GROUP DELAY AND AM-TO-PM MEASUREMENT TECHNIQUES

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ABSTRACT:

Group Delay and AM/PM conversion measurements are used as signal distortion indicators:

In many practical measurement applications we deal with quasi linear systems which pass information with a high degree of fidelity.

In order to verify the transmission performance through these systems, we can either approach them as linear networks and look for deviations from an ideal distortionless transfer function $H(j\omega)$ in terms of deviations from constant amplitude and linear phase, or we can stimulate the network with a simulation of an actual information carrying signal (modulated signal) and observe the distortion on the information itself.

In the first case, we can compute the expected signal distortion from static measurements of the deviation from constant amplitude $\Delta A(\omega)$ and linear phase $\Delta \emptyset(\omega)$ via conversion tables for a variety of commonly used signals.

In the second case, we are using a dynamic measurement approach by applying a modulated signal as a test signal and then deriving the expected signal distortion from the signal modifications observed on the test signal. The most commonly used parameters are deviations from constant gain over the information bandwidth and dynamic range of the system. This figure of merit is sometimes called gain linearity or differential gain when evaluated in the presence of another signal. Deviation from constant group delay is another distortion parameter which is used to detect transmission anomalies in the system. And again, when the phase change of the modulated signal is observed as a function of another signal, it is referred to as differential phase. Furthermore, there is the effect of modulation conversion in the form of, for example, Amplitude Modulation to Phase Modulation conversion which has to be characterized on networks and systems.

The static as well as the dynamic characterization methods are related and can often be substituted for each other.

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OBJECTIVES

- RELATE SIGNAL DISTORTION TO MEASURABLE NETWORK PARAMETERS.
- INTRODUCE BASIC MEASURING PRINCIPLES FOR GROUP DELAY AND AM/PM CONVERSION.
- DISCUSS MEASUREMENT LIMITATIONS AND TRADE OFFS ON PRACTICAL EXAMPLES.

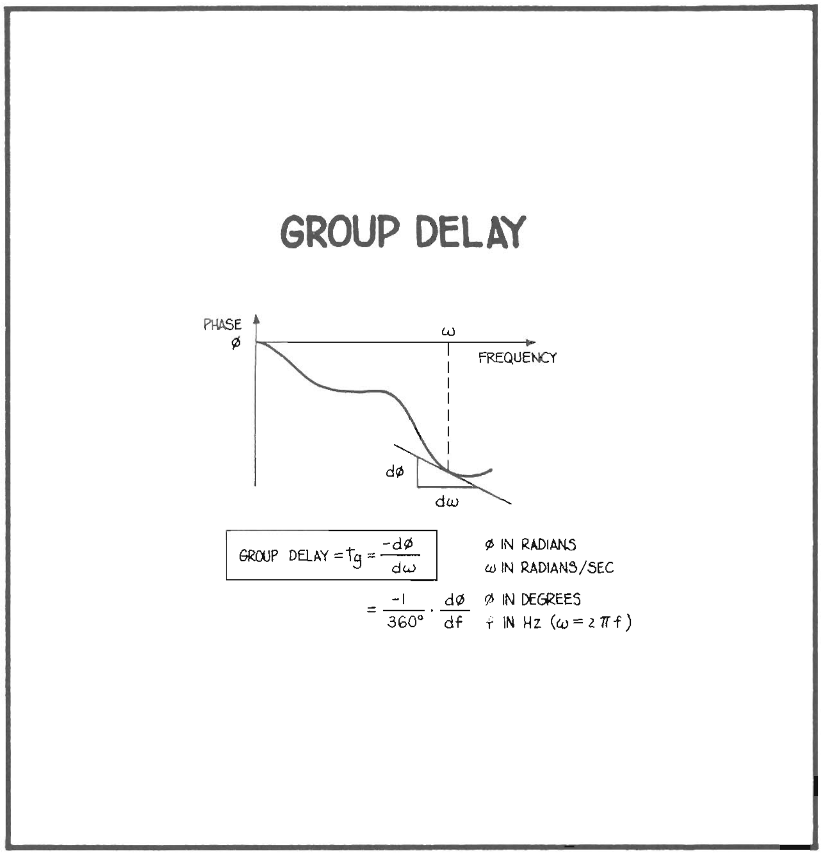
INTRODUCTION: The parameters group delay (tg) amd AM/PM conversion are used to predict/specify distortion in basically linear systems and components. The objectives of this presentation are:

- To relate signal distortion to measurable network parameters.
- Introduce basic measurement principles.
- Discuss measurement limitations.

DEFINITION OF TERMS: Starting out, it is useful to give some definition of the terms we will be dealing with:

Group Delay: First related to electrical networks by Nyquist in 1928 and defined as $t_g = \frac{d\emptyset}{d\omega}(\omega)$. tg has basically two parts:

1. a constant value tg at ω_o and
2. a variable part tg (ω) over the frequency interval. (1)(2)(3)



The two parts relate to two different aspects of group delay

1 constant → propagation delay
2 variable → distortion

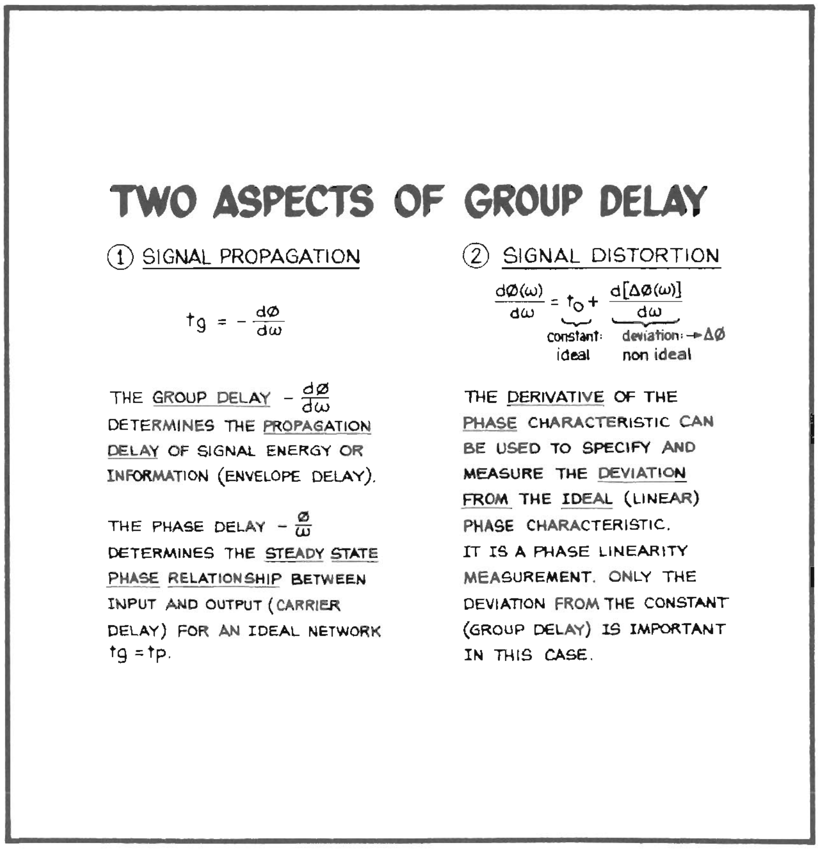
SIGNAL DELAY

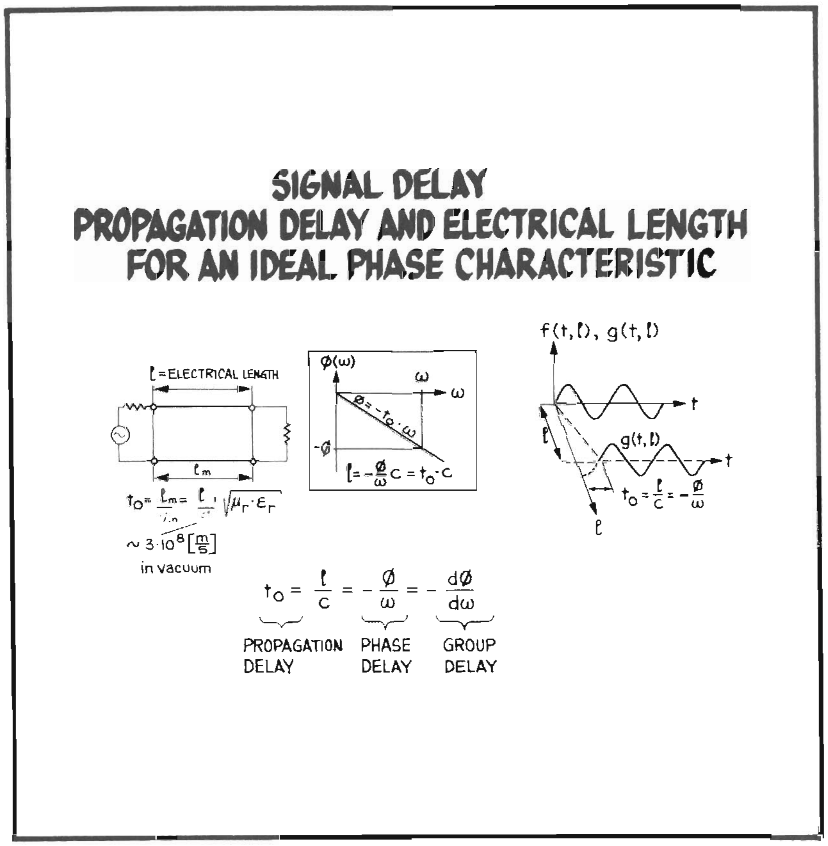
Definitions:
From the phase characteristic of a network we can determine the delay or propagation time of the various signal components. In a network with a strictly linear phase characteristic each signal component experiences the same delay t_o and it is given by the physical length l_m of the network (or propagation medium) and the propagation velocity in the medium V_m. Since the propagation velocity is a function of the premeability u_r and the dielectric constant ε_r of the medium, the propagation delay becomes:

$$t_o = \frac{l_m}{V_m} = \frac{l_m\sqrt{u_r \varepsilon_r}}{c} = \frac{l}{c}$$

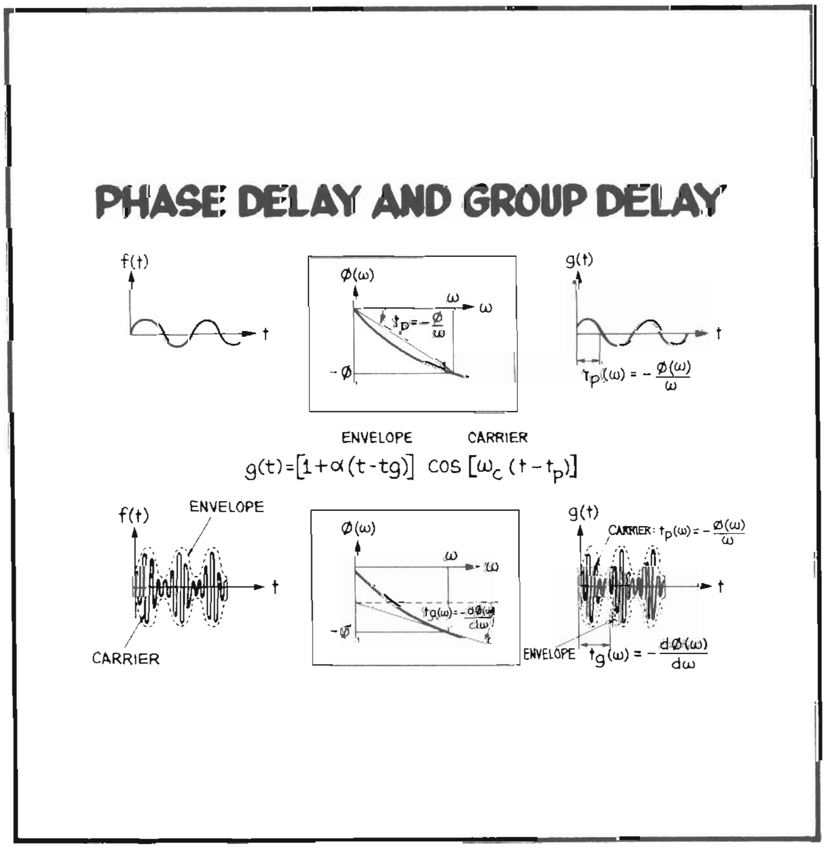
where l is the equivalent electrical length in vacuum and c the velocity of light. The electrical length of a network can be determined from the phase characteristic and is proportional to the phase slope:

$$l = -\frac{\theta}{\omega} \cdot c = t_o \cdot c.$$





The figure shows the propagation delay and the electrical length of a transmission line having an ideal phase characteristic. In a practical network, the phase characteristic is not a strictly linear function of frequency anymore. The signal delay, therefore, varies for different spectral components and other delay terms become meaningful as explained in the following paragraphs.



Phase delay:

$$t_p(\omega) = -\frac{\theta(\omega)}{\omega}$$

The phase delay of a network at any frequency is given by the slope of the vector starting at the origin (ω=0) and ending at the point ω, $-\theta$.

The phase delay does not indicate a propagation delay, it describes the steady state phase relationship between input and output for a specific frequency component ω.

Group delay:

$$t_g(\omega) = -\frac{d\theta(\omega)}{d\omega}$$

The group delay of a network at any given frequency ω is given by the dervative or the slope of the phase characteristic at that point ω.

In general, the group delay describes the propagation delay of a narrow frequency group $\Delta\omega$ with center frequency ω under the assumption that the amplitude characteristic is constant and the phase is linear over the same frequency interval $\Delta\omega$. However, a certain group delay time t_g does not mean that no energy arrives at the output of the network before that. Half the energy of the frequency group $\Delta\omega$ arrives before the time t_g. The group delay can also be interpreted as the time it takes for a sinusoidal signal of constant frequency ω applied to the network at t=0 to build up to 50% of its final value.

Envelope delay:

$$t_e(\omega) = \frac{\Delta\theta_e(\omega)}{\omega_m}$$

The envelope delay is the steady state delay of the signal envelope of a modulated signal and approaches the definition of group delay for a narrow signal spectrum ($\omega_m \to 0$) relative to the slope changes of the phase characteristic. Envelope delay is often used interchangeably with group delay for this reason.

