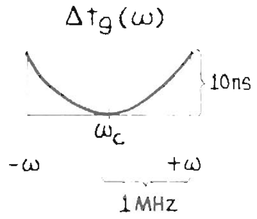


$\Delta\phi$ (rad) =
$-1\text{ns}\cdot 2\pi\cdot 1\,\text{MHz} = 2\pi\cdot 10^{-3}$ [rad]

$\Delta\phi$ (deg) =
$\frac{360}{2\pi}\cdot\Delta\phi(\text{rad}) = 0.36$ [deg] over 1 MHz

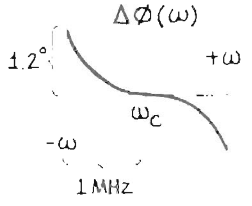
CONT. CONVERSIONS
$\Delta tg(\omega) \rightarrow \Delta\phi(\omega)$

b) PARABOLIC DELAY SPEC:



SAME STEPS AS IN EXAMPLE a):

1. $\Delta tg(\omega) = c_2 \omega^2$
2. $\Delta\phi(\omega) = -\frac{1}{3} c_2 \omega^3$
3. $\Delta\phi(\omega) = \frac{-1}{3} \cdot \frac{\Delta tg}{\omega^2} \cdot \omega^3 = \frac{-1}{3} \Delta tg \cdot \omega$
4. $\Delta\phi[rad] = \frac{2\pi}{3} \cdot 10^{-2} [rad] = 1.2 [deg]$



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