Delay of the center of gravity:

$$t_e(\omega) = -\frac{d\theta(0)}{d\omega} = t_g(0)$$

In a dispersive network, the signal envelope gets distorted and the signal delay cannot be well defined at the end points. In this case, it is more convenient to reference the delay time to the center of gravity of the signal. The propagation time of the center of gravity is the group delay time for $\omega=0$ or $\omega\ \omega_0$ in the bandpass case.

Signal front delay:

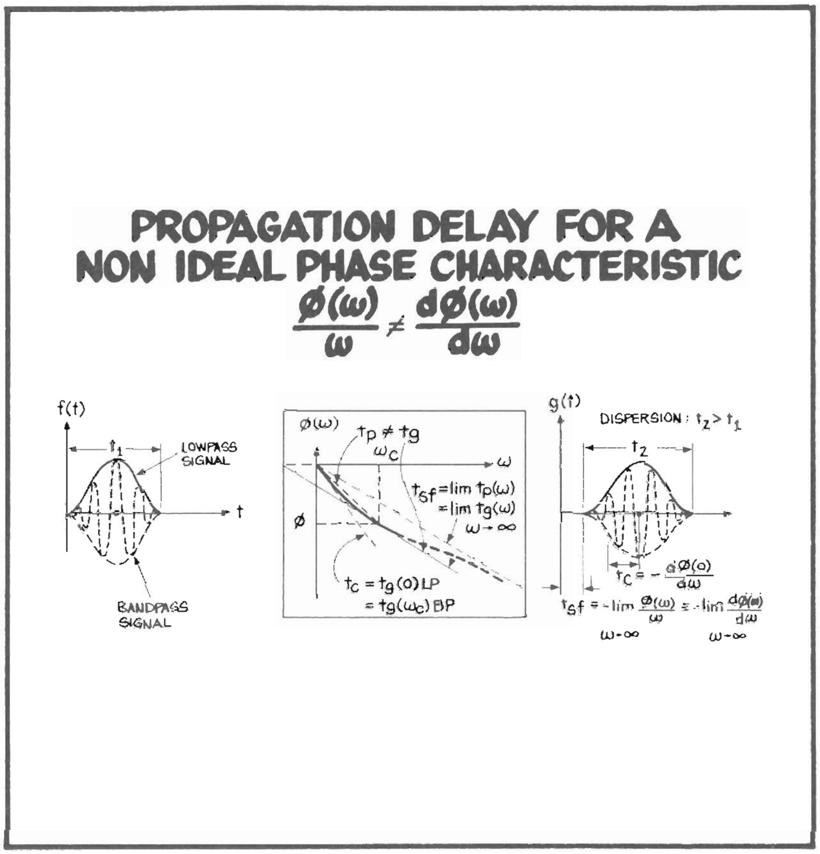
$$t_{sf} = -\lim_{\omega\to\infty} \frac{\theta(\omega)}{\omega} = \frac{l_{min}}{c}$$

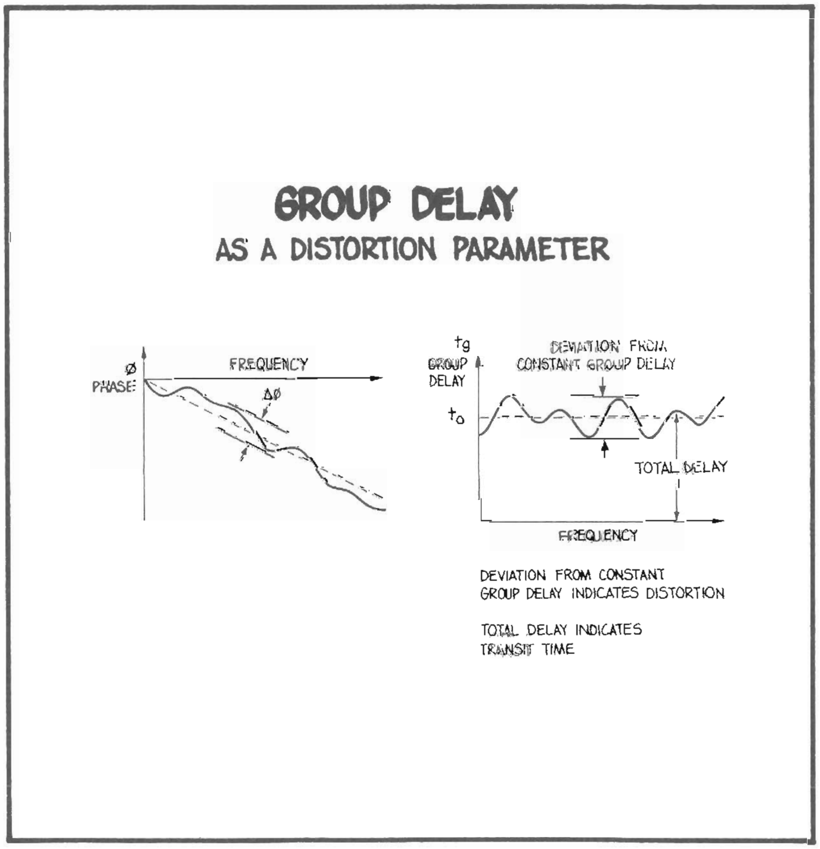
As the signal disperses due to nonideal network characteristics, the front of the signal obviously propagates faster than the center of gravity and, therefore, the front is less delayed. As shown in (4) the signal front delay is given by the asymptotic phase slope as the frequency approaches infinity.

From this it may be concluded, that the phase delay approaches the constant l_{min}/c where l_{min} is the minimum electrical length of the propagation path. In practive, l_{min} is not necessarily the electrical length through the network, but the closest physical connection between input and output which supports a mode of propagation.

SIGNAL DISTORTION: The derivative $\frac{d\phi}{d\omega}$ also indicates a deviation from an ideal (distortion free) linear phase and becomes a figure of merit for the phase linearity of a network:

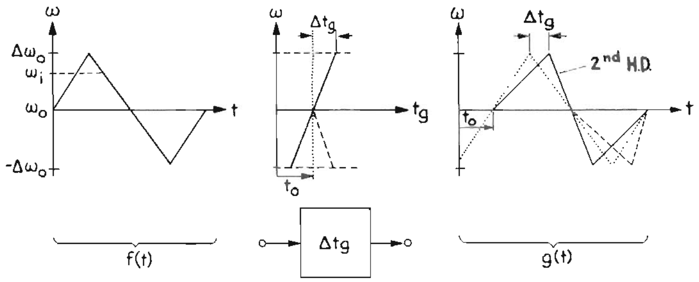
- The more $\Delta\phi$ or Δtg the more dispersion and potentially distortion the network generates.
- However, there is no simple and direct relationship between $\Delta\phi$ or Δtg and signal distortion.
- On the contrary, the relationship is fairly complex and a function of the signal, nevertheless very important, such that we take some time to explain some relationships in the next paragraph.





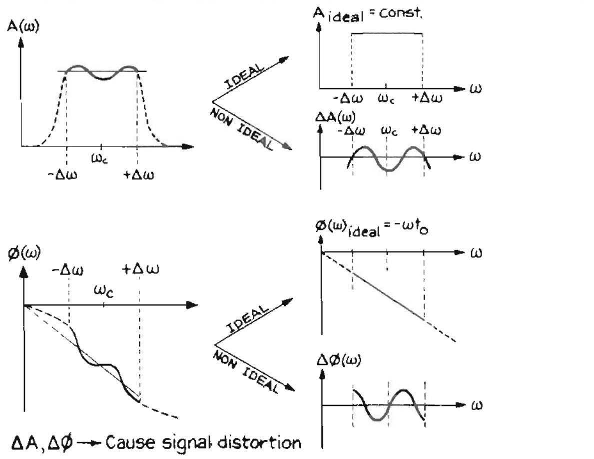
EFFECT OF GROUP DELAY VARIATIONS ON AN FM SIGNAL

$$\Delta\phi = b_2\omega^2 \rightarrow \Delta tg = 2b_2\omega$$



An example of signal distortion due to non-constant group delay is shown here. A wide deviation angle modulated signal (FM) is passed through a linear system with non-ideal phase characteristic. Each component of the modulated signal may be considered as a narrow band signal (centered around the instantaneous frequency ω_i). After passing through the network, the signal components are shifted according to the group delay characteristic and the output signal shows obvious harmonic distortion. This looks like a contradiction to the linearity postulate above and therefore it appears to be an irreversible signal alteration. However, this is not the case and as long as the signal is not demodulated or followed by another nonlinear process, the distortion can be eliminated.

NON IDEAL TRANSFER CHARACTERISTIC



DISTORTIONLESS TRANSMISSION

The criteria for distortionless transmission falls in two parts. First, the amplitude (magnitude) response must be flat over the bandwidth of interest. This means all frequencies within the bandwidth will be attenuated identically. Second, the phase response must be linear over the bandwidth of interest. This means that the relative phase relationships between frequencies within the bandwidth will be preserved after transmission.

A realizable network or system will only approximate the ideal transfer characteristic and the residuals $\Delta A(\omega)$ and $\Delta\phi(\omega)$ will cause signal distortion. $\Delta A(\omega)$ is the deviation from the ideal (constant) amplitude characteristic and $\Delta\phi(\omega)$ is the deviation from the ideal (linear) phase.

For most practical networks, the deviations $\Delta A(\omega)$ and $\Delta\phi(\omega)$ can sufficiently and accurately be approximated by representing them as a power series expansions of the form

$$\Delta A(\omega) = a_1\omega + a_2\omega^2 + \ldots$$

$$\Delta\emptyset(\omega) = b_2\omega^2 + b_3\omega^3 + \ldots$$

around the center of the passband (ω_c).

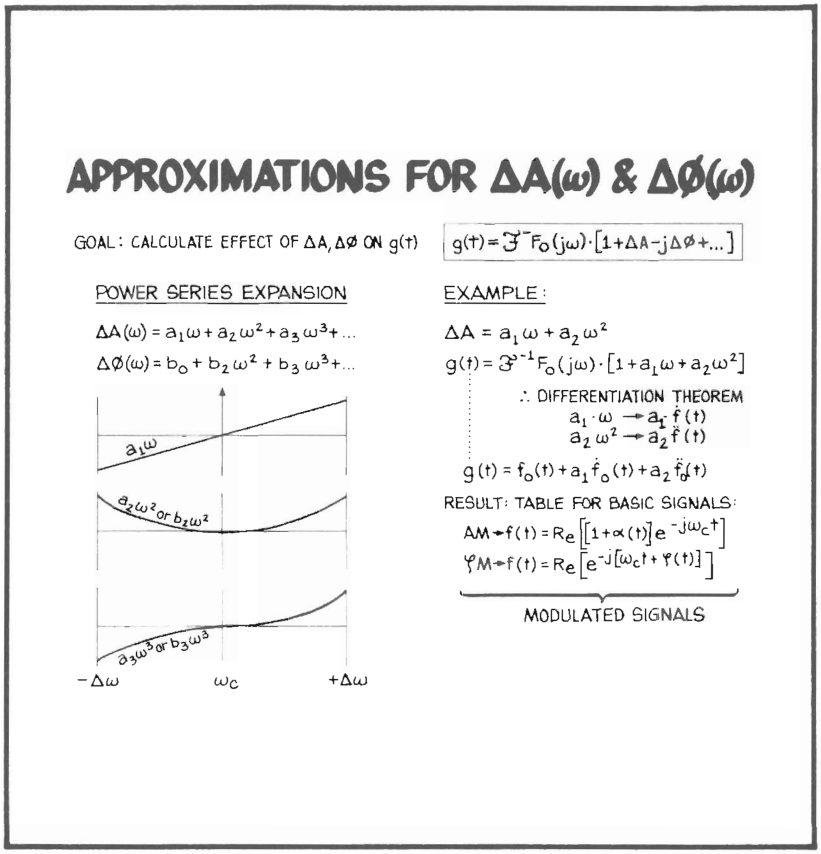
By substituting for ΔA, $\Delta\phi$ in the equation describing the input/output signal relationship; the signal modifications due

$$g(t) = \mathfrak{F}^{-1}F(j\omega).\ [1+\Delta A(\omega)-j\Delta\phi(\omega)+\ldots]$$

to the amplitude and phase coefficients a_i and b_i can be calculated and tabulated for standard signals.

This makes the Fourier transform easy because of the differentiation theorem.

- A table can be calculated for different classes of signals.
- As can be seen from the first relationship, the magnitude of ΔA is not going to cause a proportional error term on the modulation information. (5)(6)(7)(8)

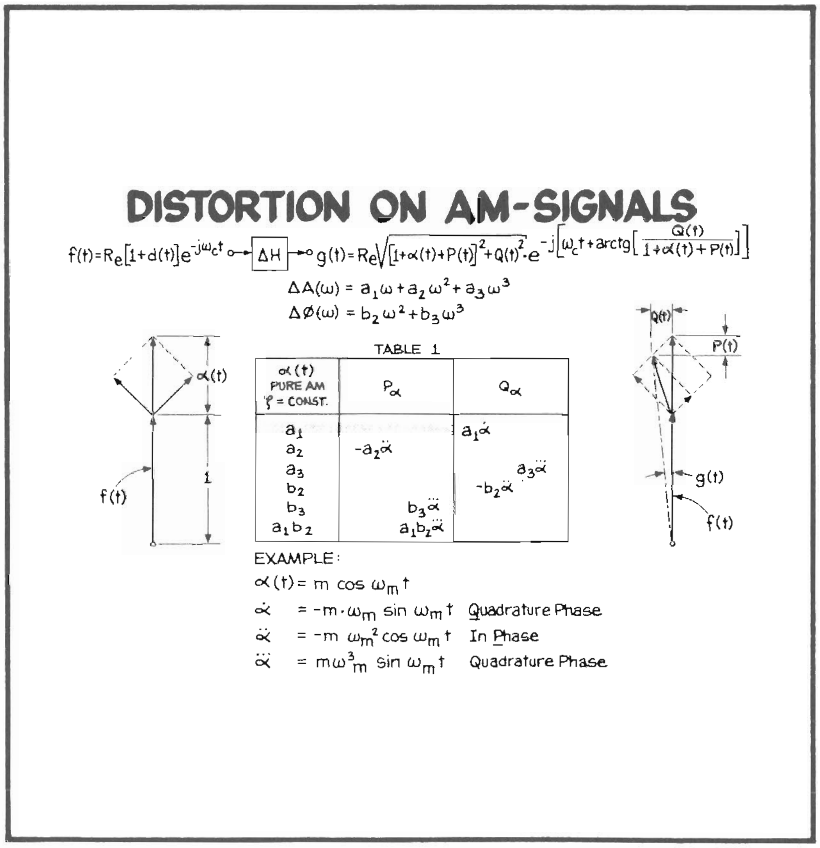


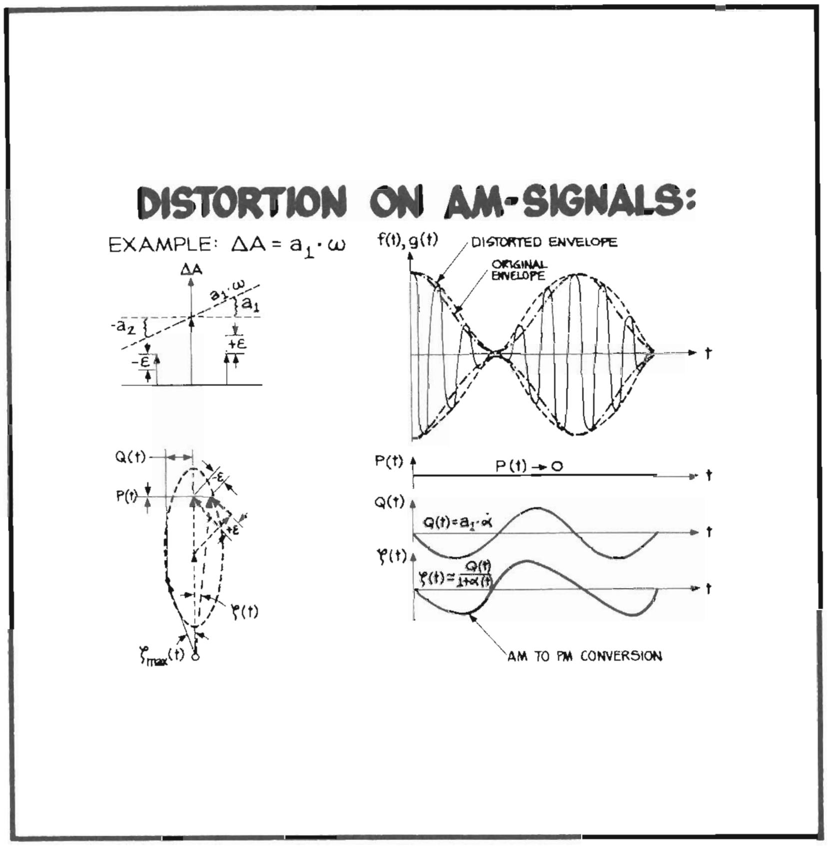
DISTORTION ON MODULATED SIGNALS

Looking at an amplitude modulated signal of the form:

$$f(t) = \mathrm{Re}[1+\alpha(t)]e^{-j\omega_c t}$$

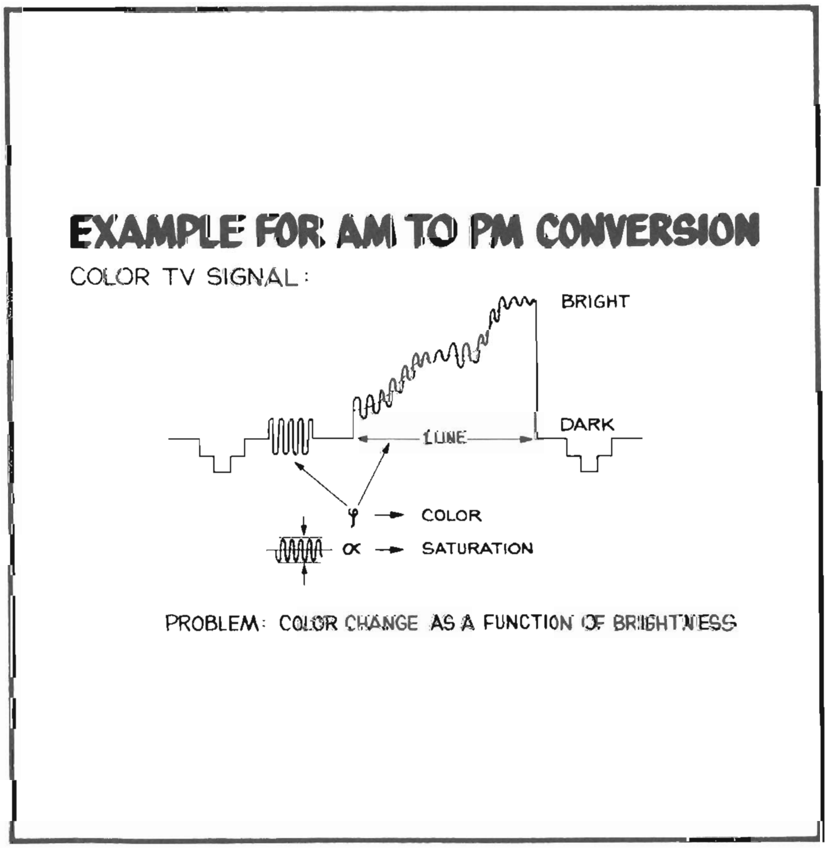
The table shows the signal or distortion components which will be generated as a function of the coefficients a_i, b_i. The resulting signal contributions are grouped in the direction of the carrier as P_α'S and perpendicular to it as Q_α'S. In the case of AM-signals, the P-component adds to or modifies the original information $\alpha(t)$ whereas the Q-component introduces parasitic phase modulation. (9)





For example, a linear component a_i in the amplitude characteristic will introduce no change in the index of modulation of the AM-signal, however, additional phase modulation is introduced which will distort the signal envelope somewhat.

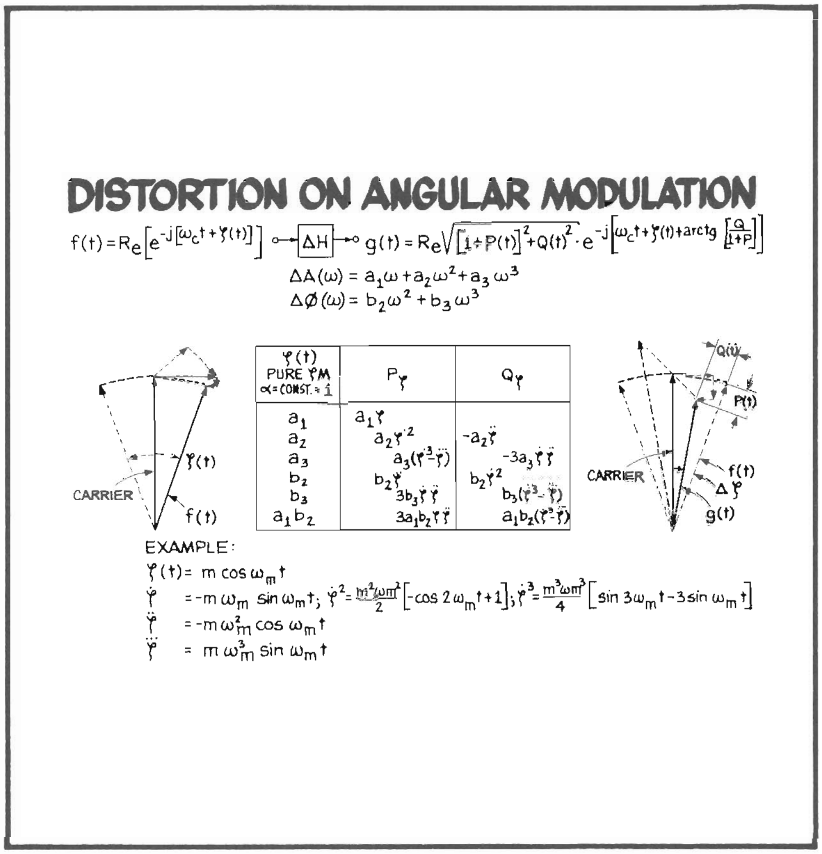
This form of distortion is called modulation conversion or AM to PM conversion, and it can adversely affect some of the composite signals like color TV.



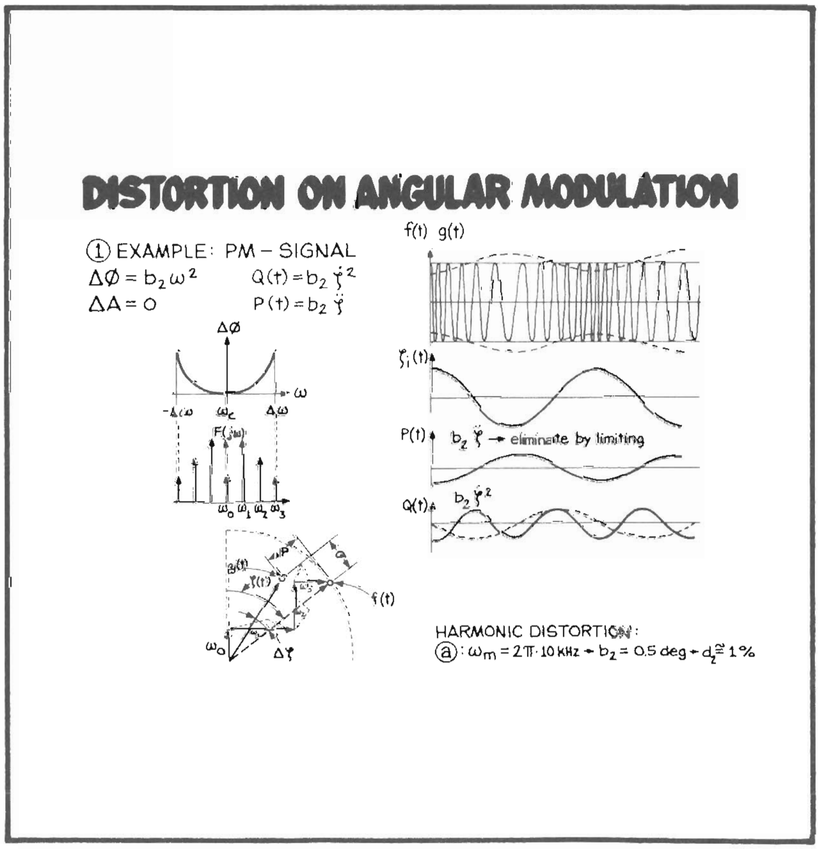
A practical example would be the color change (phase) as a function of the brightness (level) on a TV-signal.

A slope in the amplitude response ($a_1 \cdot \omega$) can convert the AM-component (dark to bright) into a phase modulation component (Q) which affects the color interpretation as a function of the signal brightness.

A similar table as for amplitude modulated test signals can be devised for angular modulation. In this case, the Q-components are the ones which directly interfere with the original modulation information, whereas the P-components only affect the amplitude of the signal. The residual AM due to the P-component can often be eliminated by a subsequent limiter without affecting the angular information.



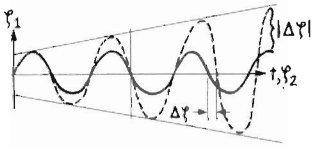
A parabolic phase component b_2, for example, will generate second harmonic distortion due to the $b_2 \dot{\varphi}^2$ term which cannot be separated from the original information content.



WHAT HAPPENS WHEN MORE THAN ONE SIGNAL IS PRESENT:

$$f(t) = e^{-j[\omega_c t + \psi_1(t) + \psi_2(t)]} \rightarrow g(t) = \sqrt{(1+P)^2+Q^2} \cdot e^{-\ldots \text{arctg}\frac{Q}{1+P}}$$

TABLE	P_ψ	Q_ψ
a_3	$a_3[\dot{\psi}^3 - \dddot{\psi}]$	$-3a_3\dot{\psi}\ddot{\psi}$



$$\dot{\psi}\ddot{\psi} = (\dot{\psi}_1+\dot{\psi}_2)(\ddot{\psi}_1+\ddot{\psi}_2) = \dot{\psi}_1\ddot{\psi}_2 + \dot{\psi}_2\ddot{\psi}_1 + \ldots$$

INTERMODULATION → $Q_1(\psi_2)$ etc, → $P_1(\psi_2)$ etc $\} \Delta\psi_1(\psi_2)$

$$\dot{\psi}^3 = [\dot{\psi}_1+\dot{\psi}_2]^3 = \ldots$$

$|\Delta\psi|(\psi_2) \approx P_1(\psi_2)$ = DIFFERENTIAL GAIN

$\Delta\psi(\psi_2) \approx Q_1(\psi_2)$ = DIFFERENTIAL PHASE

The presence of another signal ψ_2 in addition to ψ_1, leads to a form of intermodulation which is often referred to as differential phase. In this case, the magnitude or the phase relationship of the original signal $\psi_1(t)$ will change as a function of $\psi_2(t)$ in the presence of a deviation in the transfer characteristic, for example $a_{3\omega}{}^3$.

The magnitude change due to ψ_2 is usually referred to as differential gain and the phase change as differential phase.

SUMMARY AND CONCLUSIONS

DISTORTION EFFECTS ON MODULATED SIGNALS:

- CHANGE OF INDEX MODULATION
  → CHANGE IN BRIGHTNESS (TV) OR LOUDNESS VS. FREQUENCY
- NON-LINEAR ENVELOPE DISTORTION
  → HARMONIC DISTORTION
- MODULATION CONVERSION (AM TO PM)
  → COLOR CHANGE VS. INTENSITY, CROSSTALK, ETC.
- INTERMODULATION: DIFFERENTIAL GAIN AND PHASE
  → MORE THAN ONE SIGNAL COMPONENT PRESENT

RESPONSIBLE PARAMETERS:

- $\Delta A(\omega)$, $\Delta\phi(\omega)$ OR $\Delta tg(\omega)$

MEASUREMENT PRINCIPLES

TRANSFER FUNCTION: $H(j\omega)$, $A(\omega)$, $\phi(\omega)$.

DEVIATION FROM CONSTANT AMPLITUDE, LINEAR PHASE AND CONSTANT GROUP DELAY: $\Delta A(\omega)$, $\Delta\phi(\omega)$, Δtg.

GROUP / ENVELOPE DELAY

PHASE AND GAIN LINEARITY

AM TO PM CONVERSION

MEASUREMENT PRINCIPLES

Approach:
Philosophically (theoretically) we can either measure and specify the cause responsible for signal distortion (ΔA, $\Delta\emptyset$) or the effect which is the actual distortion on a signal. In practice, many of the measuring approaches are very similar and only the intent may make the difference. For example:

CAUSE for distortion in a network is the deviation from the ideal transfer function. In case of a linear N.W. we would measure:

$$\Delta A(\omega),\ \Delta\emptyset(\omega)$$

For a non-linear N.W. (AGC, etc.) we would have to measure the transfer characteristics as a function of signal level (power), which is sometimes referred to as describing function.

EFFECTS on information carrying signals is a widly used approach to characterize distortion. In essence, the signal modification is observed on a specific test signal. This can be done steady state or dynamically, depending on what kind of signals the network will see in its actual environment.

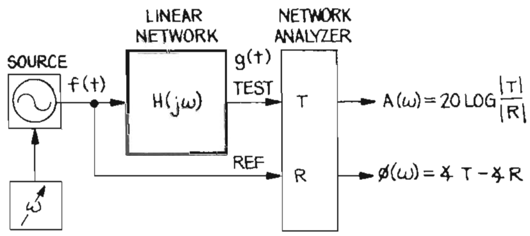
From these observations, several measurement principles have evolved:

In the "CAUSE" area, the mostly used approaches are: $\Delta A(\omega)$, $\Delta\emptyset(\omega)$, $\Delta tg(\omega)$. Principles which observe "EFFECTS" are phase/gain linearity, envelope delay, AM/PM conversion.

MEASUREMENT APPROACHES FOR DISTORTION ANALYSIS

CAUSE		EFFECT	
DEVIATION FROM THE IDEAL TRANSFER CHARACTERISTIC		OBSERVE SIGNAL MODIFICATIONS ON SPECIFIC SIGNAL	
LINEAR	NON LINEAR	LINEAR	NON LINEAR
TRANSFER FUNCTION $H(j\omega)$, $A(\omega)$ AND $\phi(\omega)$ VERSUS FREQUENCY	DESCRIBING FUNCTION $H(j\omega_0, P)$, $A(P)$, AND $\phi(P)$ VERSUS POWER LEVEL	CW-NETWORK ANALYSIS OR DYNAMIC ANALYSIS	CW-SPECTRUM ANALYSIS OR DYNAMIC ANALYSIS
MEASUREMENTS OF $H(j\omega)$ YIELD $\Delta A(\omega)$, $\Delta\phi(\omega)$, $\Delta tg(\omega)$ AND STATIC AM/PM CONVERSION	MEASUREMENTS OF $H(P)$ YIELD $\Delta A(P)$, $\Delta\phi(P)$, $\Delta tg(P)$ AND AM/PM CONVERSION	· NOISE MEASUREMENTS, CORRELATION, ETC. · MODULATION TRANSFER CHARACTERISTIC · PULSE MEASUREMENT	· GAIN AND PHASE LINEARITY · AM/PM CONVERSION · DIFFERENTIAL GAIN/PHASE · NOISE INTERMODULATION

TRANSFER FUNCTION $H(j\omega)$



TRANSFER FUNCTION $H(j\omega)$

Before we discuss the deviations from the transfer function, lets look at the transfer function measurement itself:

The transfer function $H(j\omega)$ of a linear network can be measured by applying a sinusoidal signal of variable frequency to the network and observing the change in amplitude and phase of the output signal g(t) with respect to the input signal f(t).

$$f(t) = \cos\omega t$$
$$g(t) = A(\omega)\cos(\omega t + \phi(\omega)) \text{ where}$$
$$A(\omega) = |H(j\omega)| = A_o + \Delta A(\omega)$$
$$\phi(\omega) = \measuredangle H(j\omega) = -t_o\omega + \Delta\phi(\omega)$$

Instruments which measure $H(j\omega)$ are usually referred to as Network Analyzers or Frequency Response test sets.

MEASUREMENT PRINCIPLES FOR: ΔA, $\Delta\emptyset$ & Δtg

Ⓐ SEPARATION BY SUBSTITUTION

$$\underset{\text{REAL}}{A(\omega)} = \underset{\text{IDEAL}}{A_o} + \underset{\text{NON IDEAL}}{\Delta A(\omega)}\Big\}\text{ SEPARATE FROM } A_o$$

$$\Delta A(\omega) = A(\omega) - \underset{\text{SUBSTITUTED CONSTANT}}{A_o}$$

$$\Delta\emptyset(\omega) = \emptyset(\omega) - \overset{\text{SUBSTITUTED LINEAR TERM}}{\omega t_o}$$

$$\Delta tg(\omega) = tg(\omega) - \overset{\text{SUBSTITUTED CONSTANT}}{t_o}$$

Ⓑ SEPARATION BY DIFFERENTIATION

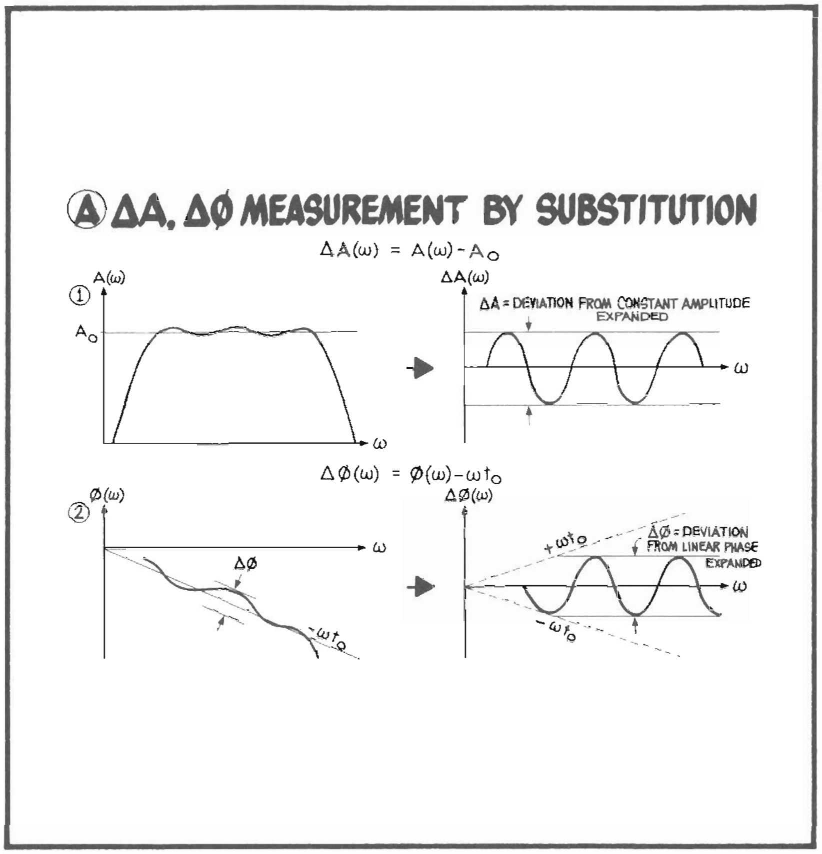
$$\frac{dA}{d\omega} = 0 + \underbrace{\frac{d[\Delta A(\omega)]}{d\omega}}_{\text{AMPLITUDE LINEARITY}}$$

$$\frac{d\emptyset}{d\omega} = \underbrace{\frac{d[\Delta\emptyset(\omega)]}{d\omega}}_{\text{PHASE LINEARITY}} + t_o$$

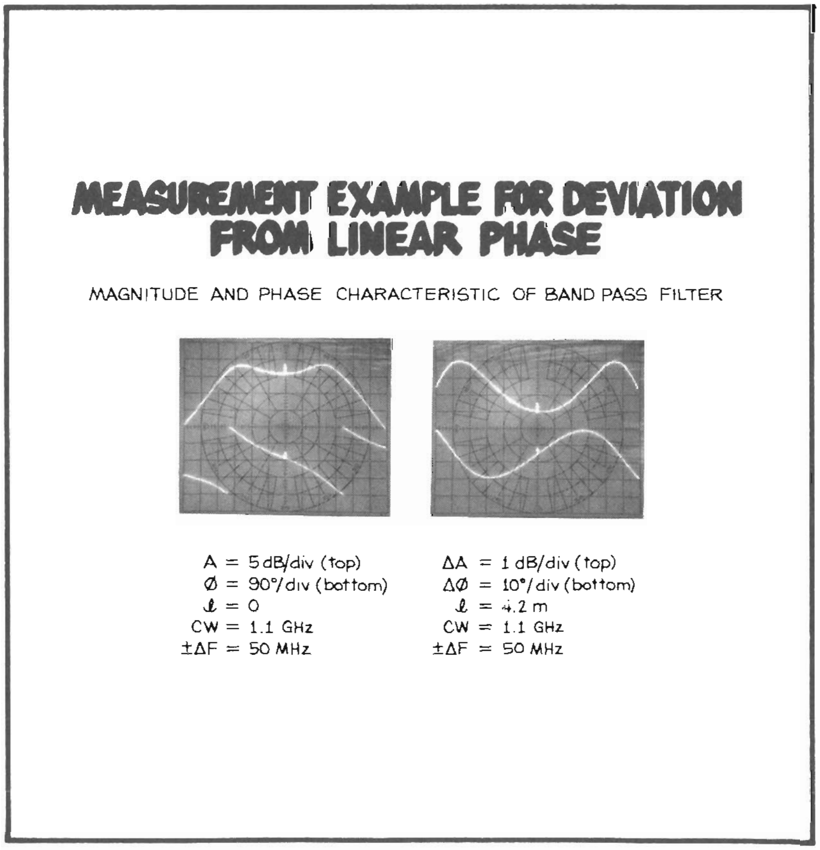
$$\frac{-d[\Delta\emptyset(\omega)]}{d\omega} = t_g(\omega) - t_o = \Delta t_g$$

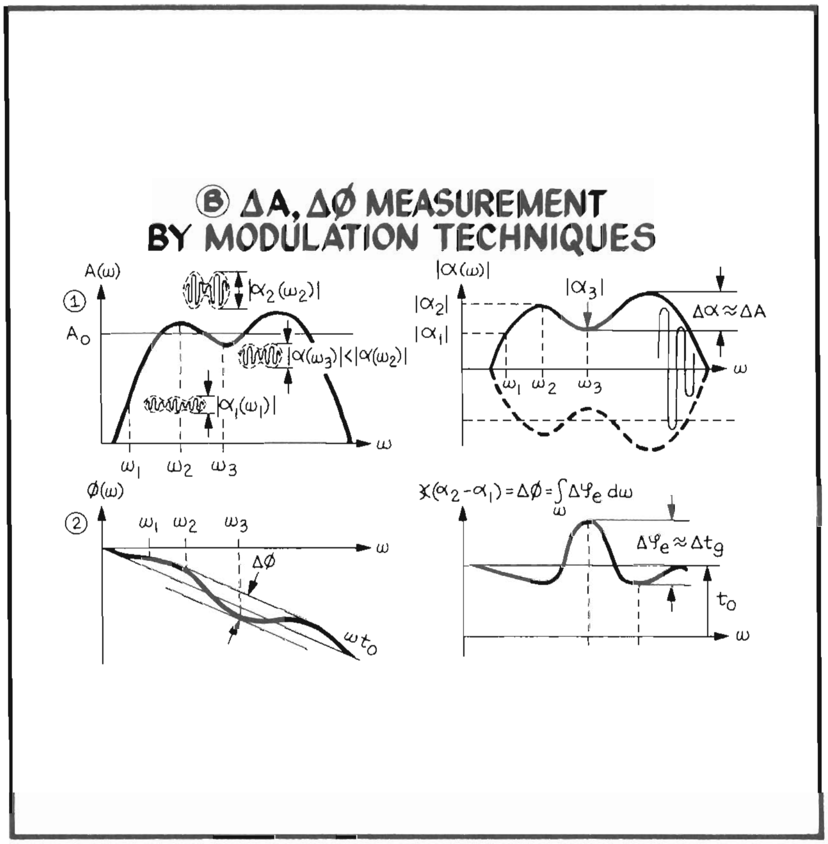
Let's first look at the deviation $\Delta A(\omega)$, $\Delta\emptyset(\omega)$ and $\Delta tg(\omega)$. These parameters can be measured with a standard Network Analyzer and are associated with the transfer function measurements. The deviations are superimposed on the ideal parts A_o, $\emptyset_o$ and can be separated by substituting for the ideal part or by eliminating the ideal part by a differentiation process.

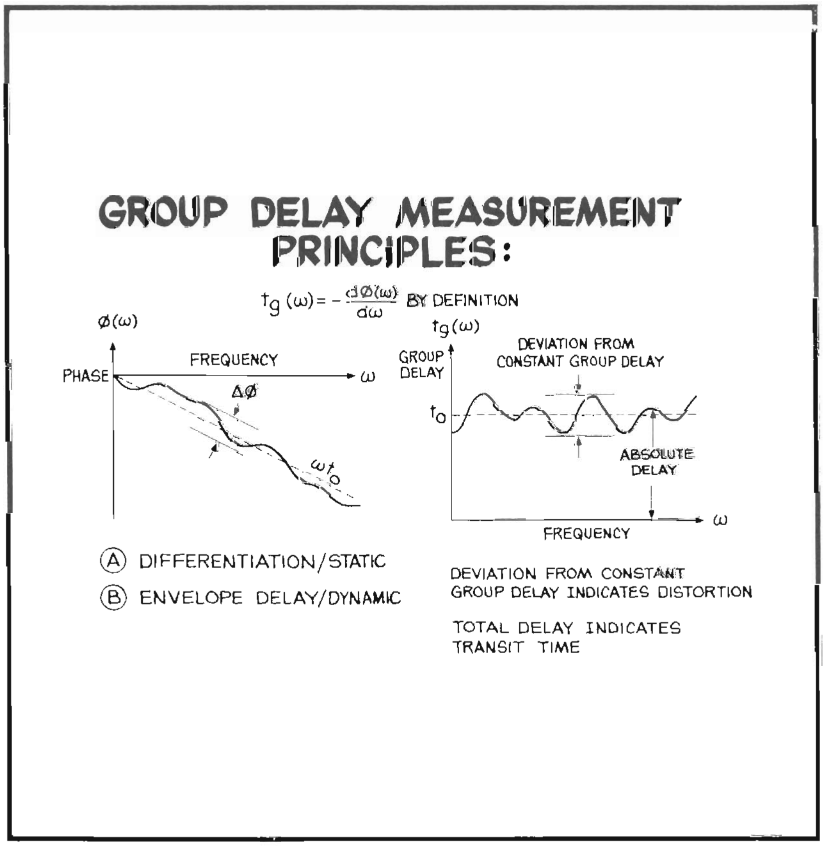
By subtracting out the ideal part of the transfer function, the non ideal part ΔA and $\Delta\phi$ can be made visible. In the case of the amplitude response, the ideal part is a constant (A_0) and can readily be subtracted out, thus showing the deviation $\Delta A(\omega)$. In case of the phase response, the ideal part is a linear function of frequency ($t_0\omega$), which has to be generated somehow and then subtracted out to show the deviation $\Delta\phi(\omega)$.



A practical measurement taken on a band-pass filter shows on the left the amplitude (top) and phase characteristic (bottom). By subtracting out the ideal part in both characteristics, the distortion causing deviation from constant amplitude and linear phase can be made visible: $\Delta A(\omega)$ (top) and $\Delta\phi(\omega)$ (bottom).







LINEARITY

In contrast to the static measurement approach, where a slowly swept sinusoidal signal is applied to the network in order to extract the transfer function, it is quite possible to modulate that very test signal and observe the change of the modulation envelope. It can be shown very readily that the magnitude of the envelope $|\alpha(t)|$ will also be scaled by the magnitude of the transfer function $|H(j\omega)| = A(\omega)$ whereas the phase shift of the envelope $\measuredangle(\alpha(t))$ has an integral relationship with the phase characteristic $\measuredangle H(j\omega) = \phi(\omega)$.

Therefore, both $|\alpha(t)|$ as well as $\measuredangle\alpha(t)$ can be related to the deviations $\Delta A(\omega)$ and $\Delta\phi(\omega)$ in the following manner:

Since for an input signal: $f(t) = \alpha(t)\cos\omega t$ the output of a linear network becomes:

$$g(t) = \underbrace{A(\omega)\,\alpha\left(t - \frac{d\phi(\omega)}{d\omega}\right)}_{\text{envelope}} \underbrace{\cos(\omega t - \phi(\omega))}_{\text{carrier}}$$

The change in the envelope magnitude $\Delta|\alpha|$ becomes proportional to $\Delta A(\omega)$ and is a measure for the gain linearity of the network. The envelope phase shift ψe is not directly related to the deviation from linear phase $\Delta\phi(\omega)$. However, $\psi e = \frac{d\phi(\omega)}{d\omega}$ is a measure for phase linearity and the phase characteristic can be constructed by integration:

$$\int_\omega \psi e\, d\omega = \Delta\phi(\omega)$$

GROUP DELAY

Group delay is an equivalent way of measuring deviations from linear phase. It is defined as the derivative of the phase response with respect to frequency. Since the derivative is basically the instantaneous slope or rate of change of the phase response, a perfectly linear phase shift would have a constant slope and, therefore, a constant group delay.

Nevertheless in most cases, the envelope phase shift or envelope delay is used directly as an indicator in lieu of the deviation from linear phase when a modulated test signal is applied. The envelope delay te is a function of the first derivative of the phase characteristic, which in turn is defined as group delay tg.

$$te(\omega) = -\frac{d\phi(\omega)}{d\omega} = tg(\omega) = \frac{\Delta\psi e}{\omega_m}$$

Therefore, the terms group delay tg and envelope delay are interchangeable as long as the phase characteristic curvature $\frac{d\phi(\omega)}{d\omega}$ stays constant over the spectrum of the modulated signal: $2\omega_m$.

Several measurement principles have evolved for group delay and we will look at the two most commonly used ones next: Following the definitions of tg, it is understandable that it can be derived from the phase characteristic by differentiation with respect to frequency.

Another approach is, of course, the one based on the envelope delay of a modulated signal.

GROUP DELAY MEASURING METHODS

STATIC MEASUREMENT

t_g IS DERIVED FROM AN ACCURATE MEASUREMENT OF THE PHASE CHARACTERISTIC BY APPROXIMATING THE LIMIT :

$$t_g(\omega) = \frac{d\phi(\omega)}{d\omega} = \lim_{\Delta\omega \to 0} \frac{\phi(\omega+\Delta\omega)-\phi(\omega)}{\Delta\omega}$$

PROBLEMS

- DIFFERENCE BETWEEN TWO LARGE NUMBERS
- CALIBRATION

DYNAMIC MEASUREMENT

t_g IS MEASURED AS PHASE-SHIFT OF THE ENVELOPE OF A MODULATED SIGNAL (ENVELOPE DELAY):

$$t_e(\omega) = \frac{\Delta\phi_e}{\omega_m}$$

$$t_e = t_g \text{ FOR } \omega_m \to 0$$

PROBLEMS

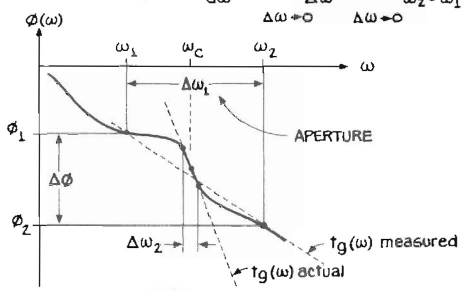
- MODULATED SIGNAL: APERTURE, ENVELOPE DISTORTION

The first approach, the differentiation is also referred to as a static measurement, because it is based on a sequence of steady state phase measurements, which are then numerically (or in an analog fashion) processed to approximate the definition in the limit. This group delay measurement principle points out a basic limitation (of all the known techniques) which is the cause for most measurement errors: Aperture trade-offs: For a finite Ø-resolution, $\Delta\omega$ cannot be arbitrarily small otherwise $\Delta\emptyset$ goes to zero because of the truncation effect when we subtract large numbers. Increasing $\Delta\omega$ leads to an average $\Delta\emptyset$ which is not the true slope anymore.

GROUP DELAY MEASUREMENT PRINCIPLES

Ⓐ STATIC METHOD:

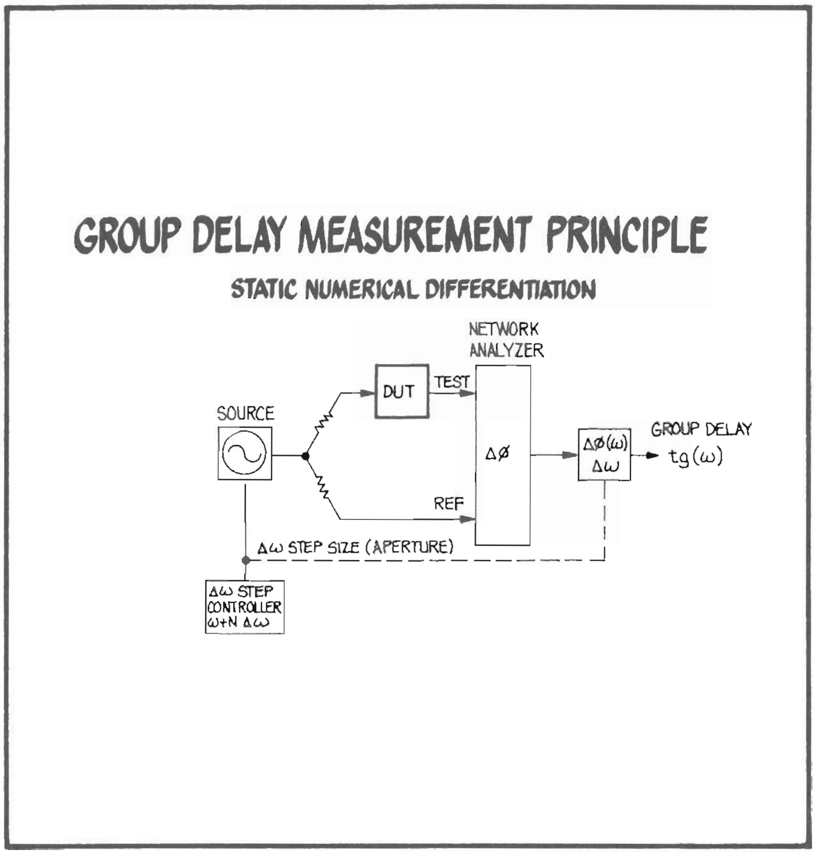
NUMERICAL DIFFERENTIATION $t_g(\omega) = -\frac{d\phi(\omega)}{d\omega} = -\lim_{\Delta\omega \to 0} \frac{\Delta\phi}{\Delta\omega} = -\lim_{\Delta\omega \to 0} \frac{\phi_2-\phi_1}{\omega_2-\omega_1}$



$$\frac{d\phi(\omega_c)}{d\omega} \neq \frac{\Delta\phi(\omega_c)}{\Delta\omega} = \text{FUNCTION OF } \Delta\omega, \phi(\omega)$$

TRADE OFFS:

- For a finite resolution in ϕ, $\Delta\omega$ cannot be arbitraraly small, otherwise $\Delta\phi$ goes to zero.
- Increasing $\Delta\omega$ leads to an average value $\Delta\phi$ over the interval $\Delta\omega$ (aperture) rather than the true value at ω_c.

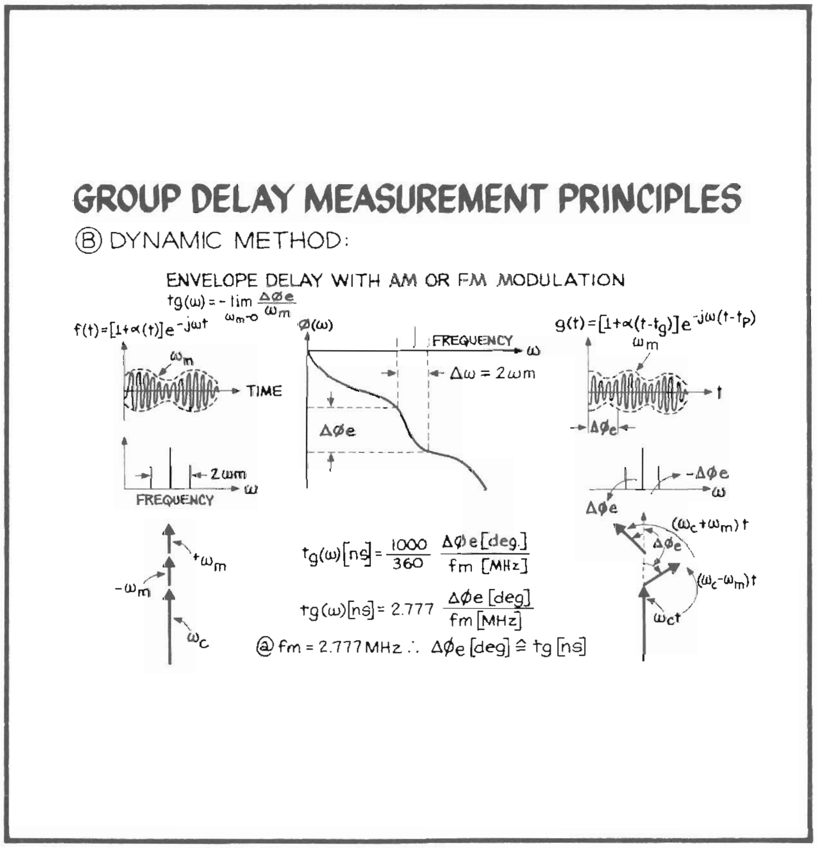


Of the several group delay measurement techniques, the numeric one is probably the oldest and most straight forward. It is a static or CW technique that involves measuring the phase at two closely spaced frequencies and then computing the slope (approximation of the derivative). Since the measurements are made at CW frequencies and computation of the slope is required, this technique usually does not produce real time results but is used extensively in automatic systems. Depending on the source, the user has almost complete control over the frequency step $\Delta\omega$ (aperture).

Phase Slope Technique

HP Models 8542B
8409A
3042A
8507B OPT 005

- o Computation Required
- o Not Dynamic
- o Variable Aperture



The scaling relationship between group delay and an envelope phase shift measurement allows a relatively simple measurement setup as shown.

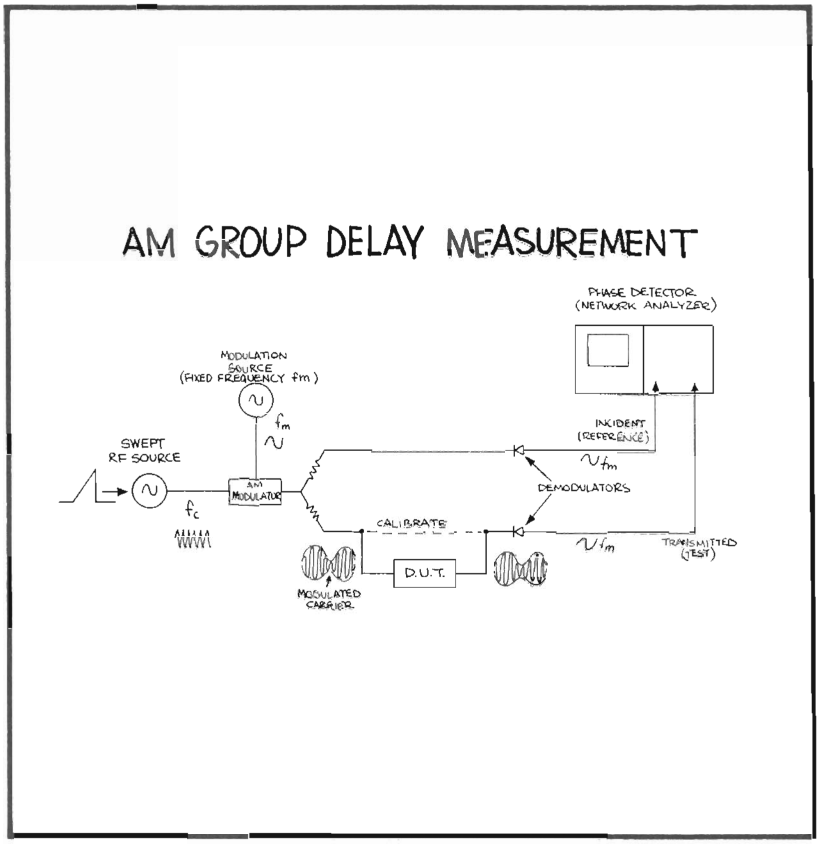
The aperture problem is also present when using an envelope delay method, because of the spectral width of the test signal. The phase resolution limitation criteria then apply to the envelope phase shift $\Delta\phi e$. Increasing the modulation frequency (aperture) increases the delay resolution for a given phase resolution but also potentially averages the measurement.

In the AM group delay measurement, the source is AM modulated and the envelope phase shift (group delay) caused by the test device is measured with a Network Analyzer. The aperture is determined by the modulation frequency and must be within the operating range of the network analyzer. This allows the user to make the tradeoffs between aperture, resolution, and measurement range. Measurements through frequency translators (mixers) are possible, however, limiting type test devices can remove the AM modulation. Typical network analyzers used are the HP 8407A and HP 8405A.

AM Technique

HP Model 8405A
8407A

- Range of Selectable Aperture
- Measurement through Frequency Translators
- Limiting Problem
- Complicated Set-Up
- Dynamic (AM to PM) (explained later)



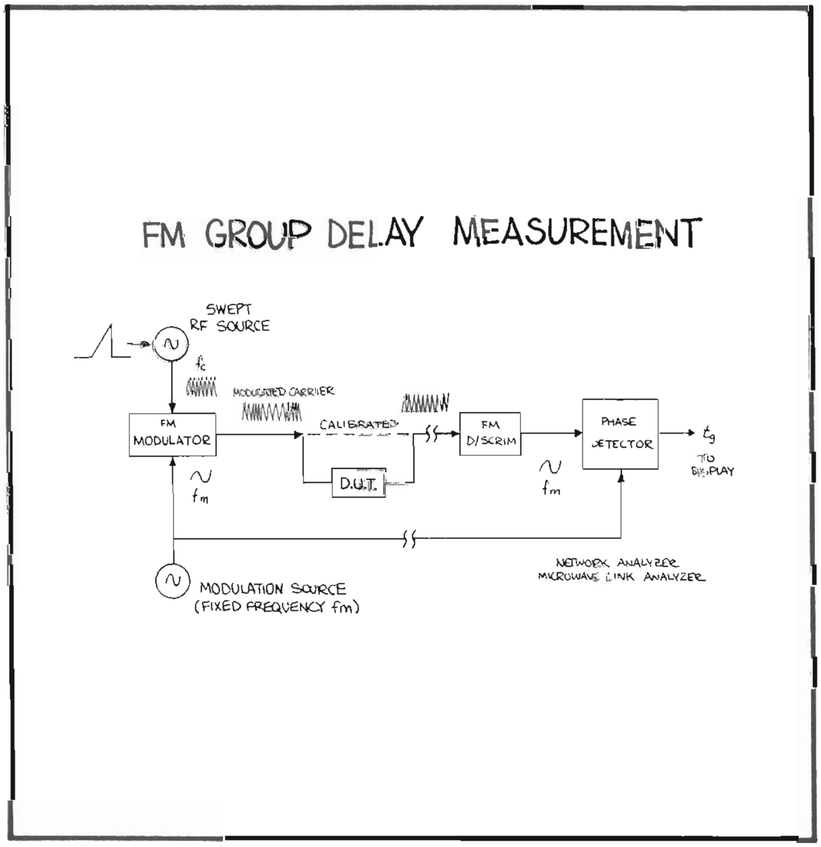
Angular modulation (FM or PM) lends itself also for dynamic group delay measurements and so do other than sinusoidaly modulated signals as can be readily verified.

FM delay measurements are made in basically the same fashion as AM delay measurements. Frequency translators such as mixers and doublers can be measured as well as limiting test devices (FM modulation is not affected by limiting). This technique is employed with the HP Microwave Link Analyzer and is often used to make measurement of links where the source is remote from the receiver. It is also used extensively in the testing of systems composed of frequency translators and limiting power amplifiers.

FM Technique

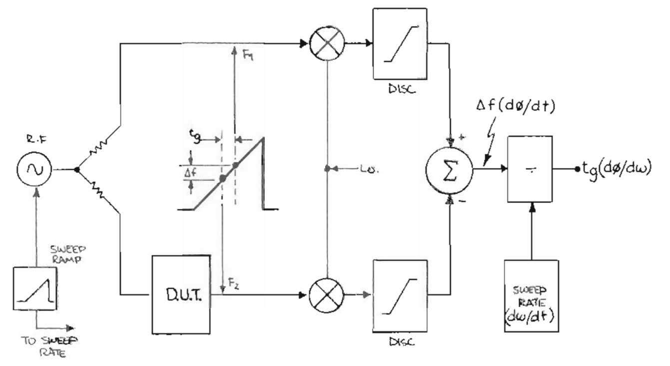
HP Models 3702B/3703B or 3705A
Microwave Link Analyzer

- Selection of Several Apertures (Resolution vs Range and Noise)
- Measurements through Frequency Translators
- Source Remote From Receiver
- No Limiting Problem
- Limited Sweep Widths
- Dymanic (AM to PM) (explained later)



LINEAR FM DELAY MEASUREMENT

8505A NETWORK ANALYZER



A linear FM technique, for example, is used in a HP instrument (8505A). It is based on recovering and evaluating the envelope phase shift of a linearly swept signal.

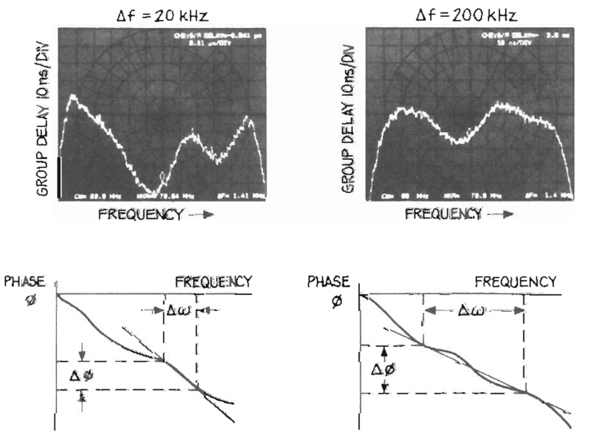
If the source is swept in a linear fashion, the delay throught the test device will cause a difference in the transmitted and incident channel frequencies. The magnitude of this frequency difference is determined both by the device's delay and the source's sweep downconverted to the IF, discriminated, and the effects of the source sweep rate removed, yielding the device's group delay. Both absolute delay (up to 80 μsec) and deviation from constant group delay can be measured. Fully calibrated, real time CRT displays are used to display the results. (11)

Linear FM Technique

HP Models 8505A
8507

- Real Time Display
- Directly Calibrated Display
- Total Delay and Delay Variation
- Measure Total Delays up to 80 μs
- No Limiting Problems
- Limited Choice of Apertures

EFFECTS OF INCREASING APERTURE



MEASUREMENT LIMITATIONS

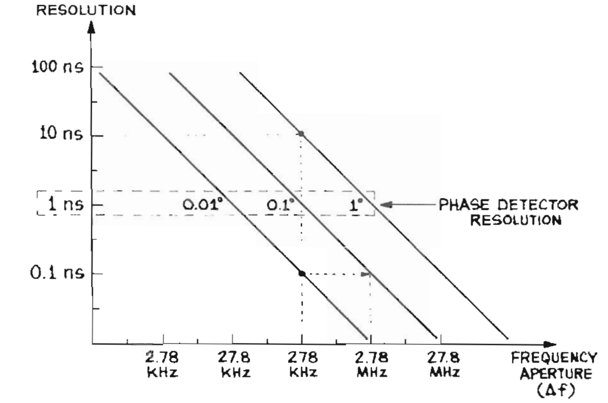
The effect of increasing aperture is shown on the two measurements in the picture. Changing the aperture, $\Delta\omega$, can cause different values of group delay to be measured as indicated by these CRT photos. Narrowing the $\Delta\omega$ will increase the frequency resolution. However, eventually phase differences can no longer be resolved and only noise will result.

Increasing the $\Delta\omega$ permits better group delay resolution for a given phase resolution. However, the measurement accuracy has been reduced as shown by the diagram which has averaged out the fine grain variations. Tradeoffs between aperture and delay resolution are fundamental to any group delay measurement.

The resolution is mainly given by the phase detector resolution and the maximum allowable frequency aperture (ωm). The latter is a function of the device frequency response.

GROUP DELAY RESOLUTION

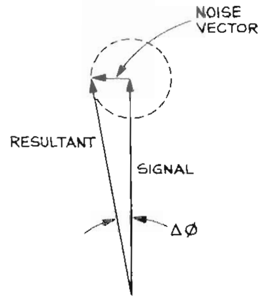
AS A FUNCTION OF APERTURE AND PHASE RESOLUTION



The signal to noise ratio will also introduce a measurement ambiguity or limitation by introducing a phase ambiguity. Residual FM, noise figure of device, differentiation noise, etc.

NOISE LIMITATIONS

GROUP DELAY MEASUREMENTS



SIGNAL/NOISE (FOR A GIVEN BANDWIDTH)	ΔØ	Δt_g*
80 dB	±0.006°	±0.06 ns
60 dB	±0.06°	±0.6 ns
40 dB	±0.6°	±6.0 ns
20 dB	±6.0°	±60.0 ns

*FOR Δf = 278 KHz

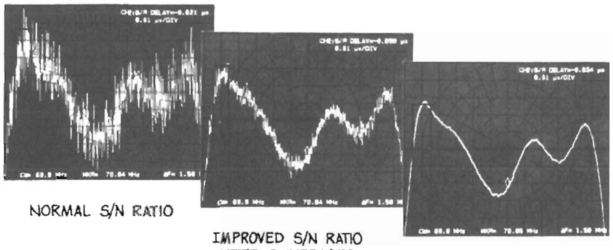
WHERE $t_g = \frac{-1}{360°} \cdot \frac{\Delta Ø}{\Delta f}$

NOTE: AS Δf → 0, THE SIGNAL/NOISE → 0 dB

CAUSES OF NOISE

- SOURCE RESIDUAL FM
- D.U.T NOISE FIGURE
- PHASE DETECTOR NOISE

NOISE REDUCTION USING SIGNAL AVERAGING



NORMAL S/N RATIO

IMPROVED S/N RATIO AFTER 10 AVERAGES

S/N 20dB AFTER 100 AVERAGES

One way of reducing the noise in group delay measurements, is to use digital signal averageing. The exponentially weighted running average results in a reduction of noise. (HP 8501A Storage-Normalizer)

SUMMARY: MEASUREMENT LIMITATIONS IN GROUP DELAY MEASUREMENTS

APERTURE: MEASUREMENT APERTURE DUE TO $\Delta\omega$

NOISE: S/N RATIO IN SOURCE, RECEIVER, DEVICE, ETC.

HARDWARE LIMITATIONS: DETECTOR RESOLUTION, SOURCE RESOLUTION, ETC.

PRINCIPLE RELATED LIMITATIONS: DIFFERENTIATION, NON LINEAR DISTORTION

HOW DOES Δt_g COMPARE TO $\Delta\phi$

$\Delta\phi$: WHEN EVER POSSIBLE SINCE IT HAS MUCH LESS AMBIGUETY.

Δt_g: S/N CAN BE IMPROVED BY AVERAGING.

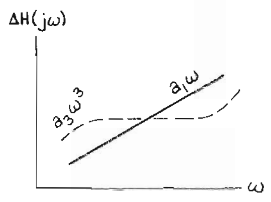
As mentioned previously, there are some measurement limitations associated with group delay measurements which we have to be aware of to best utilize the equipment: Resolution due to aperture and noise.

AM TO PM CONVERSION

In order to maintain signal fidelity, it is important to characterize the phase relationship of the various signal components through the networks. In non-ideal systems, the signal level as well as the frequency of the signal components will affect the phase relationship of the signal. As discussed in the previous paragraph, envelope phase shift or group delay variations indicate a phase change between the various frequency components. The level depending phase variations, however, are commonly addressed as AM to PM conversion. Linear as well as non-linear networks are able to generate AM/PM conversion.

AM TO PM CONVERSION

LINEAR NETWORK



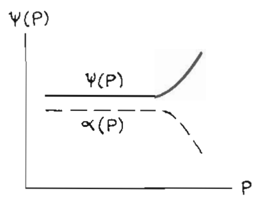
AMPLITUDE: $A(\omega)$

PHASE: $\phi(\omega)$

GROUP DELAY: $t_g(\omega) = \frac{-d\phi(\omega)}{d\omega}$

AM/PM: $M \propto \psi = \Delta\psi(\alpha_i)$

NON-LINEAR NETWORK



GAIN COMPRESSION: $\alpha(P)$

AM/PM CONVERSION: $\Delta V(P)$

INTERMODULATION

HARMONIC DISTORTION

As shown earlier, AM/PM conversion can be generated by a network which exhibits an amplitude deviation $\Delta A(\omega)$ of odd order amplitude coefficients or even order phase coefficients or both.

DISTORTION ON AM-SIGNALS

$$f(t) = R_e[1+d(t)]e^{-j\omega_c t} \rightarrow \boxed{\Delta H} \rightarrow g(t) = R_e\sqrt{[1+\alpha(t)+P(t)]^2 + Q(t)^2} \cdot e^{-j\left[\omega_c t + \mathrm{arctg}\left[\frac{Q(t)}{1+\alpha(t)+P(t)}\right]\right]}$$

$$\Delta A(\omega) = a_1\omega + a_2\omega^2 + a_3\omega^3$$

$$\Delta\phi(\omega) = b_2\omega^2 + b_3\omega^3$$

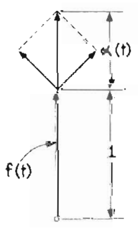
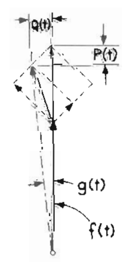


TABLE 1

α(t) PURE AM Ψ = CONST.	P_α	Q_α
a_1		$a_1\dot{\alpha}$
a_2	$-a_2\ddot{\alpha}$	
a_3		$a_3\dddot{\alpha}$
b_2		$-b_2\ddot{\alpha}$
b_3	$b_3\dddot{\alpha}$	
$a_1 b_2$	$a_1 b_2\ddddot{\alpha}$	



EXAMPLE:

$\alpha(t) = m\cos\omega_m t$

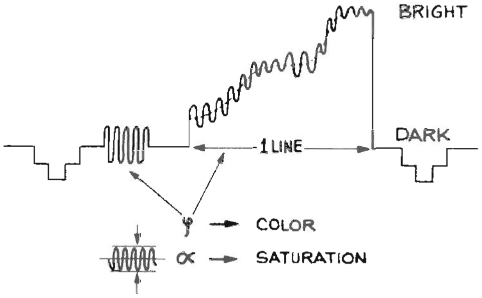
$\dot{\alpha} = -m\cdot\omega_m \sin\omega_m t$ Quadrature Phase

$\ddot{\alpha} = -m\,\omega_m^2\cos\omega_m t$ In Phase

$\dddot{\alpha} = m\omega_m^3\sin\omega_m t$ Quadrature Phase

EXAMPLE FOR AM TO PM CONVERSION

COLOR TV SIGNAL:



PROBLEM: COLOR CHANGE AS A FUNCTION OF BRIGHTNESS

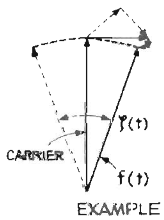
For an AM signal which is demodulated by a phase insensitive demodulator, AM to PM conversion is not a problem. However, for mixed (or hybrid) modulated signals AM to PM does affect the PM component (color TV).

DISTORTION ON ANGULAR MODULATION

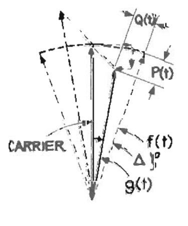
$$f(t) = R_e\left[e^{-j[\omega_c t + \varphi(t)]}\right] \circ\!\!-\!\!\boxed{\Delta H}\!\!-\!\!\circ\; g(t) = R_e\sqrt{[1+P(t)]^2 + Q(t)^2}\cdot e^{-j\left[\omega_c t + \varphi(t) + \operatorname{arctg}\left[\frac{Q}{1+P}\right]\right]}$$

$$\Delta A(\omega) = a_1\omega + a_2\omega^2 + a_3\omega^3$$

$$\Delta\emptyset(\omega) = b_2\omega^2 + b_3\omega^3$$



$\varphi(t)$ PURE φM α= CONST. = 1	P_φ	Q_φ
a_1	$a_1\dot{\varphi}$	
a_2	$a_2\dot{\varphi}^2$	$-a_2\ddot{\varphi}$
a_3	$a_3(\dot{\varphi}^3-\ddot{\varphi})$	$-3a_3\dot{\varphi}\ddot{\varphi}$
b_2	$b_2\ddot{\varphi}$	$b_2\dot{\varphi}^2$
b_3	$3b_3\dot{\varphi}\ddot{\varphi}$	$b_3(\dot{\varphi}^3-\dddot{\varphi})$
$a_1 b_2$	$3a_1b_2\dot{\varphi}\ddot{\varphi}$	$a_1b_2(\dot{\varphi}^3-\ddot{\varphi})$



EXAMPLE:

$$\varphi(t) = m\cos\omega_m t$$

$$\dot{\varphi} = -m\omega_m \sin\omega_m t;\quad \dot{\varphi}^2 = \frac{m^2\omega_m^2}{2}\left[-\cos 2\omega_m t + 1\right];\quad \dot{\varphi}^3 = \frac{m^3\omega_m^3}{4}\left[\sin 3\omega_m t - 3\sin\omega_m t\right]$$

$$\ddot{\varphi} = -m\omega_m^2\cos\omega_m t$$

$$\dddot{\varphi} = m\omega_m^3\sin\omega_m t$$

Angular modulation tends to be converted into amplitude modulation at just about any deviation ΔA and/or $\Delta\emptyset$. Contrary to popular belief, the AM component cannot be easily removed (limiting, etc.) since exactly these functions tend to show AM to PM conversion which affects the original phase information by superimposing a component of the same frequency with different phase relationship and therefore, changes or distorts the signal.

AM TO PM MEASUREMENT PRINCIPLES FOR:

LINEAR (NON-IDEAL) NETWORKS

(AM/PM CONVERSION AS A FUNCTION OF FREQUENCY)

A) STATIC VS FREQUENCY:
THE PRESENCE OF A_i, B_i COMPONENTS INDICATES AM/PM

B) DYNAMIC VS FREQUENCY:
AM/PM CONVERSION WILL BE INDICATED BY THE PHASE SHIFT OF THE MODULATION ENVELOPE (tg)

NON LINEAR NETWORKS

(AM/PM AS A FUNCTION OF SIGNAL LEVEL)

A) STATIC VS LEVEL:
CARRIER PHASE SHIFT VS SIGNAL LEVEL INDICATES AM/PM

B) DYNAMIC VS LEVEL:
ENVELOPE PHASE SHIFT VS SIGNAL LEVEL INDICATES AM/PM

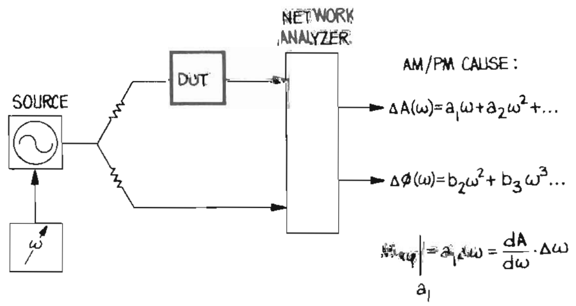
AM/PM MEASUREMENT FOR LINEAR NETWORKS

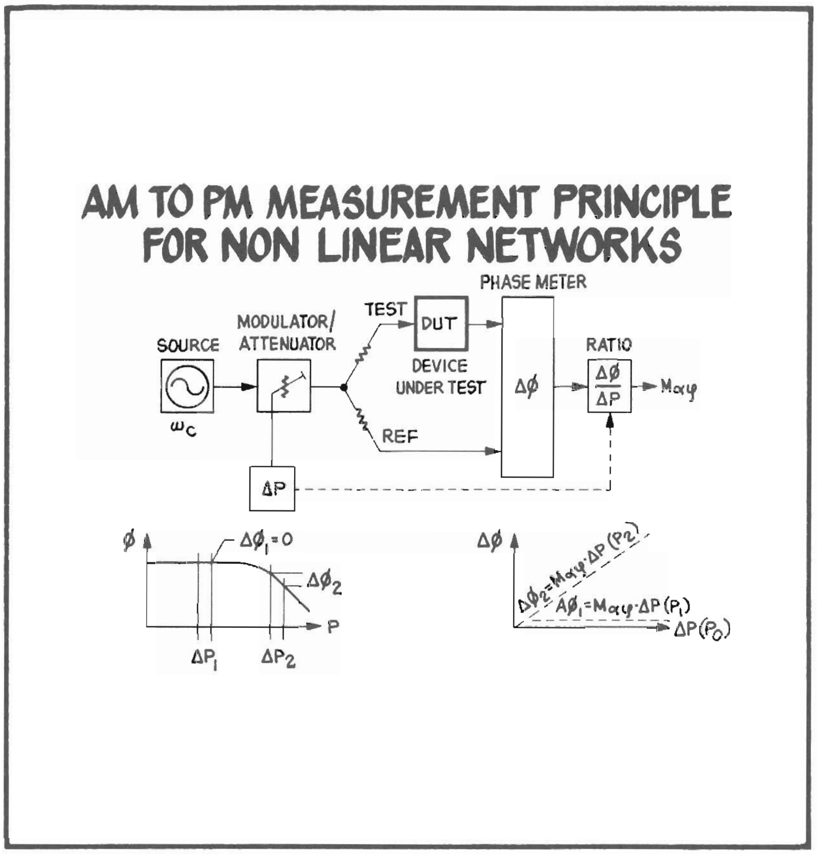
This measurement system (basic Network Analyzer) allows us to measure AM/PM conversion (as a function of frequency) by determining the amplitude and phase coefficients causing it.

However, this is not the only way AM to PM conversion takes place. Another possible cause is a non-linear transfer characteristics, where the gain of a network changes as a function of signal level (e.g. Compression). In this case, a signal of constant frequency and variable power level can be applied as a test signal.

AM TO PM MEASUREMENT PRINCIPLE

FOR LINEAR NETWORKS



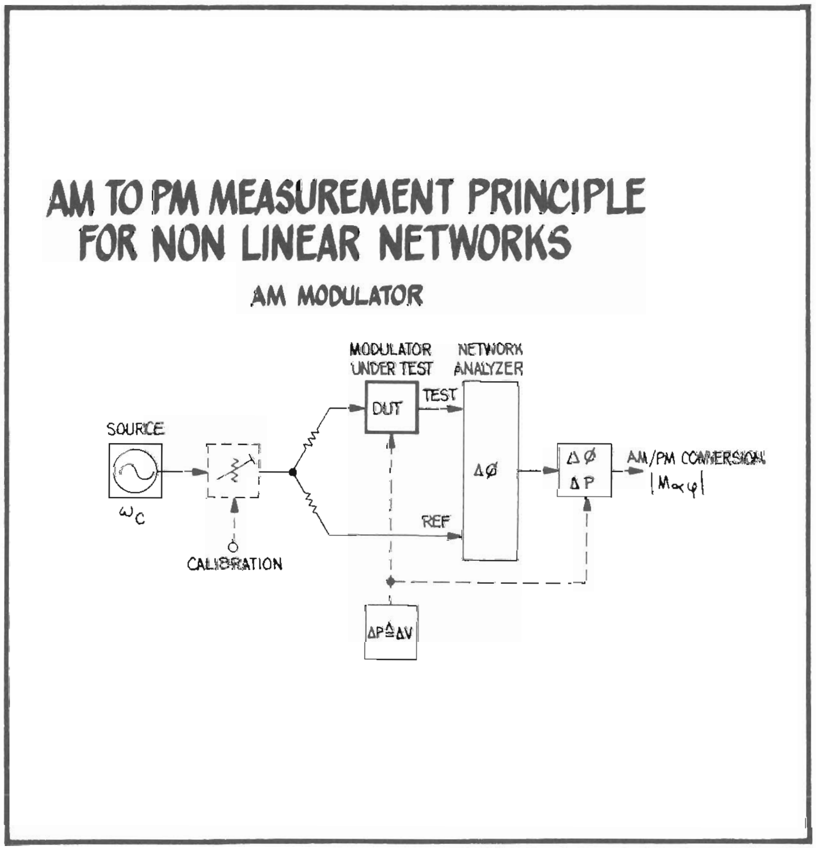


AM/PM MEASUREMENTS FOR NON-LINEAR NETWORKS

This form of AM/PM conversion is conceptually the easiest one to assess from a measurement point of view, therefore, we will start with a measurement principle for it. Assuming the DUT is an amplifier, it is reasonable to assume that the phase shift (Ø) through it would basically remain constant over most of its dynamic range.

For higher signal levels, the amplifier would probably start to compress, therefore, generate harmonic distortion and consequently, change the phase of the original signal.

An amplitude change ΔP would, therefore, create a phase change ΔØ thus AM/PM conversion in the region of P2. The ratio Δφ/ΔP is called AM/PM coefficient and the dimension is usually rad/dB or degr/dB for a given ΔP(1dB) and at a certain power level P.

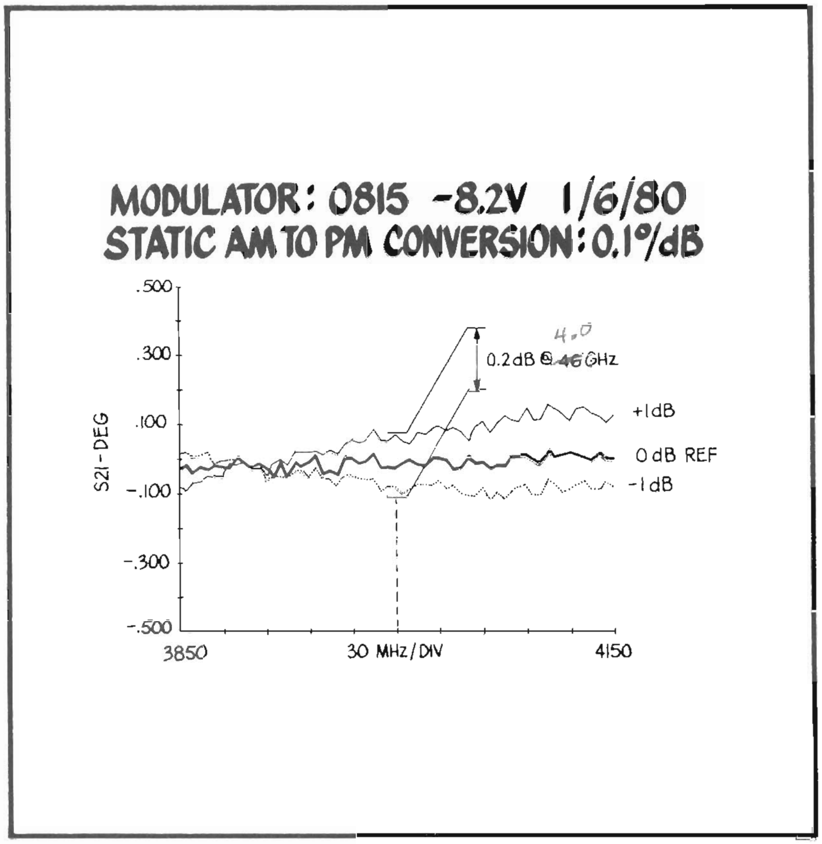


Sounds simple, but a modulator is probably the main contributor to AM/PM conversion in any system. Therefore, it is important to characterize and specify AM/PM conversion on modualtors either as part of a system or stand-alone components.

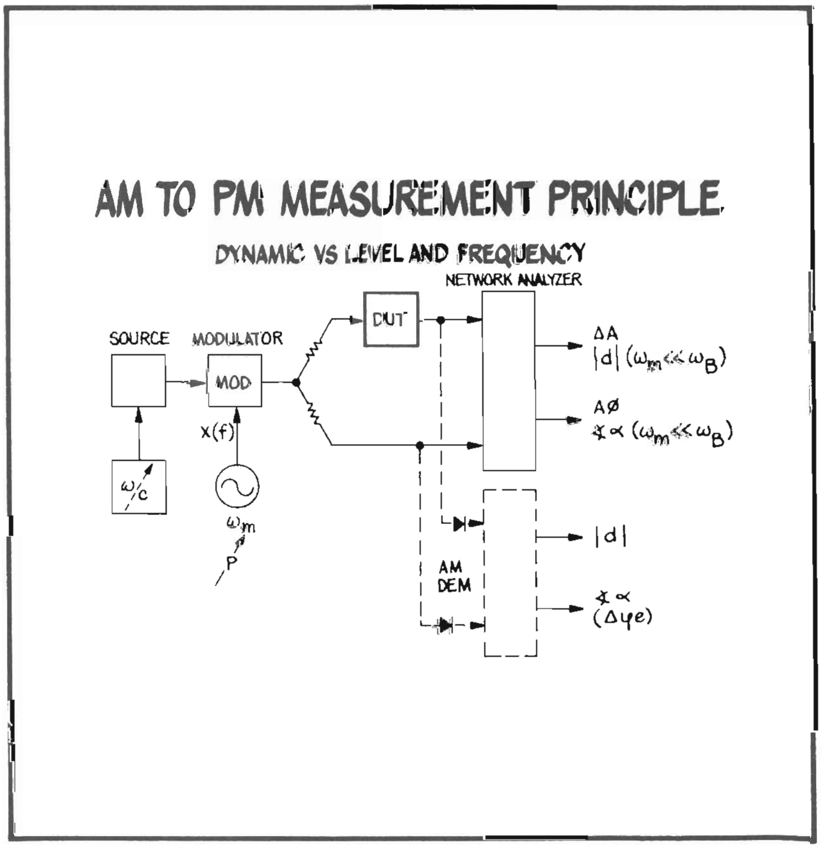
The easiest way to characterize modulators is to pair them up with an ideal demodulator and then compare magnitude and phase relationship over the operating parameters of the modulator.

This static measurement concept can be expanded to a dynamic one by amplitude modulating the test signal by @1dB. When swept vs. power, the carrier phase shift would indicate the AM/PM conversion and when swept vs frequency, the envelope phase shift would indicate the group delay. This block scheme seems probably contrieved, but it is a standard instrument (8505) expanded for an AM/PM measurement by adding an AM - modulator either before or after the power splitter.

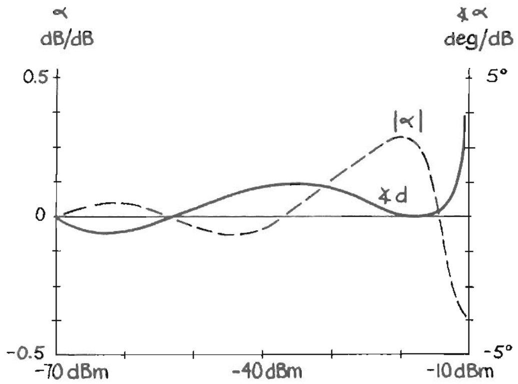
Here is a simple example of a static AM/PM conversion on an amplitude modulator for leveling purposes. Power level is parameter.



This measurement set-up allows us to measure AM/PM conversion on an actual AM signal. The demodulated AM envelope is phase compared with the reference in magnitude and phase. The phase shift with respect to the depth of modulation (e.g. 1dB) give the dynamic AM/PM coefficient $M_{\alpha\psi}$. (The actual phase shift change is the same parameter which would be interpreted as envelope or group delay for a linear network.) The results of such a measurement on a power amplifier are shown in the next slide.

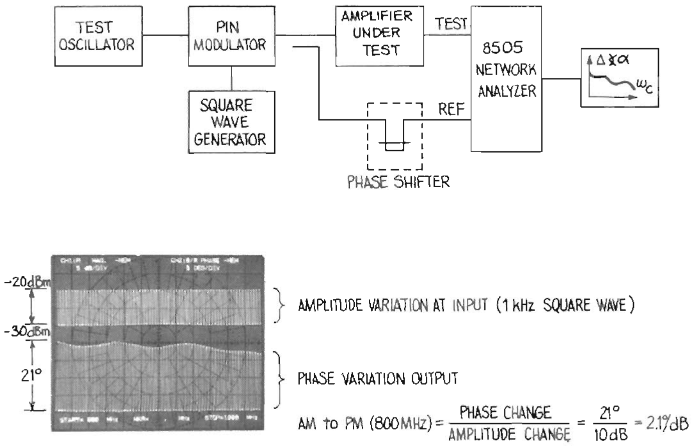


DYNAMIC AM TO PM CONVERSION VS SIGNAL LEVEL AMPLIFIER



The amount of AM/PM of an amplifier is dependent on the power of the test signal so the absolute levels are critical. Thus, there may not be much of a difference in phase change between a 5 dB or a 20 dB amplitude variation. The placement of the amplitude difference to include a rapidly changing portion of the gain compression curve of the device under test can be crucial.

AM TO PM CONVERSION MEASUREMENT



$$\text{AM to PM (800 MHz)} = \frac{\text{PHASE CHANGE}}{\text{AMPLITUDE CHANGE}} = \frac{21^\circ}{10\,\text{dB}} = 2.1^\circ/\text{dB}$$

This slide shows the test set-up and a picture of the results.

SUMMARY AND CONCLUSIONS

GROUP DELAY
SIGNAL DELAY, DISTORTION

DISTORTION ON MODULATED SIGNALS
HARMONIC DISTORTION, INTERMODULATION, MODULATION CONVERSION

MEASUREMENT PRINCIPLES
ΔA, $\Delta \phi$, Δt_g VS FREQUENCY

AM TO PM CONVERSION
CAUSES: LINEAR AND NON-LINEAR NETWORKS

MEASUREMENT PRINCIPLES
AM/PM CONVERSION VS FREQUENCY AND SIGNAL LEVEL, STATIC AND DYNAMIC

EXAMPLE: 1

$f(t) = Re[\alpha(t) \cdot e^{-j w_c t}]$

$f(t) = (1 + m\cos w_m t)\cos w_c t$

THEN:

$g(t) = \mathfrak{F}^{-1} F(jw) \cdot H(jw)$

BY COMPONENT (STEADY STATE):

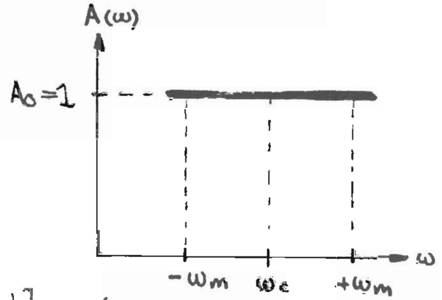
$g(t) = \cos[w_c t + \phi(w_c)]$

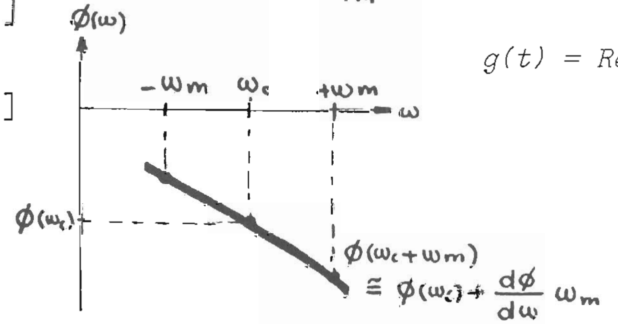
$g(t)_{-1} = \cos[(w_c - w_m)t + \phi(w_c - w_m)]$

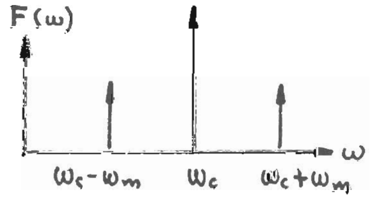
$g(t)_{+1} = \cos[(w_c + w_m)t + \phi(w_c + w_m)]$

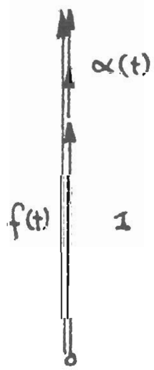
SUBSTITUTE FOR:

$$\phi(w_c - w_m) \approx \frac{d\phi}{dw}(w_c) \cdot w_m = \phi'(w_c) \cdot w_m$$









SUBSTITUTE FOR:

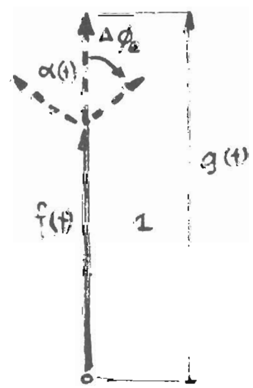
$$-\frac{\phi(w_c)}{w_c} = t_p(w_c)$$

$$-\frac{d\phi(w_c)}{dw} = t_g(w_c)$$

BRING $g(t)$ IN THE SAME FORM AS $f(t)$:

$g(t) = A(w_c)[1 + m\cos w_m(t - t_g)\cos w_c(t - t_p)]$

$g(t) = Re\, A(w_c)\, \alpha(t - t_g) e^{-j w_c (t - t_p)}$



EXAMPLE:

$$f(t)=Re[\alpha(t) \cdot e^{-jw_c t}]$$

$$f(t)=(1+m\cos w_m t) \cos w_c t$$

THEN:

$$g(t)= \mathfrak{F}^{-1} F(jw)\cdot H(jw)$$

BY COMPONENT (STEADY STATE):

$$g(t)= A(w_c) \cos [w_c t + \phi(w_c)]$$

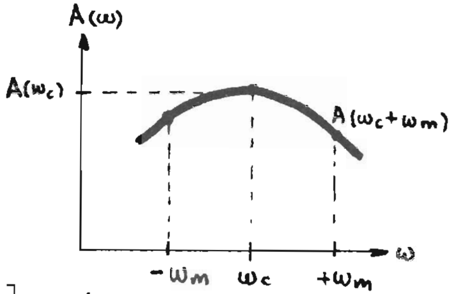
$$g_{-1}(t)= A(w_c-w_m) \cos[(w_c-w_m)t+\phi(w_c-w_m)]$$

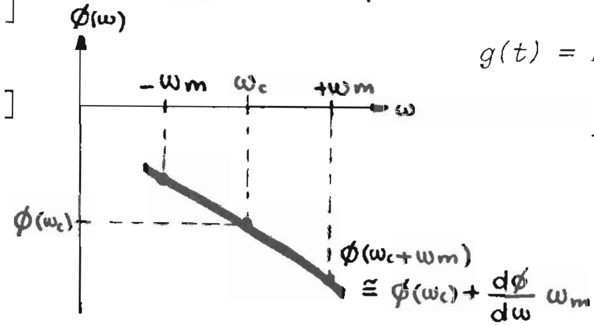
$$g_{+1}(t)= A(w_c-w_m) \cos[(w_c+w_m)t+\phi(w_c+w_m)]$$

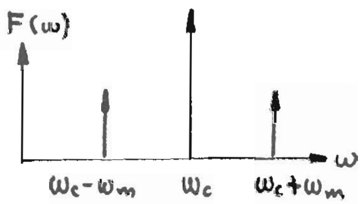
SUBSTITUTE FOR:

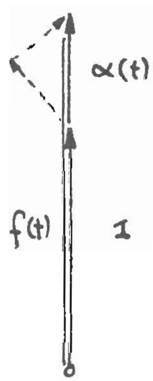
$$A(w_c-w_m) \simeq \frac{dA}{dw}(w_c)\cdot w_m = A'(w_c)\cdot w_m$$

$$\phi(w_c-w_m) \simeq \frac{d\phi}{dw}(w_c)\cdot w_m = \phi'(w_c)\cdot w_m$$









SUBSTITUTE FOR:

$$-\frac{\phi(w_c)}{w_c} = t_p(w_c)$$

$$-\frac{d\phi(w_c)}{dw} = t_g(w_c)$$

BRING $g(t)$ IN THE SAME FORM AS $f(t)$:

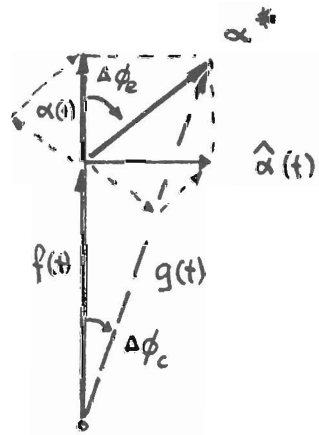
$$g(t) = A(w_c)[1+m\cos w_m(t-t_g)\cos w_c(t-t_p)]$$

$$+A'(w_c).w_m\,[\sin w_m(t-t_g)\sin w_c(t-t_p)]$$

+. . .HIGHER ORDER TERMS

$$g(t) = Re[A(w_c)\,\alpha(t-t_g)e^{-jw_c(t-tp)}$$

$$+A'(w_c)w_m\,\hat{\alpha}(t-t_g)e^{-jw_c(t-tp)}\cdot e^{-j\frac{\pi}{2}}$$



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